A STUDY OF TRANSVERSE RESONANCE CROSSING IN FFAG

Masamitsu Aiba, The Tokyo University, Tokyo, 113-0033, Japan
Shinji Machida, Yoshiharu Mori, KEK, Ibaraki, 305-0801, Japan

Abstract

Non-scaling and semi-scaling FFAG [1,2], in which tune varies during acceleration, are proposed and studied as muon accelerators. For those accelerators, we study beam dynamics of resonance crossing, namely what happens when tunes cross resonance. Emittance growth and beam loss are evaluated quantitatively. In this paper, we discuss island trapping due to third order resonance crossing.

INTRODUCTION

FFAG accelerators have in principle constant tune due to zero chromaticity condition, in other words a scaling law is satisfied. However, it allows different design of FFAG accelerators, once we give up constant tunes. Those machines are called non-scaling and semi-scaling FFAG studied as muon accelerators [1,2]. When tune is varied during acceleration and crosses a resonance, emittance growth or beam loss may occur. We need to make sure growth or loss is small enough to be permitted. To treat beam dynamics of resonance crossing, two ways are known. One is to take an integral of equation of motion, the other is to consider an islands trapping. Basically, the former is for fast crossing and the latter is for slow crossing. In this paper, we discuss island trapping for third integer crossing specifically.

ISLAND TRAPPING

It was shown by A. W. Chao and M. Month in Ref.[3] that island trapping is occurred when tune crosses resonance with amplitude dependence due to non-linear term and driving term, which is small compared to non-linear one. Remarkable equation developed in Ref.[3] is the relation among trapping efficiency, crossing speed, non-linear detuning and driving term. They treat specifically fifth integer resonance crossing.

The condition for non-linear term and driving term corresponds to a guiding field of FFAG accelerator that has large intrinsic non-linear term and small accidental driving term. Semi-scaling FFAG has similar field to scaling FFAG, therefore the model of island trapping is suitable. Contrary, some non-scaling FFAG does not have non-linear component, so it may not be applicable for the model. In first place, scaling FFAG has constant tunes, but practically tunes tend to vary slightly and then the model is of course suitable.

THIRD INTEGER RESONANCE CROSSING

We will treat island trapping due to third integer resonance crossing. In addition, an opposite direction crossing, that is, islands move inward in phase space and disappear, is considered.

The equation for trapping efficiency of third integer resonance crossing is

$$P_T = \frac{\pi}{\sqrt{2}} \kappa^{-1/2} \alpha_s^{-1/4} \exp(-\alpha_1) \quad , \qquad (1)$$

where

$$\alpha_s = \alpha_1, \ if \ \alpha_1 > 1$$
$$= 1, \ \ if \ \alpha_1 < 1$$
$$\alpha_1 \approx \alpha_0, \ when \ \kappa >> 1$$
$$\alpha_0 = \left(\frac{\varepsilon}{4\pi \Delta_{NL} \Delta_e} \right)^{\frac{2}{3}} : the\ adiabatic\ parameter$$

ε : *the change of tune* ν *per one revolution*

$$\xi = \frac{3\Delta_L}{2\Delta_e}$$
$$\kappa = \frac{3\Delta_{NL}}{4\Delta_e}$$
$$\Delta_L = \frac{1}{3} p - \nu \ (p : integer) : the\ linear\ tune\ shift$$

Δ_{NL} : *the nonlinear tune shift*

Δ_e : *the excitation width*.

Above symbols are same to Ref. [3], but Δ_L, Δ_{NL} and Δ_e need to be arranged for third integer resonance. And ξ is defined for further convenient. We can find easily that a constant of motion near third integer resonance is

$$C = \xi\alpha + \kappa\alpha^2 + \alpha^{3/2} \cos 3\psi \ . \qquad (2)$$

A phase space topology can be obtained with Eq. 2. Figure 1 shows it near third integer resonance for various ξ and constant κ. It is seen that a phase space topology changes during tune crosses resonance as follows.

(a) When tune is far from resonance, there is no fixed point
(b) At $\xi = 9/32\kappa$, three islands are created
(c) Three islands get larger and stretch to the origin as tune closes to resonance
(d) Tune is just on resonance, three islands meet at the origin
(e) As leaving resonance, three islands move outward keeping a closed three piece chain

As is seen in Fig. 1, a phase space topology changes dynamically. Consequently, island trapping is occurred during tune crosses resonance.

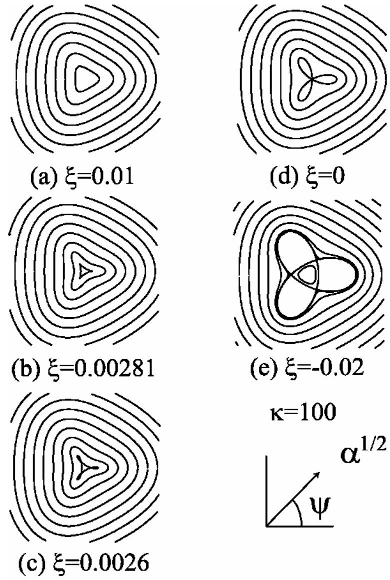
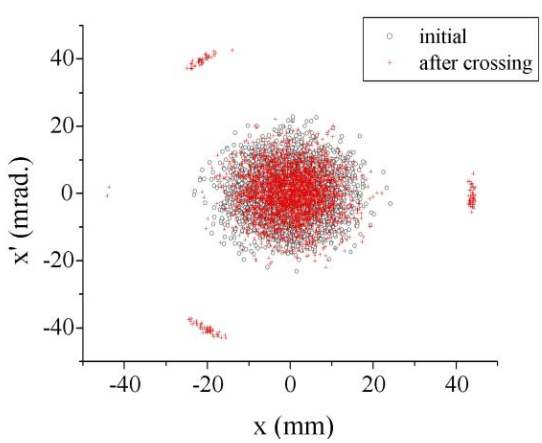


Figure 1 The phase space topology for third integer resonance crossing

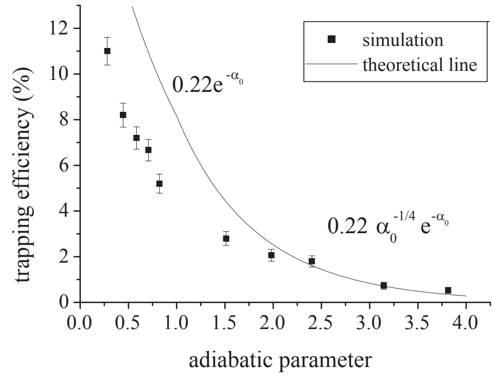
A simulation, one-dimensional tracking with 2*2 matrix, is performed to verify these equations. Results for normal crossing are shown in Fig. 2 and input parameters are summarized in Tab. 1.

Table 1 Parameters of simulation

Crossing Resonance	3n=11
Adiabatic Parameter	various
Nonlinear Detuning	4.77*10^-4
Excitation Width	3.58*10^-6
k	100
Twiss Parameter at Observation Point	α=0 β=1(m)
Number of Test Particle	3000
Average Emittanece	100π (mm-mrad.)



(a) The particle distribution before and after crossing (adiabatic parameter =1)



(b) The calculated trapping efficiency with theoretical line

Figure 2 The simulation results for normal crossing

Parameters in Tab. 1 are determined to be a condition specific to FFAG accelerator, whose non-linear term is very larger than a driving term. Fig. 2(a) is one of simulation results showing particle distribution before and after crossing resonance. It is seen that three islands trap some particle. When tune leave resonance further, trapped particles are brought to infinite amplitude. Trapping efficiency means a ratio of trapped particle to total particle. It is calculated for various crossing speed and plotted in Fig 2(b). We can see strong correlation between the trapping efficiency of simulation result and theoretical line in Fig. 2(b).

Another interesting case is opposite crossing. If the sign of octupole or the direction of tune variation is changed, islands move inward in phase space and finally disappear.

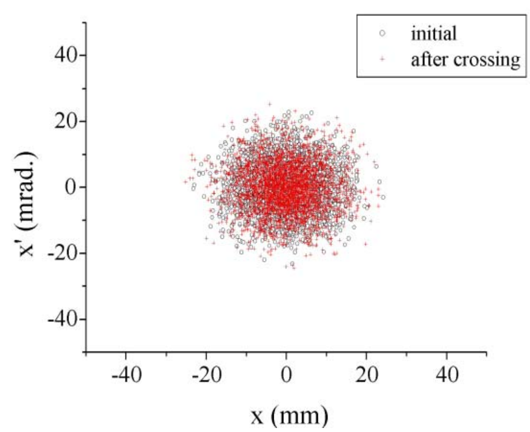


Figure 3 The particle distribution before and after opposite crossing

Figure 3 is contrasted with Fig. 2(a). Only crossing direction is different in these two results. In Fig. 3, a beam seems not to take an influence at all. In order to enhance an influence, larger driving term is introduced.

Figure 4 shows snapshots of phase space during tune crosses resonance. A driving term is now fifty times to previous. Then a blow-up during opposite crossing is observed.

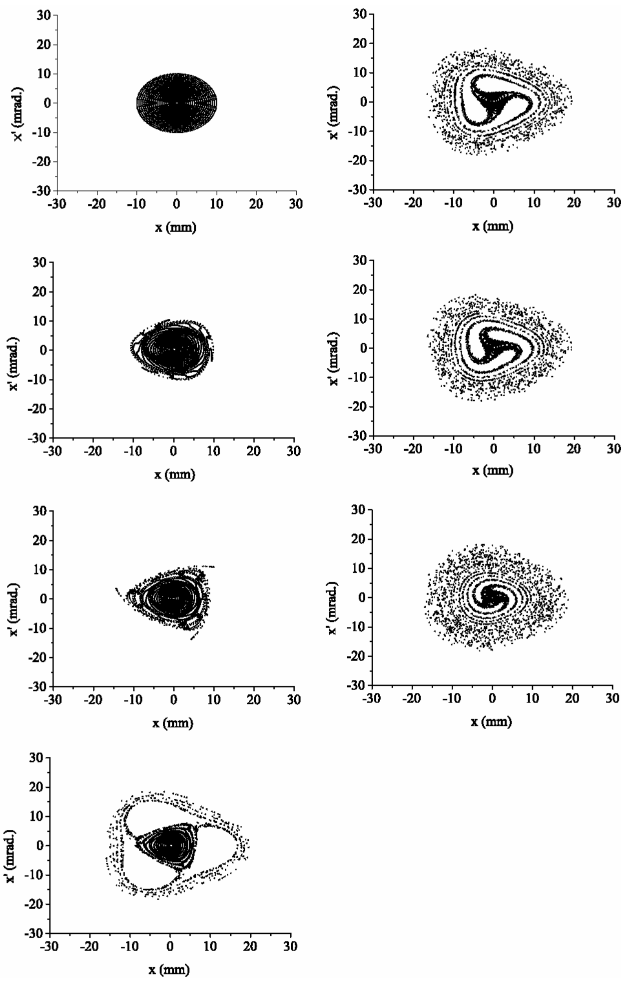


Figure 4 Snapshots of particle distribution during crossing (ε=2x10^{-7})

A phase space topology with Fig. 1 and the results in Fig. 4 lead behaviour of particles as follows.

1. When islands close to a beam, particle distribution is distorted from an ellipse to a triangle.
2. Right after islands touch a beam, surface particles begin to leak through three west points of the islands chain.
3. As islands move inward, leak particles moves along islands and occupy the space that islands pass.
4. When tune becomes on resonance, islands touch the origin to be like a clover, then particle leaking finishes.
5. After on resonance, island centers moves to disappearance point and some particle, depending on crossing speed, settle down in the central stable area.
6. Islands disappear finally and the particle distribution after crossing is fixed.

Considering the behaviour of particles during opposite crossing, we can find the fact of the following statement. When a speed of islands moving, in other word crossing speed, is slow compared to particle flow, the final beam radius is almost same to the outline of islands chain of the moment that islands touch a beam.

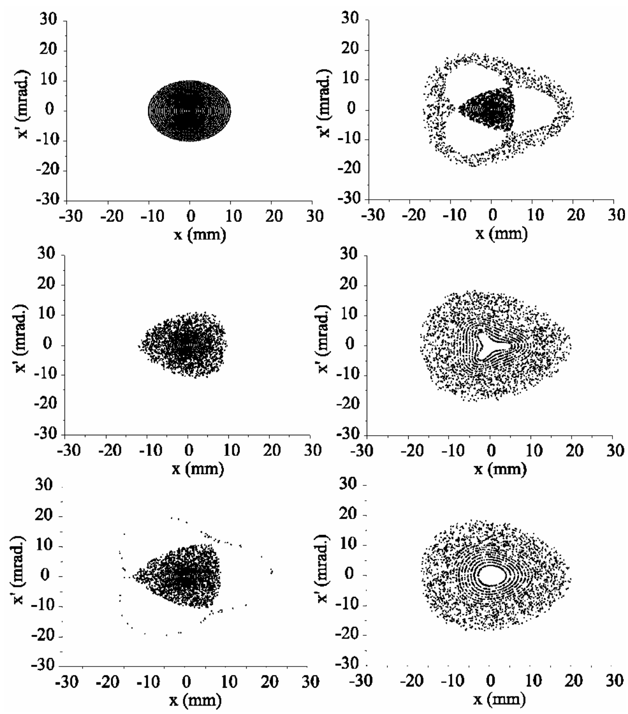


Figure 5 Snapshots of particle distribution during crossing (very slow crossing / ε=1x10^{-10})

Figure 5 shows the case of very slow opposite crossing. As shown in Fig. 5, when crossing speed is very slow, no particle settle down in the central stable area. However, maximum beam radius is limited by the outline of islands chain of the moment that islands touch a beam. This statement would be available for infinitely slow crossing. Since phase space trajectories are always close during third integer resonance crossing, no particle can be brought to infinite amplitude even if crossing speed is infinitely slow.

SUMMARY

Resonance crossing is studied for semi-scaling and non-scaling FFAG. Island trapping is discussed specifically for third integer resonance crossing. It is shown that the theory developed in Ref.[3] is also applicable for third integer crossing. Furthermore, we find that in one direction growth is limited to a finite value even if crossing speed is infinitely slow.

REFERENCES

[1] J.S. Berg et. al., Proc. of Pac'03, pp3413-3415
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[3] A.W. Chao et. al., NIM, 121(1974), pp129-138