NONLINEAR EVOLUTION OF THE BEAM IN PHASE SPACE AT ELETTRA

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Abstract

Phase space in the Elettra storage ring has been investigated. The beam is kicked and the coordinates of the bunch centroid are acquired for at least 1000 turns. The Hilbert transform [1] has been used to deduce the evolution of beam phase space from position coordinates. Several nonlinear effects have been detected, such as the amplitude dependence of the betatron tune, the presence of high order and coupling resonances. Fixed points have been evidenced as well as the behaviour of the beam in their neighbourhood. Scans in lifetime versus tune confirm the limiting effect of the observed resonances on the region of regular motion.

MEASUREMENT TOOL SYSTEM AND DATA ANALYSIS

The Transverse Multibunch Feedback System (TMFB) of ELETTRA is used in anti-damping mode to excite coherently a single bunch in the horizontal or vertical plane. Only the inner region of the dynamic aperture has been investigated (x,y < 5 mm). The measurements presented here have been performed for a 0.3 mA single bunch at 2.0 GeV and the nominal sextupole configuration for Elettra.

The turn by turn bunch centroid's position is recorded at a beam position monitor (BPM) and its profile is analysed. A Frequency Map Analysis (FMA) [2], applied to the first 1000 turns, allows to map the physical space of the amplitudes of the betatron oscillations into the tune diagram and so to recognise resonance conditions.

The application of the Hilbert transform to the acquired data allows the reconstruction of the experimental phase space and resonance conditions can be verified by observing periodical structures, such as fixed points, islands, sepratrices.

Finally, scans in lifetime versus tune confirm the limiting effect of the detected resonances on the dynamic aperture.

HILBERT TRANSFORM

The Hilbert transform permits the observation of the transverse phase space using only one BPM. If $x = \{x_1, x_2, x_3, \ldots, x_n\}$ is the vector position of the bunch centroid at a given azimuth in the storage ring, where $x_i \sim A_i\cos\phi_i$, the Hilbert transform rotates it by 90° in the frequency domain.

Returning to the physical space, $x'_i \sim A_i\sin\phi_i$ is obtained, namely the analogous angular divergence rescaled with the same amplitude of x_i. The implemented divergence is not physically true, but the topology of the phase space is still preserved, as well as the information on the nonlinear dynamics that it contains.

NONLINEAR EFFECTS

After the bunch is coherently excited, because of the nonlinearities, each particle in the bunch oscillates with its own betatron frequency, which depends on the amplitude of the oscillation (tune shift with amplitude). The resulting decoherence of the motion causes the centre of mass of the bunch to collapse towards the reference orbit.

The decay rate N_c of the centroid's amplitude of oscillation, in terms of number of turns, depends on the optics, on the initial amplitude of oscillation (z_0, with z=x,y) and on the nonlinear coefficient c_{ii} (i=1,2) [3]:

$$N_c = \sqrt{\beta_z \varepsilon_z}\,\frac{1}{(2\pi\,|c_{ii}|\,z_0)} \qquad (1)$$

where c_{ii}, being itself a function of the optics and of the sextupoles' strength, is the coefficient of the 2nd order tune shift with amplitude:

$$\begin{aligned}\Delta\nu_x &= 2\,J_x\,c_{11} + J_y\,c_{12}\\ \Delta\nu_y &= 2\,J_y\,c_{22} + J_x\,c_{12}\end{aligned} \qquad (2)$$

where $J_z = \sqrt{\beta_z \varepsilon_z}$.

The decay rate of decoherence, i.e. c_{11} for the horizontal plane and c_{22} for the vertical one, becomes an index of the nonlinearities present in the machine: the faster the collapse, the stronger are the nonlinearities. In practice, strong nonlinearities limit the observation of the centroid motion at large amplitudes, but, at the same time, they offer the possibility of studying a rich pattern of resonances and large regions of orbit distortion in phase space.

PHASE SPACE

The turn by turn linear transverse motion of a single particle generates in phase space an ellipse, that is transformed into a circle by the Hilbert transform. The experimental points, representing the bunch centroid's position and its pseudo-divergence, lie on that circle. If the centroid oscillates at large amplitudes, nonlinearities distort the circle and the points will spiral towards the centre due to the decoherence of the motion of the single particles. Furthermore, as the tune approaches a resonance condition, periodic structures appear. In particular, in the absence of a resonance, the phase space is full of points, whose density depends on the nonlinearities: a fast collapse, that is a strong nonlinear motion (Eq.1), reduces the points' density in the outer region of phase space (Fig.1a,1b). In general, because of the nonlinear nature of

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the motion, the topology of the observed phase space depends on the initial excited amplitude of oscillation of the bunch centroid.

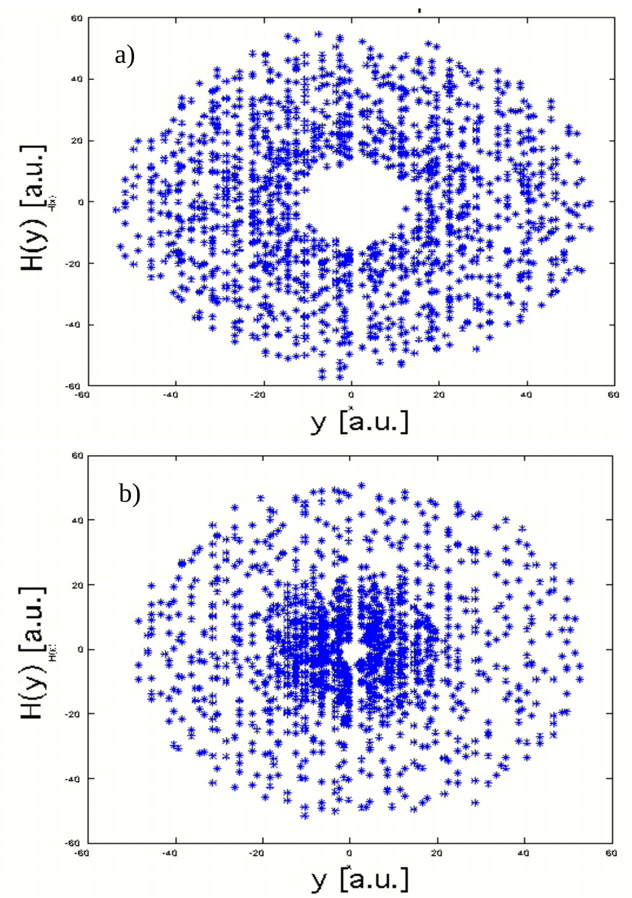

Fig.1a,1b Vertical phase space for $\nu_y \approx 0.190$ (1500 turns). The maximum amplitude y is about 2.5 mm (a,b). After ID10 (Figure-8 permanent magnets undulator) is closed to the minimum gap of 19 mm (b), the nonlinearities in Elettra increase, doubling the nonlinear coefficient c_{22}.

When a resonance condition $m\nu_{x,y} \approx p$ (m,p integers) is close to being satisfied, after m turns the particle returns to the same point in phase space and m fixed points are observed, which represent stable motion (Fig.2). Then the decoherence makes the points spiral along m branches (Fig.3a,3b).

The area near a fixed point generally allows quite regular motion, too; here the particle can be trapped into islands – represented by m smaller ellipses around the m fixed points – or, in general, into a region of stochastic motion (Fig.4).

The boundary between regions of different dynamical behaviour (regular orbits, fixed points, unstable motion, etc.) is defined by the separatrix: it may be a direction for particle's escape and organises phase space via partitioning it in regular and non-regular areas of motion (Fig.5) [3].

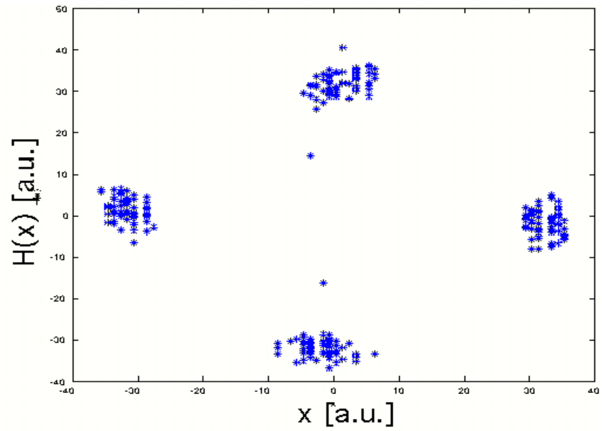

Fig.2 Fourth order "on-resonance" condition $4\nu_x$=57 (250 turns). The smearing around the fixed points is due to the inherent betatron coupling. The very fast decay (N < 500 turns) is not shown.

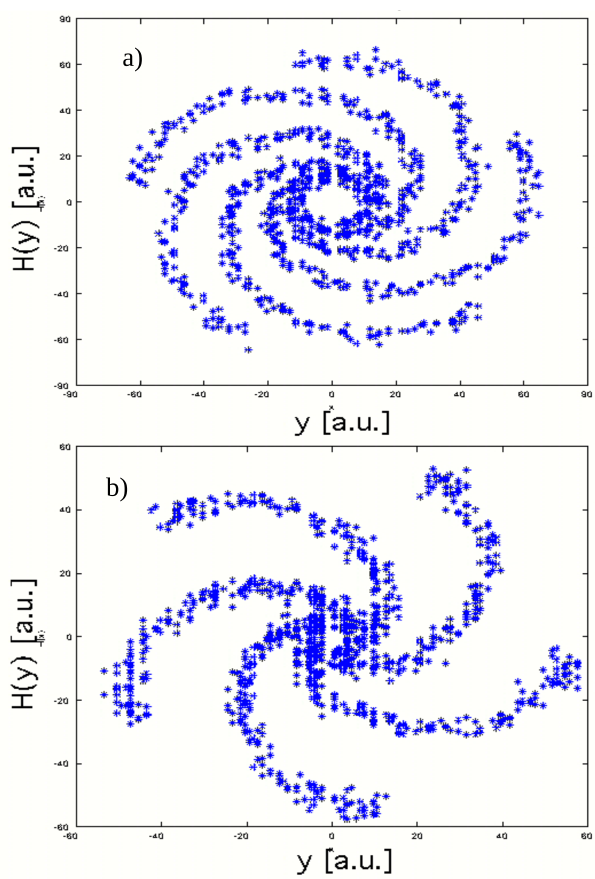

Fig.3a,3b Vertical phase space for ν_y near the 5th order resonance condition $5\nu_y$=41 (1000 turns), with D10 open (a) and closed (b). After the closure of ID10 to the minimum gap, c_{22} is doubled, that is the decay is twice faster. However, the "near resonance" condition still exists and it preserves the 5 branches structure.

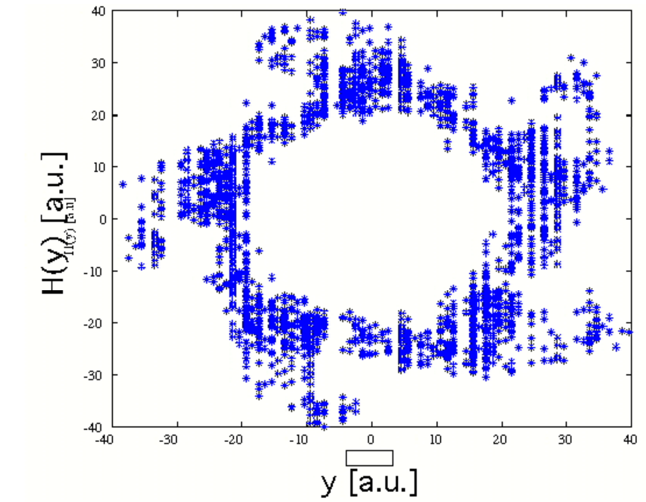

Fig.4 Fifth order resonance $5\nu_y$=41 in the vertical plane (3300 turns). Stochastic motion is around 5 islands, at large amplitudes. The decoherence moves the effective tune far from the "on-resonance" condition and destroys the periodic structure (inner circle).

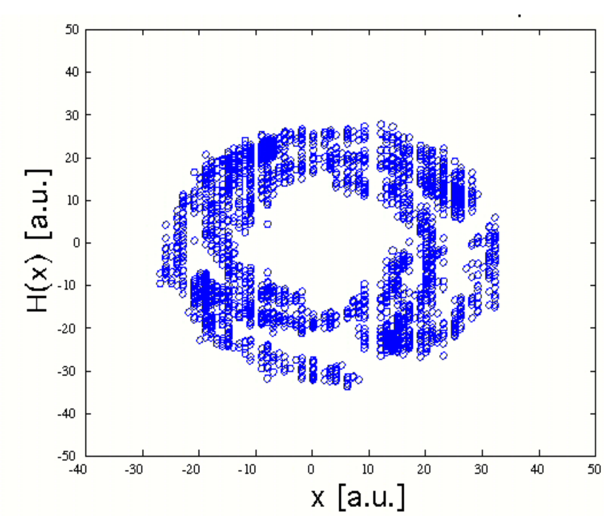

Fig.5 Fourth order resonance $4\nu_x$=57 in the horizontal plane. There is a change in the sense of rotation along the branches as the centroid approximates an unstable fixed point: this is the crossing point of the separatrices, which define the contour of 4 islands.

LIFETIME SCAN VERSUS TUNE

A beam lifetime scan versus the horizontal tune is shown in Fig.6, with ν_y=0.174 for the yellow and blue lines, ν_y=0.210 for the pink one. The beam current is about 300 mA during all the measurements, but drops to few milliAmperes when the lifetime is less than 10 hours.

The scan shows some saw-tooth in the region of high order resonances ($0.20 \le \nu_x \le 0.30$), confirming the influence of the 4th and 5th orders, already observed in phase space. The coupling resonances $\nu_x - \nu_y = 6$ and $2\nu_x + 2\nu_y = 23$ are detected, too. However, the most damaging effects come only from the lowest orders ($n \le 3$).

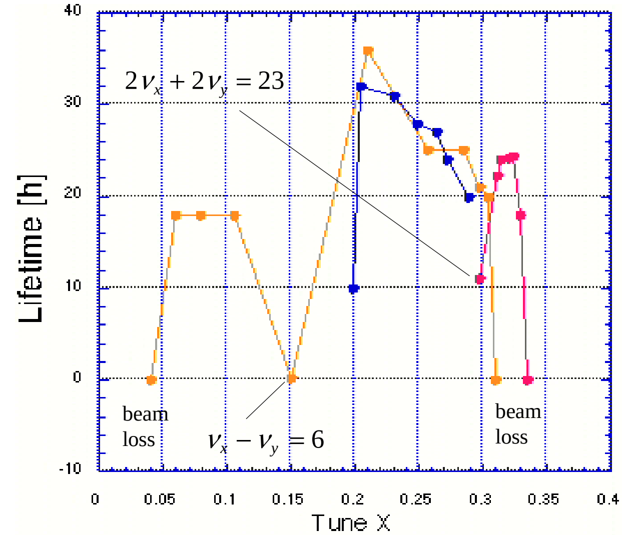

Fig.6 Lifetime scan versus horizontal tune.

CONCLUSIONS

The transverse phase space has been experimentally observed, finding qualitative differences between the "on resonance" and the "out of resonance" condition for the tunes. A semi-quantitative estimation of the nonlinear effect of a Figure-8 undulator on the vertical motion has been provided through the coefficient c_{22} (Eq.1), with a direct impact on the 2nd order tune shift with amplitude (Eq.2).

The observation of islands and fixed points prooves the existence of 4th and 5th order resonances, which generate stochastic motion and reduce beam lifetime.

The resonance pattern results to be rich but not critical for stability and beam losses are due only to the 1st and 3rd order resonances.

Since phase space investigation has been performed only in the inner part of the dynamic aperture, improvements in the diagnostics system could permit a more thorough investigation of stronger nonlinear effects at larger amplitudes.

REFERENCES

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