ON BEAM DYNAMICS OPTIMIZATION*

D.A. Ovsyannikov, S.V. Merkuryev, St. Petersburg State University, Bibliotechnaya pl.2, Petergof, St. Petersburg, 198504, Russia

Abstract

Mathematical optimization methods are widely used in designing and construction of charged particles accelerators. In this paper new approach to beam dynamics optimization is considered. Suggested approach to the problem is based on the analytical representation for variation of examined functionals via solutions of special partial differentional equations. The problem of optimization is considered as a problem of mutual optimization chosen synchronous particle motion and charged particles beam in whole. This approach was applied to the beam dynamics optimization for RFQ structure.

PROBLEM STATEMENT

Let us consider the following mathematical model of control described by a system of differential equations [1]:

$$\frac{dx}{dt} = f(t,x,u), \tag{1}$$

$$\frac{dy}{dt} = F(t,x,y,u), \tag{2}$$

with initial conditions

$$x(0) = x_0, \tag{3}$$

$$y(0) = y_0 \in M_0, \tag{4}$$

Here $t \in T_0 = [0,T]$ — is independent variable, $x \in R^n$ and $y \in R^m$ — vectors of phase variables, $u = u(t)$ — r-dimensional function, T — is fixed number. Set $M_{t,u} = \{y_t \mid y_t = y(t, x(t), y_0, u), y_0 \in M_0\}$ is a cross-section of beam of trajectories $y(t, x(t), y_0, u)$, $y_0 \in M_0$ at a moment t under fixed control $u = u(t)$ and according program motion $x(t)$. In other words, $M_{t,u}$ is a shift of set M_0 along trajectories of system (2). The set $M_0 \subset R^m$ is compact, with nonzero measure; n-dimensional vector-function $f(t,x,u)$ is supposed to be continuous with its partial derivatives of the first order, m-dimensional vector-function $F(t,x,y,u)$ is supposed to be continuous together with partial derivatives of second order inclusive.

We assume that admissible controls $u = u(t)$, $t \in T_0$, form some class D of piecewise continuous function with values from a compact set $U \in R^r$, R^n, R^m, R^r are n, m and r-dimensional Euclidean spaces.

Let us agree that subsystem (1) describes dynamics of program motion, and subsystem (2) describes dynamics of motions disturbed with respect to initial conditions. In particular, subsystem (2) can be considered as equations in deviations from program motions.

Solution of system (1),(2) with initial conditions (3),(4) and fixed control function $u = u(t)$ will be a trajectory $x(t) = x(t, x_0, u)$ and a beam of trajectories $y(t, y_0) = y(t, x(t), y_0, u)$, $y_0 \in M_0$ turning equations (1),(2) into identity. Let us note that solution of subsystem (1) can be considered independently of subsystem (2).

Let us introduce the following functionals:

$$I_1(u) = c_1 \int_0^T \varphi_1(t, x(t), u(t))dt + c_2 g_1(x(T)), \tag{5}$$

$$I_2(u) = c_3 \int_0^T \int_{M_{t,u}} \varphi_2(t, x(t), y_t, u(t)) dy_t dt + c_4 \int_{M_{T,u}} g_2(y_T) dy_T. \tag{6}$$

Here $\varphi_1, \varphi_2, g_1, g_2$ are nonnegative, continuously differentiable functions, c_1, c_2, c_3, c_4 — nonnegative constants. Functional (5) characterizes program motion dynamics, and functional (6) estimates behavior of beam trajectories.

Let us introduce the following functional:

$$I(u) = I_1(u) + I_2(u), \tag{7}$$

simultaneously estimating dynamics of program motion and particles beam dynamics.

VARIATIONS OF FUNCTIONALS

Consider special partial differential equations

$$\frac{\partial \nu_1}{\partial t} + \frac{\partial \nu_1}{\partial x} f(t,x,u) + c_1 \varphi_1(t,x,u) = 0 \tag{8}$$

$$\frac{\partial \nu_2}{\partial t} + \frac{\partial \nu_2}{\partial x} f(t,x,u) + \frac{\partial \nu_2}{\partial y} F(t,x,y,u) + \nu_2 \operatorname{div}_y F(t,x,y,u) + c_3 \varphi_2(t,x,y,u) = 0 \tag{9}$$

with terminal conditions

$$\nu_1(T, x) = c_2 g_1(x),$$

$$\nu_2(T, x, y) = c_4 g_2(x, y).$$

Here

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$$v_1 = v_1(t,x),$$
$$v_2 = v_2(t,x,y).$$

Introduce the following functions:

$$w_1(t,x,u) = \frac{\partial v_1}{\partial t} + \frac{\partial v_1}{\partial x} f(t,x,u) + c_1\varphi_1(t,x,u),$$

$$w_2(t,x,y,u) = \frac{\partial v_2}{\partial t} + \frac{\partial v_2}{\partial x} f(t,x,u) +$$

$$+\frac{\partial v_2}{\partial y} F(t,x,y,u) + v_2 div_y F(t,x,y,u) + c_3\varphi_2(t,x,y,u).$$

Evidently

$$w_1(t,x,u(t)) \equiv 0,$$
$$w_2(t,x,y,u(t)) \equiv 0.$$

Hence we obtain new representation for functionals (5),(6), namely,

$$I_1(u) = v_1(0,x_0),$$
$$I_2(u) = \int_{M_0} v_2(0,x_0,y_0)dy_0.$$

Let

$$\tilde{u} = u + \Delta u,\ \tilde{x}(t) = x(t,x_0,\tilde{u}),\ \tilde{y}(t) = y(t,y_0,\tilde{u}).$$

Then we obtain

$$\Delta I(u,\Delta u) = I(\tilde{u}) - I(u) = \int_0^T w_1(t,\tilde{x}(t),\tilde{u}(t))dt +$$

$$+\int_0^T \int_{M_{T,\tilde{u}}} w_2(t,\tilde{x}(t),\tilde{y}(t),\tilde{u}(t))d\tilde{y}_t dt.$$

Using the results of the works [2-4], the increment of functional (7) can be presented in the following form:

$$\Delta I = \delta I + \circ(\mu),$$
$$\delta I = \delta I_1 + \delta I,$$

where

$$\delta I_1 = \int_0^T w_1(t,x(t),\tilde{u}(t))dt,$$

$$\delta I_2 = \int_0^T \int_{M_{t,u}} w_2(t,x(t),y_t,\tilde{u}(t))dy_t dt,$$

$$\circ(\mu)/\mu \to 0 \text{ as } \mu \to 0,$$

$$\mu = \max_{y_0 \in M_0} (\|\Delta_u f\|_L + \|\Delta_u F\|_L + \|\Delta_u div_y F\|_L),$$

$$\Delta_u f = f(t,x,u+\Delta u) - f(t,x,u),$$

$$\Delta_u F = F(t,x,y,u+\Delta u) - F(t,x,y,u),$$

$$\Delta_u div_y F = div_y \Delta_u F,$$

$$\|\Theta\|_L = \int_0^T |\Theta(t)|\, dt.$$

Present variation δI_2 in the following form:

$$\delta I_2 = \int_0^T \int_{M_{t,u}} \left\{ \frac{\partial v_2}{\partial x}\Delta_u f + \right. \tag{10}$$
$$\left. + \frac{\partial v_2}{\partial y}\Delta_u F + v_2 div_y \Delta_u F + c_3 \Delta_u \varphi_2 \right\} dy_t dt.$$

Introduce new functions

$$q_i(t,x,y,u) =$$
$$= \int_0^{y_i} \frac{\partial v_2(t,x,y_1,\cdots;y_{i-1},\zeta_i,y_{i+1},\cdots;y_n)}{\partial x} d\zeta_i f_i(t,x,u).$$

Let there exists function Φ_2 that

$$\varphi_2(t,x,y,u) = div_y \Phi_2(t,x,y,u),$$

then variation (10) may be represented as

$$\delta I_2 = \int_0^T \int_{M_{t,u}} div_y(\Delta_u q + v_2 \Delta_u F + c_3 \Delta_u \Phi_2) dy_t dt$$

or

$$\delta I_2 = \int_0^T \int_{S_{t,u}} \{\Delta_u q + v_2 \Delta_u F + c_3 \Delta_u \Phi_2\} n(y_t) dS_{t,u} dt. \tag{11}$$

Here $S_{t,u}$ is the boundary of $M_{t,u}$ and $n(y_t) = \dfrac{\Psi(y_t)}{\|\Psi(y_t)\|}$ is an external unity normal.

Vector-function Ψ satisfies the equation

$$\frac{d\Psi(y(t))}{dt} = -\left(\frac{\partial F(t,x(t),y(t),u(t))}{\partial y}\right)^* \Psi(y(t))$$

with initial condition

$$\Psi(y_0) = \Psi(y(0)) = \partial B_0(y_0)/\partial y_0,$$

where $B_0(x) = 0$ is the equation of M_0 boundary..

Let there exists vector-function G_2 that

$$div_y G(x,y) = g_2(x,y),$$

then we obtain new representation for functional I_2:

$$I_2(u) = c_3 \int_0^T \int_{S_{t,u}} \Phi_2(t,x,y_t,u)\, n(y_t) dS_{t,u} dt +$$

$$+c_4 \int_{S_{T,u}} G_2(x_T, y_T)\, n(y_T) dS_{T,u}.$$

So the set control problem is reduced to the boundary set control problem for functional under consideration.

OPTIMIZATION OF LONGITUDINAL MOTION IN RFQ

The developed algorithm was applied to optimization of longitudinal motion of beam particles in RFQ structure.

The longitudinal motion of beam particles in RFQ accelerator is described by the following system of differential equations [5,6]:

$$\frac{d}{d\tilde{\tau}}(L/L_0)^2 = 2k\cdot\eta(\tilde{\tau})\cdot\cos\varphi_s(\tilde{\tau}), \quad (12)$$

$$\psi'' + \frac{\left((L/L_0)^2\right)'}{(L/L_0)^2}\psi' + \frac{(L/L_0)''}{(L/L_0)}\psi - \\ -\frac{\eta(\tilde{\tau})}{(L/L_0)^2}\left(\cos\varphi_s - \cos(\psi+\varphi_s)\right) = 0 \quad (13)$$

Here $\tilde{\tau} = \Omega_0\tau$, $\Omega_0^2 = \frac{4\pi\, e(U_L\Theta)_{\max}}{W_0 L_0^2}$, $\tau = ct$ — is the reduced time, e — is the charge of electron, U_L — is the electrode voltage, Θ — is the efficiency of acceleration parameter, W_0 — is the self-energy of accelerated particle, $L = \beta_s\lambda$ — is the length of period, β_s — is the reduced velocity of synchronous particle, λ — is the wave-length of accelerating field, c — is the light velocity, $k = \Omega_0/\tilde{\omega}$, $\tilde{\omega} = 2\pi\omega/c$, ω — is the frequency of accelerating filed, $\psi(\tilde{\tau}) = 2\pi\,(z_s - z)/L$ — is the deviation of longitudinal coordinate of each beam's particle from the longitudinal coordinate of the synchronous particle.

So equations (12),(13) determine synchronous particle dynamics and the motion of beam particles in whole, respectively. The control functions $\eta(\tilde{\tau})$ and $\varphi_s(\tilde{\tau})$ are the acceleration efficiency and the phase of synchronous particle — the piecewise linear functions on integration time interval $[0, T]$.

Introduce functionals

$$I_1(u) = c_1\int_0^T \hat{\varphi}\,(A_{def}{}^2)\,d\tilde{\tau} + c_2(x(T) - \bar{x})^2, \quad (14)$$

$$I_2(u) = c_3 \int_{M_{T,u}} \left(\psi_T^2 + \psi'^2_T\right) d\psi_T\, d\psi'_T, \quad (15)$$

where A_{def} is the defocusing factor, calculated by the formulae

$$A_{def} = 2k^2\eta(\tilde{\tau})\left|\sin\varphi_s(\tilde{\tau})\right| / (L/L_0)^2,$$

and value $x(T) = (L/L_0)^2\,|_T = (\beta_s/\beta_0)^2\,|_T$, where $\bar{x}$ is a fixed parameter, setting the synchronous particle velocity at the output of accelerator; $\hat{\varphi}$ is a penalty function that could be defined differently.

The approach of optimization described above was used for RFQ structure designing. The following functional was considered:

$$I(u) = I_1(u) + I_2(u),$$

were $I_1(u)$ and $I_2(u)$ are determined by (14),(15). On the basis of variation (11) the methods of longitudinal motion optimization were constructed. For dynamics calculation and optimization the Matlab v.6.5 codes were used. The realization of developed methods shows their effectiveness.

The following table represents some of the accelerating structure parameters:

Some parameters of obtained RFQ structure.

Type of particles	Protons
Input Energy	95 KeV
Output Energy	5 MeV
Frequency	352 MHz
Electrode Voltage	100 KV
Structure's Length	5,5 meter
Final Synchronous Particle Phase	-24.39 radian

CONCLUSION

The paper suggests new approach allowing joint optimization of program motion and an ensemble of perturbed motions.

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