FREQUENCY MAP ANALYSIS WITH THE INSERTION DEVICES AT ELETTRA

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Abstract

Frequency map analysis (FMA) [1] is a very efficient technique for the understanding of the resonances which may affect the stability of the electrons. Measurements correlated to simulations may provide a method to improve beam lifetime and the injection efficiency, particularly important in case of top up operation. In this paper, the results of frequency map measurements and simulations for the ELETTRA storage ring are presented both for the bare lattice as well as for the case in which the insertion devices (IDs) are operational.

INTRODUCTION

Frequency Map Analysis (FMA) is a numerical method, which can be generally applied to any Hamiltonian system or symplectic map. FMA studies are a possible way to obtain a vision of the resonances affecting the dynamic aperture and of the nonlinear characteristics of insertion devices. Theoretical simulations and experimental measurements to investigate the FMA for Elettra have been carried out; several software codes have been developed both for the post processing and for simulations.

CODES FOR FMA

Since the experimental data is the resulting centre of mass motion of a kicked beam, the codes were developed to track both single particles as well as a gaussian beam composed of many particles, from which the centre of mass may be extracted [2]. In both treatments, the single particle transverse dynamics are modelled by the canonical coordinates (x,p_x,y,p_y), where x,y are the horizontal and vertical positions and p_x,p_y are the transverse normalised momenta.

For some magnetic field configurations, resonances appear for an integer linear combination of the transverse tunes ν_x,ν_y, i.e. $p\nu_x + q\nu_y = m$, where p,q and m are integer numbers, and |p| + |q| is often called the resonance order.

FMA constructs the so-called frequency map from the space of initial conditions to the tune space by searching for a quasi-periodic approximation of the transverse motion over a finite time span T. More precisely, two of the initial conditions are fixed as $p_x = p_y = 0$ and x,y are taken on a grid of initial values with step size of 0.25 mm in each plane. In order to extract the linear tunes, an initial amplitude of 0.001 mm is used as the first value of the grid. Then the test particle is tracked for each set of initial conditions and the four-dimensional variables (x,y,p_x,p_y) are recorded for each turn for the time span T. By using a numerical algorithm based on a refined Fourier technique (NAFF) [3], which provides a tune accuracy of $1/T^4$, we search for a quasi-periodic approximation of $z_x(t) = x(t) + ip_x(t)$ and $z_y(t) = y(t) + ip_y(t)$ of the form:

$$z_w(t) = a_w e^{i\nu_w t} + \sum_{k=1}^{N} a_k e^{i(m_k,\nu)t}$$

where w = x,y, $\nu=(\nu_x,\nu_y,1)$ is the fundamental frequency vector, $m_k = (m_{xk},m_{yk},m_{3k})$ is a multi-index and the complex amplitude a_k is ordered by decreasing magnitudes.

Each particle is tracked over 2000 turns and for the surviving particle the transverse tunes ($\nu_x^{(1)},\nu_y^{(1)}$) and ($\nu_x^{(2)},\nu_y^{(2)}$) are computed with the FMA over two consecutive samples of 1000 turns. The diffusion rate D is then calculated as [3]:

$$D = \log_{10}\sqrt{\left[\left(\nu_x^{(1)}-\nu_x^{(2)}\right)^2 + \left(\nu_y^{(1)}-\nu_y^{(2)}\right)^2\right]}$$

and may be used as a stability index. The diffusion rate is coded by a colour scale from blue for vary stable orbits to red for unstable and chaotic ones. This diffusion rate gives a good long-term stability criterion

2ND ORDER NONLINEAR COEFFICIENTS

The sextupolar nonlinearities acting in Elettra generate a 2nd order tune shift with amplitude:

$$\Delta\nu_x = 2J_x c_{11} + J_y c_{12}$$

$$\Delta\nu_y = 2J_y c_{22} + J_x c_{12}$$

where $J_z = \sqrt{\beta_z\varepsilon_z}$ for z=x,y and the nonlinear coefficients c_{ij} are functions of the optics and of the sextupolar strengths.

Experimentally the tune shift with amplitude is revealed by the collapse through decoherence of the bunch centroid towards the reference orbit [4]. The decay rate N_c of the centroid's amplitude of oscillation, in terms of turn number, is inversely proportional to the nonlinear coefficients, c_{11} for the decay in the horizontal plane and c_{22} for the vertical case. The decoherence has been measured for various working points (Fig.1); a fit of the decay rate [4] then gives the analogous nonlinear coefficient (Fig.2a and Fig.2b).

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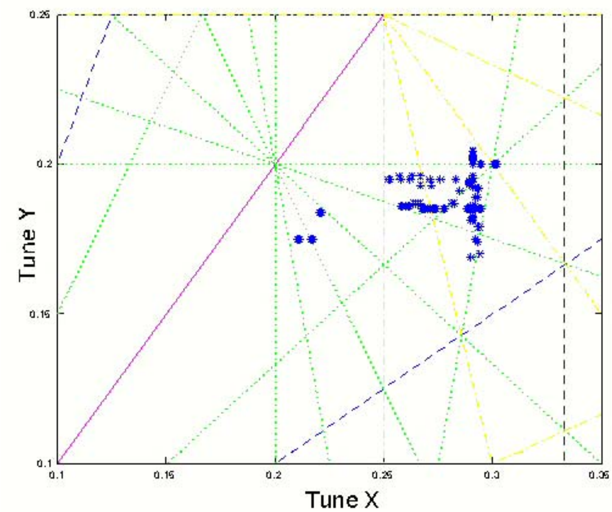

Fig.1 Tune diagram. The experimental points intercept 4th (yellow) and 5th order (green) resonances. Some of them result from the simulated FM in Fig.4

Thus c_{ij} becomes an index of the nonlinearity of the machine: the faster the collapse, the larger is the coefficient's value, and the stronger are the nonlinearities.

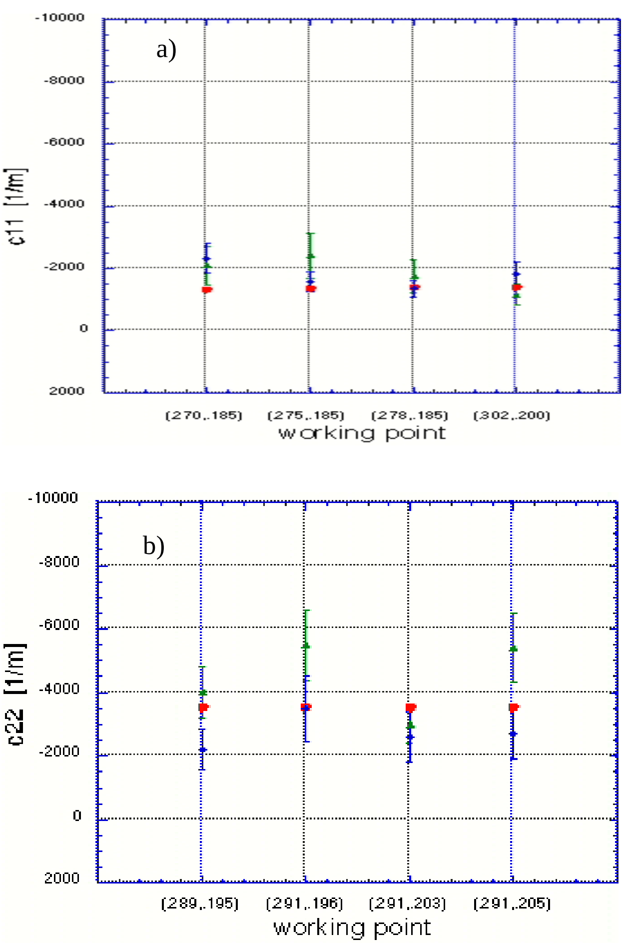

Fig.2 Horizontal (a) and vertical (b) nonlinear coefficients.

Fig.2a and Fig.2b compare the c_{11} and c_{22} values obtained with a fit (green), a direct measure of the effective tune versus amplitude of the excitation (blue) and the theoretical values computed by Mad (red). The two larger values of c_{22} correspond to measurements corrupted by mulfunctioning of the diagnostics system.

RESONANCE PATTERN

Tracking, without synchrotron motion, has been performed over 1000 turns. The two families of chromatic sextupoles are set in order to adjust a slightly positive chromaticity, while the family of harmonic sextupoles is included and set to operating conditions. The working point is (14.25, 8.20).

The lattice is characterised by an optical asymmetry experimentally evaluated in 20% for the horizontal plane and 13% for the vertical plane. This is the only source of the 12-fold symmetry breaking in the simulations.

The dynamic aperture (Fig.3) shows highly diffusing elliptic trajectories which reflect the nonlinear distortion of the phase space.

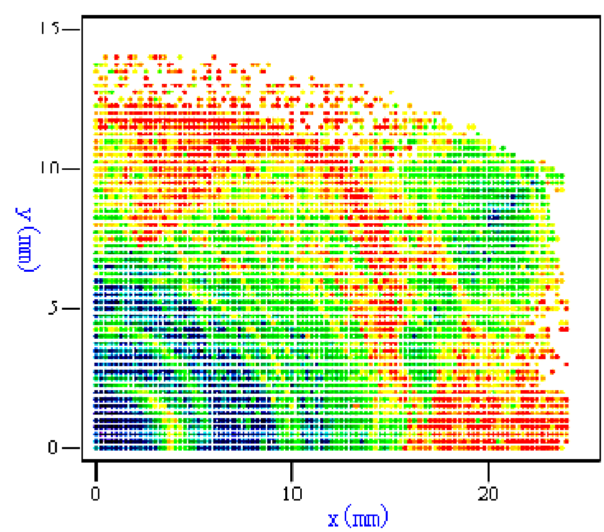

Fig.3 Dynamic aperture resulting from single particle tracking after 1000 turns in the bare lattice.

These lines are mapped in the tune diagram (Fig.4), where clustering of amplitude dependent tunes with high diffusion rate (distortions of the FM) can be observed. The absence of dots in the tune diagram means that the particle was lost. The systematic resonance $\nu_x - \nu_y = 6$ acts like an attractor: tune regions near the resonance line are forbidden, while the on-resonance condition allows a strongly irregular motion which however is still bound.

The machine's symmetry break excites non-systematic resonances of 4th and 5th order which may also be of the coupling types. Phase space for each of the experimental working points (Fig.1) has been investigated to confirm their existence in the machine [5].

The FM boundaries are determined by the tune shift with amplitude. The vertical tune shift defines the larger bottom boundary, being an index of stronger nonlinearities in the vertical plane with respect to the horizontal. This is also confirmed by the measurements of the nonlinear coefficients (Fig.2).

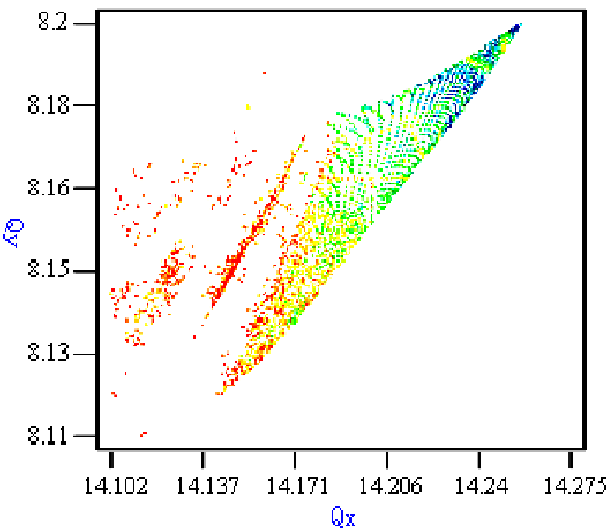

Fig.4 FM from tracking in the bare lattice with optical asymmetries. Showed resonance lines till to the 5th order.

INSERTION DEVICES

Three insertion device (ID) configurations (Tab.1) have been experimentally investigated through the FMA; the beam decoherence has been analysed in order to obtain the nonlinear coefficients for both the transverse planes, too. The IDs are closed to the minimum gap in all the cases.

Tab.1 IDs contributions to the nonlinear coefficients.

	(ν_x, ν_y)	Δc_{11}	Δc_{22}
ID9	(.250,.190)	+ 16%	-
ID10	(.290,.194) (.250,.195) (.200,.195)	+25%	+ 82% + 88% + 40%
All IDs	$.252<\nu_x<.290$, $\nu_y=.195$	– 60%	-
All IDs	(.290,.169) (.292,.174) (.293,.186)	-	+ 175% + 97% + 140%

Tab.1 shows the percentile variation of the coefficients induced by the three configurations, to be compared with the values of c_{11} and c_{22} measured with the bare lattice and already presented in Fig.1a and Fig.1b. A total relative error for the values of the coefficients has been estimated as ± 20%: it includes uncertainties in the real optics, the fitting error for the calculation of the coefficients and the experimental error in the calibration of the system exciting the bunch.

It is interesting to notice that the effective nonlinearities resulting from the closure of all the IDs reduce by a large amount the horizontal tune shift, while it amplifies the vertical one.

Tab.2 Insertion device parameters .

ID - type	B_{max} [T]	gap_{min} [mm]
ID9 – APPLE ID	0.8	18.6
ID10 - Perm. Magn. Figure-8	0.9(v) / 0.1(h)	19
All IDs in Elettra	1.0	18.6

CONCLUSIONS

A series of codes for FMA have been developed in order to study the inner structure of the tracked dynamic aperture and the resonance pattern in the tune diagram. Simulations show non-systematic resonance to 4th and 5th order, excited only by the optics asymmetry, but particle loss appears only for the 2nd order difference resonance.

The FM reveals stronger nonlinearities in the vertical plane with respect to the horizontal. This is confirmed by the measurements of the 2nd order tune shift with amplitude evaluated through the nonlinear coefficients. The experimental data are consistent with the theoretical predictions.

The ID contributions to the tune shift have been measured in terms of the percentile variation of the nonlinear coefficients. It depends on the optics, but in general it shows a very large nonlinear effect in the vertical plane with respect to the horizontal, as expected from theory. The closure of most of the IDs in Elettra results in a reduction (– 60%) of the horizontal tune shift, but in a very large increase in the vertical coefficient (+ 175% in the worse case).

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