LASER COOLING OF ELECTRON BUNCHES IN COMPTON STORAGE RINGS

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Abstract

Self–consistent dynamics of a bunch circulating in the Compton storage ring has been studied analytically. Disturbances from both the synchrotron and Compton radiations were taken into account. The emittances in laser–dominated rings (where the synchrotron energy losses are much smaller then the Compton ones) were evaluated. The resultant emittances (synchrotrons plus Comptons) were compared with the synchrotrons. As were shown, the longitudinal degree of freedom is heated up due to Compton scattering. Almost the same conclusion is valid for the vertical uncoupled betatron emittance. Since it is impossible in principle to get zero dispersion in the banding magnets, the radial emittance almost always cooling down by laser. Therefore in practical cases of coupled transverse oscillations with the horizontal emittance determining the vertical one, the laser will cool down the transverse degrees of freedom.

INTRODUCTION

Efficiency of Compton storage rings — number of emitted X–ray quanta per circulating electron per second — exhibit strong dependence upon the electron bunch dimensions: smaller the bunch sizes larger the yield, main figure of merit.

In this report we are going to derive minimal beam sizes as functions on the ring and laser parameters. Another goal is to reveal the problem whether interactions of electrons with the laser splash lead to heating or cooling of the bunch.

We will make use the term *cooling* in the sense of *decrease of the emittances.* We will use emittance in the meaning of *RMS phase space emittance* $\epsilon_{\rm rms}$:

$$\epsilon_{\rm rms} \equiv \sqrt{\langle x^2\rangle \langle p_x^2\rangle - \langle x p_x\rangle^2}\ , \qquad (1)$$

where (x, p_x) are the conjugated coordinate and momentum.

STATIONARY EMITTANCES

In the first approximation, the particle motion in a bunch can be suggested as 3–dim oscillator with uncoupled degrees of freedom. The motion of oscillator without damping and excitation is governed by an equation:

$$\ddot{x} - U_x'(x) = 0\ , \qquad (2)$$

where $U(x)$ is the potential function (potential well).

The potential function of the harmonic oscillator (approximation for the transverse oscillations) is $U(x) = Q^2x^2/2$, for the mathematical pendulum (synchrotron oscillations) $U(x) = \cos Qx$. Further we will restrict our study to harmonic oscillators as small amplitude synchrotron oscillations reduce to harmonic ones.

Conservative Hamilton system (ensemble of oscillators) holds occupied by particles phase volume conservative. Thus, initial emittance preserved in time (saying nothing of its magnitude). To get the equilibrium specific emittance, the system should open to externals: become nonconservative.

Perturbations of the conservative system in general can be decomposed into (a) perturbation of the potential function, (b) excitation, and (c) damping. The equilibrium state of the system (distribution in phase space) is settled due to balance of excitation and damping.

Perturbed canonical equations have a form

$$\begin{aligned} \dot{x} &= p\ ; \\ \dot{p} &= -U_x'(x) + F(p,t)\ . \end{aligned} \qquad (3)$$

Here $F(p,t)$ is a random function describing perturbation from interactions of electrons with the laser splash.

The random function $F(p,t)$ contains a stochastic component $F(p,t) - \langle F(p,t)\rangle$ and a regular one $\langle F(p,t)\rangle$. By expanding $\langle F(p,t)\rangle$ into power series of p around $p = 0$, the equations (3) can be deduced to that corresponding to Kramers equation describing motion of the damped nonlinear oscillator excited by white noise:

$$\ddot{x} + \alpha\dot{x} + U_x'(x) = \sqrt{S}\xi(t)\ ,$$

where $\xi(t)$ is the unity white noise; α the friction factor.

The stationary density in the phase space can be written in the form:

$$\rho_{\rm st}(\epsilon) = N \exp\left(-\frac{\epsilon}{\epsilon_{\rm rms}}\right)\ ,$$

with $\epsilon_{\rm rms}$ being the root mean square emittance,

$$\epsilon_{\rm rms} = \frac{S}{2Q\alpha}\ .$$

Hence, the stationary emittance is proportional to excitation power and inversely — to the damping term.

In the electron storage rings, both the excitation and damping are caused by acting of short impulses (with typical duration less than 10^{-10} s. Therefore, as it follows from the Campbell theorem, the excitation and damping terms can read as:

$$\alpha = \sum_i \nu_i\alpha_i'\ ; \qquad S = \sum_i \nu_i S_i'\ ,$$

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where primed are statistical properties of single impulse of i species; ν_i the frequencies of impulses.

Two important consequences follow from it. First: if both the excitation and damping are caused by the same and only process then the partial emittance ε_i is independent on the frequency:

$$\epsilon_i = \frac{S_i}{2Q\alpha_i} = \frac{\nu_i S_i'}{2Q\nu_i\alpha_i'} = \frac{S_i'}{2Q\alpha_i'}$$

Second: the total emittance of a system undergo several processes, can be decomposed into a weighted sum of partial emittances:

$$\epsilon_{\rm rms} = \frac{\sum_i \nu_i S_i}{2Q\sum_i \nu_i\alpha_i} = \frac{\sum \epsilon_i\nu_i\alpha_i'}{\sum \nu_i\alpha_i'} . \qquad (4)$$

This expression is a basis in learning if the Compton interactions result in cooling or heating. Let us decompose resultant emittance (4) into two partial ones: the "synchrotron" which is the emittance of bunch with the laser switched off, and the "laser" partial emittance. Actually, the synchrotron emittance is settled due to balance of sum excitations and damping (the only source of damping with the laser off is the synchrotron radiation emission, that is why we refer to it as "synchrotron").

From (4), it follows that if the synchrotron emittance is smaller than the laser $\epsilon_{\rm las} > \epsilon_{\rm synch}$ then heating will take place with laser on, and on the contrary: if $\epsilon_{\rm las} < \epsilon_{\rm synch}$ then cooling.

PARTIAL LASER EMITTANCES

In our estimations we will apply the Thomson cross section for Compton scattering: in the Compton X–ray source there are good margins (by several orders of magnitude) both in energy of laser photons and their density within which application of the Thomson cross section is valid.

Indeed, Thomson cross section is valid for the laser wavelength in the electron rest frame exceeding the electron Compton wavelength. It is equivalent to the energy of electrons: $\gamma < E_0/4E_{\rm las}$ ($E_0 \equiv m_{\rm e}c^2 \approx 0.511\,$eV is the electron rest energy).

The limiting density of laser photons for the Thomson formula is equivalent to unity magnitude of the deflection factor (undulator parameter): the limiting density of photons results in scattering by the bunch electron of about one photon per laser wavelength.

Statistical parameters of the electron recoil due to a Compton interaction are:

$$\langle E_{\rm x}\rangle = 2C_\varphi\gamma^2 E_{\rm las} ; \qquad (5)$$

$$\langle E_{\rm x}^2\rangle = \frac{16}{3}C_\varphi^2\gamma^4 E_{\rm las}^2 ; \qquad (6)$$

$$\left\langle (E_{\rm x} - \langle E_{\rm x}\rangle)^2 \right\rangle = \frac{4}{3}C_\varphi^2\gamma^4 E_{\rm las}^2 ; \qquad (7)$$

$$\langle E_{\rm x\perp}^2\rangle = \frac{8}{3}C_\varphi^2\gamma^2 E_{\rm las}^2 . \qquad (8)$$

Here $C_\varphi \equiv (1+\cos\varphi)/\,2$, φ is the crossing angle.

Longitudinal Emittance

Applying the described above procedure and the recoil momentum (5) and the mean squared deviation (6) or (7), we get the partial relative energy spread caused by the laser interactions (see [1]):

$$\sigma_E^2 = \frac{\sigma_\gamma^2}{\gamma^2} = \kappa\frac{\gamma C_\varphi E_{\rm las}}{E_0} , \qquad (9)$$

where $\kappa = 1/6$ if the centered noise (7) employed, or $\kappa = 2/3$ if the total deviation (6) used.

Transverse Emittances

Making use the same approach and applying a so called smooth or (η,ψ) presentation the equation of betatron oscillations cast into a form of harmonical oscillator (see [2]):

$$\frac{{\rm d}^2\eta}{{\rm d}\psi^2} + Q^2\eta = 0 ; \qquad (10)$$

$$\eta \equiv \frac{y}{\sqrt{\beta}} , \qquad \psi \equiv \int_s \frac{{\rm d}s}{Q\beta} .$$

Here $y = \{x, z\}$ is the transverse deflection of a particle from the equilibrium orbit, Q the betatron number (number of betatron oscillations per turn), $\beta = \beta(s)$ the betatron (amplitude) function, s longitudinal coordinate playing the role of time.

Substituting into this expression the mean and mean squared energy losses with account for division of the energy into two transverse degrees of freedom, we finally get an expression describing the transverse bunch dimensions at the interaction point:

$$\sigma_y^2 = \frac{\beta_y^2 E_{\rm las}}{3\gamma E_0} . \qquad (11)$$

As it can be seen from the derived expression, the transverse dimension of the bunch in the interaction point is proportional to value of the betatron function in this point and square root of ratio of the laser photon energy to the electron energy.

SIMULATION OF LONGITUDINAL DYNAMICS

The presented above analytical estimations concerning partial laser energy spread in the electron bunches were derived under assumption that:

- Damping of the synchrotron oscillations is regular, caused by viscous friction;
- Excitation is caused by a white noise with zero average value;
- Each electron is subjected to large number of exciting interactions per a period of synchrotron oscillations.

Strictly speaking, these assumptions are not true for the synchrotron dynamics in the Compton storage ring. To validate the analytical results, a Monte Carlo simulation of the synchrotron dynamics were performed.

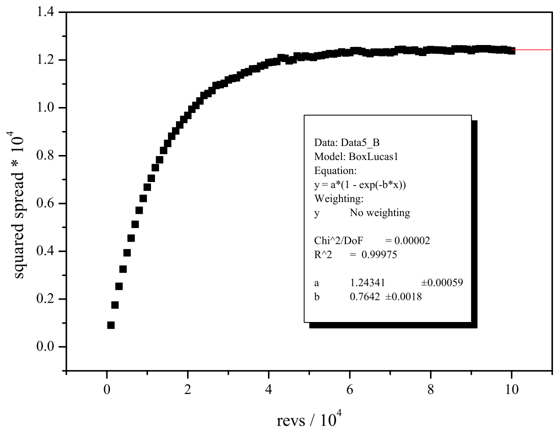

Figure 1: Temporary behavior of energy spread. Number of particles 10^5, probability of interaction per rev 0.1

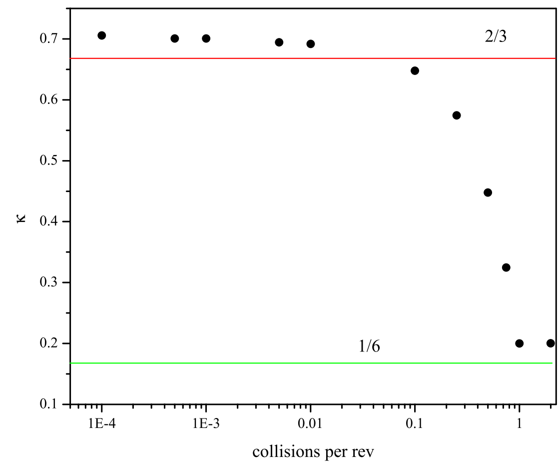

Figure 2: Dependence of κ on number of interactions per revolution.

The relative energy deviation of circulating electron p is changing due to Compton energy loss per the revolution where the interaction takes place:

$$p_f = p_i \left[1 - b\zeta(2 + p_i)\right] - b\zeta \ ,$$

with $b = 2(1 + \cos\varphi)\gamma_s E_{\rm las}/E_0$, ζ is a (random) ratio of energy loss to its maximal value.

The dynamics of a bunch consisting of $2 \times 10^3 \dots 10^5$ macroparticles with zero initial phase and energy deviation were carried out. The number of interactions per revolution was varied from 2 to 10^{-4}.

The results of simulation are as follows. The damping time (duration of transition to steady state) is equal to that predicted by Robinson–Kolomensky–Lebedev theory of synchrotron damping [3] with accuracy better than 2 %, see Fig.1

Concerning the stationary energy spread in the electron bunch (which factor κ – $1/6$ or $2/3$ – in (9) is valid), the simulation show that the energy spread is determined by the number of interactions per revolution, Fig.2. It reveals no response upon the period of synchrotron oscillations.

COOLING OR HEATING?

As longitudinal emittance is proportional to laser's photon energy then the synchrotron partial emittance is always much smaller then the laser one: *longitudinal degree of freedom is heated up due to Compton scattering.*

Almost the same conclusion is valid for the vertical uncoupled betatron emittance. Besides the energy of laser photons which is much larger than the equivalent "synchrotron photons" there squared beta function magnitude in the numerator. In principle it is possible to provide this magnitude say hundred times smaller that in bending magnets where synchrotron radiation emitted. In this particular case the cooling would be attained.

There two exciting mechanisms in the radial (horizontal) betatron oscillations. One is the same as for the vertical ones: random declining angle between the emission direction and electron's trajectory. The second mechanism much severe than fist: emission of the synchrotron quantum in dispersive section results in a skip of the equilibrium orbit. Since it is impossible in principle to get zero dispersion in the banding magnets where the synchrotron light emitting and keeping in mind larger magnitude of the beta function in there as compared with the low–beta dispersion–free interaction point, we come to conclusion: *the radial emittance almost always cooling down by laser.*

Therefore in practical cases of coupled transverse oscillations with the horizontal emittance determining the vertical one, the transverse degrees of freedom will be cooled down by the laser.

The stationary energy spread in bunches circulating in the Compton storage ring is dependent on the number of interactions per bunch revolution: with decrease in this number, the partial energy spread increase (twofold for the synchrotron dominated rings as compared with laser dominated ones).

REFERENCES

[1] Eugene Bulyak, "Theoretical Aspects of Compton Cooling in Storage Rings." Presented at *Int. Workshop on laser wire monitors and laser–Compton X–ray generation, Kyoto,* Nov 2003.

[2] E. Wilson, "Transverse beam dynamics." In *Proc. CERN Accel School CAS–92*. CERN 94–01, Geneva, 1994.

[3] R.P. Walker, "Radiation damping." In *Proc. CERN Accel School CAS–92*. CERN 94–01, Geneva, 1994.