# INVESTIGATION OF MICROWAVE INSTABILITY ON ELECTRON STORAGE RING TLS

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### INTRODUCTION

With the planned installation of a superconducting rf system, the new operation mode of TLS, the electron storage ring at NSRRC, is expected to double the beam intensity. Several accelerator physics topics need to be examined. Beam instability of single-bunch longitudinal microwave instability is one of these topics. We consider two approaches to measure the effective broad band impedance. We compare these measurement results with each other and to old data [Ref.1]. We calculate the threshold current of microwave instability with a modemixing analysis code written by Dr. K. Oide of KEK [Ref.2]. We also develop a multi-particle tracking code to simulate the instability. The results of simulation and measurement are compared and discussed. We conclude that doubling of beam current from 200 mA (1.5 mA/bunch) to 400 mA (3 mA/bunch) will not trigger the microwave instability even without a Landau cavity to lengthen the bunch. The benefit of Landau cavity is mainly for beam life time.

### **IMPEDANCE MEASUREMENT**

We use two kinds of measurements to estimate the effective broad band impedance. One is using streak camera to measure the bunch length versus beam current. We then find the microwave instability threshold current, and compare it with the Boussard criterion for microwave instability threshold [Ref.3],

$$\frac{Z}{n}I_{th} = \sqrt{2\pi} \frac{E_0}{e} \alpha \sigma_{\varepsilon}^2 \frac{\sigma_Z}{R} F, F = \begin{cases} 1 & \text{if } \sigma_Z > b \\ (\frac{b}{\sigma_Z})^2 & \text{if } \sigma_Z < b \end{cases}$$
(1)

where  $I_{th}$  is the threshold average beam current, *b* is the pipe radius with the resonance frequency  $\omega_r = c/b$ , *c* is speed of light, *R* is the circumference radius,  $E_0$  is the beam energy,  $\alpha$  is the momentum compaction factor, and  $\sigma_z$  and  $\sigma_e$  are the rms bunch length and energy spread.

The second way to obtain Z/n is by measuring the shift of vertical betatron tune  $V_y$  with single-bunch current. One may compare the data with [Ref.4]

$$\frac{edf_y}{dI} = \frac{dv_y}{dN} \approx -\frac{1}{2\gamma v_y} \frac{r_0 R^2}{b^3} \frac{Z/n}{Z_0}$$
(2)

with  $r_0$  the classical electron radius and  $Z_0$  the impedance of free space.

The results are shown in figure 1 for bunch length measurement and figure 2 for the vertical tune shift measurement. The maximum single-bunch beam current

in these two measurements is around 16 mA. Below this current we did not find obvious threshold current in figure 1, indicating the bunch lengthening is due to potential well distortion. Assuming the threshold current is 16 mA, the Boussard criterion gives  $Z/n = 0.42 \ \Omega$  for  $\sigma_z > b$  and  $Z/n = 2 \ \Omega$  for  $\sigma_z < b$ . In figure 2, the vertical betatron frequency shifts with beam current at the rate of -0.258 kHz/mA. The derived broad band impedance is 3  $\Omega$ . The same measurements had been performed when the ring operated at 1.3 GeV [Ref.1]. The bunch lengthening measurement gave  $Z/n = 1 \ \Omega$ . The rate of vertical frequency shift versus current is -0.2 kHz/mA, corresponding to a broad band impedance of 2  $\Omega$ .

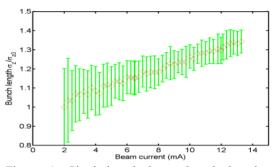


Figure 1: Single-bunch beam bunch length versus bunch current.

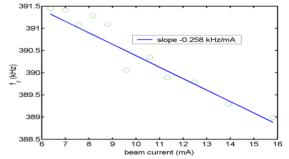


Figure 2: Single-bunch beam vertical betatron frequency shift with beam current. Circles are measurement data and solid line is a linear fit with slope of -0.258 kHz/mA.

## SIMULATION OF MICROWAVE INSTABILITY

In order to visualize the microwave instability more clearly, we develop a multi-particle tracking code to simulate the instability. We use 30000 macroparticles to represent a bunch. Each macroparticle *i* is tracked in phase space of position and energy coordinates  $(z_i, \mathcal{E}_i)$  with longitudinal equation of motion which includes effects of radiation damping, quantum fluctuation and

wake field. Here  $z_i$  is the position difference with the synchronous particle, a negative value means the particle is behind the synchronous particle,  $\varepsilon_i$  is the energy difference with synchronous particle. The particle *i* is advanced on *n*th turn according to [Ref.5]:

$$\varepsilon_{i,n} = (1 - \frac{2T_0}{\tau_{\varepsilon}})\varepsilon_{i,n-1} + 2\sigma_{\varepsilon 0}E_0 \sqrt{\frac{T_0}{\tau_{\varepsilon}}}ran(i,n-1) + eV_{rf}\sin(\phi_s - \omega_{rf}\frac{z_{i,n-1}}{c}) + eV_{ind}(z_{i,n-1})$$
(3)

$$z_{i,n} = z_{i,n-1} - \frac{\alpha c T_0}{E_0} \varepsilon_{i,n}$$
(4)

with  $T_0$  the revolution period,  $\tau_{\varepsilon}$  the damping time,  $\sigma_{\varepsilon 0}$  the nominal rms energy spread,  $V_{rf}$  the rf voltage,  $\omega_{rf}$  the rf angular frequency,  $\phi_s$  the synchronous phase and *ran()* a random function with normal distribution of zero mean and rms 1. The induced voltage  $V_{ind}$  due to the wake function on any turn is given by

$$V_{ind} = -\int_{z}^{\infty} dz' \rho(z') W_{0}'(z-z')$$
(5)

where  $\rho(z)$  is the charge distribution of the bunch. We choose the wake function as the resonator type with quality factor Q = 1 and the effective broad band impedance  $Z/n = 1 \Omega$ . This kind of wake function can be presented in phasor form as

$$\widetilde{\omega}_{0}'(z<0) = 2\kappa R_{s} e^{\kappa z/c} e^{i\frac{\overline{\omega}}{c}z} (1-i\frac{\kappa}{\overline{\omega}})$$

$$\widetilde{\omega}_{0}'(z=0) = \kappa R_{s} (1-i\frac{\kappa}{\overline{\omega}})$$

$$\widetilde{\omega}_{0}'(z>0) = 0$$
(6)

 $W_{a}' = RE(\tilde{\omega}_{0}')$ , where  $\kappa = \omega_{r}/(2Q)$ ,  $\varpi = \sqrt{\omega_{r}^{2} - \kappa^{2}}$ ,  $Z/n \approx (R_{\rm s}\omega_{\rm p})/(Q\omega_{\rm r})$ . Because the vertical vacuum beam pipe radius of NSRRC has gradually been modified from 0.02 m to about 0.01 m in the long straight sections for the insertion devices, two resonance frequencies of  $\omega_r = c / 0.02$  (case I) and  $\omega_r = c / 0.01$  (case II) were used for the tracking. Other simulation parameters used are:  $T_0 = 400$  ns,  $\tau_{\varepsilon} = 1$  ms,  $\sigma_{\varepsilon 0} = 7.5$ e-4,  $V_{rf} = 800$ kV,  $\omega_{rf}$  /(2 $\pi$ ) = 499.654 MHz,  $\phi_s$  = 2.967, and the natural bunch length  $\sigma_{z0}$  = 9.1 mm. The initial distribution of the bunch was first derived from the tracking without the effect of the wake. The 30000 particles were initially taken as synchronous particles or with a uniform distribution along  $\pm 5\sigma_{z0}$  longitudinal axis and tracked for two damping times. In both cases one gets the Gaussian distribution with the same  $\sigma_{\scriptscriptstyle z,0}$  and  $\sigma_{\scriptscriptstyle arepsilon,0}$  as shown in figure 3. Then this distribution was used as the initial condition to track the microwave instability with two wake fields case I and case II. The particles were

tracked 10000 turns, or four damping times. In each turn we recorded the average  $(z, \varepsilon)$  and the rms bunch length and energy spread. By averaging these four quantities over the last damping time we obtain the average properties and the standard deviations of the distribution. The simulation results were shown in figures 4 and 5. We also calculate the threshold current of microwave beam instability with a mode-mixing analysis code written by Dr. K. Oide of KEK [Ref.2]. The results are compared with the tracking simulation in figures 4 and 5.

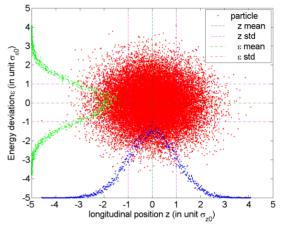


Figure 3: The natural Gaussian beam distribution in  $(z, \varepsilon)$  phase space as the initial distribution for the particle tracking of microwave beam instability. The natural bunch length  $\sigma_{z,0}$  of NSRRC is 9.1 mm and natural energy spread  $\sigma_{\varepsilon 0}$  is 7.5e-4.

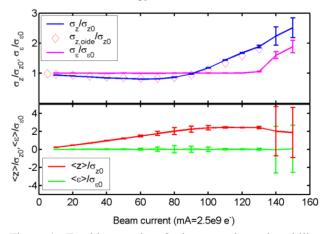


Figure 4: Tracking results of microwave beam instability of pipe radius larger than the bunch length (case I). The upper plot shows the change of bunch length and energy spread versus beam current. The variations are normalized to natural bunch length and energy spread. The solid lines represent the results of particle tracking and the diamonds are the result from Oide's code. The lower plot shows the changes of beam center and nominal energy. The vertical bar represents the standard deviation of the bunch motion during last damping period (2500 turns).

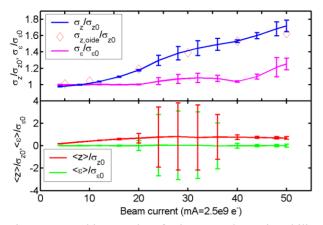


Figure 5: Tracking results of microwave beam instability of pipe radius approximately equal the bunch length (case II). The labels and units are same as in Figure 4.

### DISCUSSION

Results from particle tracking and from Oide's code agree well in both cases I and II. In case I the bunch length is less than b, the impedance can be capacitive to the bunch, therefore the bunch sometimes shows shortening in the potential well distortion region as seen in the upper plot of figure 4. In the potential well distortion region the energy spread does not change. The threshold current of the microwave instability is about 130 mA. This value is larger than that derived from the Boussard criterion of Eq.(1) which gives 32 mA. The shift of bunch center toward positive z direction as bunch current increases is the energy compensation of the bunch for the energy loss due to the resistive part of the impedance as shown in lower plot of figure 4. In the microwave instability region the bunch not only shows the bunch length and energy spread turbulence, presented by the large standard deviation, but also executes rigid dipole motion. The oscillation amplitude of the synchrotron motion is  $\sqrt{2}$  times the standard deviation shown in the lower part of figure 4. The oscillation amplitude is about 4  $\sigma_{z,0}$  and 3.675  $\sigma_{\varepsilon_0}$  for  $(z, \varepsilon)$ .

In case II the bunch length is slightly longer than the beam pipe radius, the impedance is inductive to the bunch; therefore the bunch is lengthened in the potential well distortion region as shown in the upper plot of figure 5. There are two beam instability regions in this case. The instability first onsets at beam current about 20 mA where the bunch length is equal to the pipe radius like in case I and companies rigid dipole oscillation. Beyond 40 mA, the dipole oscillation disappears and the slope of the bunch lengthening versus beam current is different. The threshold current derived from Boussard criterion is about 6.7 mA in case II. The discrepancy between simulation and Boussard criterion had been explained by Oide [Ref.2]. The potential well distortion plays a role to prevent the instability in the simulation. We count on the simulation more than on the Boussard criterion since the simulation reflects more reality of beam behaviour.

The bunch length measurement shown in figure 1 can be fitted by two resonator impedance models as shown in figure 6. The resonance frequency is  $\omega_r = c/0.005$  with Q = 0.51,  $Z/n = 2 \Omega$  and Q = 1,  $Z/n = 1.5 \Omega$ . Due to a wide range of uncertainty in the broad band impedance based on model dependent measurements, however, we have used in our simulations the model with Q = 1, Z/n $= 1 \Omega$ . Scaling to other values of Z/n can be obtained by simple scaling of the beam current.

Concluding from the above, the new measurement of effective broad band impedance is higher than the old measurement. However the determination of the broad band impedance from the bunch length measurement is sensitive to the choice of the beam pipe b. It is somewhat ambiguous to use single b value in the calculation. In a continuation of this work, we plan to include two broad band resonators to model the storage ring impedance more realistically. We also plan to explicitly include the effect of a Landau cavity.

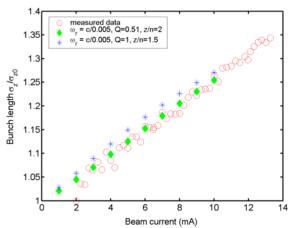


Figure 6: The bunch length measurement data compare with two impedance with the same  $\omega_r = c / 0.005$  and Q=0.51,  $Z/n = 2 \Omega$  or Q=1,  $Z/n = 1.5 \Omega$ .

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