SPACE CHARGE PROBLEM IN LOW ENERGY SUPER-CONDUCTING LINACS

N.Vasyukhin, R.Maier and Yu.Senichev, FZJ, Juelich, Germany

Abstract

At present the super-conducting option of linear accelerators is considered for low energy, and new types of RF cavities are considered for this purpose. However, together with electrodynamics problems we should solve the transverse stability problem, since in structures with external focusing elements the focusing period is longer, and in higher accelerating field the defocusing factor increases as well. In this paper we consider the transverse stability problem, taking into account the non-linear space charge problem. The fundamental mechanism of halo creation in super-conducting linear accelerators is investigated to minimize the particle losses. The theoretical results are supported by numerical simulation.

INTRODUCTION

Super-conducting linear accelerators have a number of advantages in comparison with room temperature accelerators: high accelerating gradient, low energy losses and high duty cycle. But despite of these benefits there are some problems in high intensity accelerator design.

Firstly, when the focusing elements are located between cryo-modules, the focusing period is longer and the effect of space charge is stronger. Secondly, due to a high accelerating gradient the phase advance in the longitudinal plane can be even twice higher than in the radial plane in the low energy part, and there is the possibility of transverse-longitudinal resonances. Thirdly, the high RF defocusing factor of the accelerating field affects the radial motion, and it should be compensated by quadrupoles. Therefore the working point in the Smith-Gluckstern stability diagram is smeared due to the RF phase dependence. Additionally it goes down along the same phase advance curve and the stable region decreases in size.

In the case of high intensity accelerators the linear detuning of space charge can be as high as 50%. Compensation of linear space charge forces in the center of the bunch can also be maintained by quadrupoles. However compensation of linear detuning by quadrupoles could move halo particles out of the stable region due to nonlinear detuning of these particles. One of the strongest restrictions for current in high intensity accelerators is imposed by fixed loss levels, about 5W/m. Relatively large apertures in super-conducting cavities are permitted, therefore the main part of losses is usually located in quadrupoles where the beta function has its maximum. The aperture of quadrupoles is restricted by the magnetic saturation on the pole tip (~1T) and the gradient required for the focusing. In particular for a gradient of 50T/m the permitted aperture can be estimated as 2cm. The halo particles are the majority of lost particles. Attempts to describe qualitatively halo creation mechanisms have been undertaken, but an analytical solution of the equations in this case is possible only with a significant simplification of the model. Thus there is no general theory of halo creation mechanisms. It is generally accepted that it is a more or less resonant process, in which some particles, because of resonances, increase in amplitude of transverse oscillations.

THEORETICAL MECHANISM OF HALO CREATION FROM THE MECHANICAL POINT OF VIEW

There are two different concepts explaining halo creation: mechanical and thermodynamical. In linear accelerators the acceleration time is much shorter than the relaxation time of the system. Therefore a mechanical concept is more appropriate than a thermodynamical one. The development of this concept was attempted by Yu. Senichev [1]. The point was to describe qualitatively nonlinear parametric resonances due to mismatched RMS size of bunch and forced envelope oscillations in alternative gradient focusing structure. Obviously, this estimation can be done only for “frozen” space charge or only for the initial moment of time in an accelerator, because during acceleration this distribution changes. But these resonance conditions should increase the amplitude of transverse oscillations. To solve the equation analytically the following approximations were taken: axial symmetry, binomial distribution of particles, smooth approximation, as well as number of in approximation for 2D model. Following this idea one can obtain resonant Hamiltonian for particles motion in case of mismatched beam:

$$H_{r,n}(\delta I,\psi)=\left(1-\frac{\beta\tilde{\mu}}{2n}\right)\delta I-\frac{1}{2}R_c^2C_{sc}\sum_{k=2}^{m}c_kA^{2k-4}G_{2k}(k-1)I^{k-2}\left(\delta I\right)^2-$$
$$-\frac{(n-1)}{2}(b_{n0}+nd_{n0})R_cr_mC_{sc}c_nA^{2n-4}I^{n-2}\left(\delta I\right)^2\cos(\psi)$$

where, β - beta function, $\tilde{\mu}$ - the frequency of beta function oscillation due to mismatching, n – the order of resonant term in binomial distribution, m – number of terms in binomial distribution, c_n - coefficient in distribution, C_{sc} - space charge coefficient, A^2 - emmittance, $G_{2n}=\frac{(2n)!}{2^{2n}(n!)^2}$, $d_{n0}=\frac{1}{2^{2n-2}}$, $b_{n0}=\frac{1}{2^{2n-1}}$, r_m - amplitude of mismatching, ψ - slow phase, I – invariant which similar to integral of action, δI - deviation of invariant due to nonlinear terms of space charge.

The first term is linear detuning, while the second term describes the deviation of nonlinear detuning and the third term is the resonant term. The investigation shows that there are conditions for resonance even for frozen space charge at the beginning of the linac. But this is valid only for a qualitative understanding of the phenomenon. In reality the above mentioned approximations can distort the picture especially in the low energy part of a super-conducting linac.

SPECIFIC FEATURES OF SUPER CONDUCTING LINACS AND CODE DESCRIPTION

From the simulation point of view the most important features of super-conducting linacs are:

- High accelerating gradient. In a super-conducting cavity one can achieve very high electric and magnetic fields. Therefore at low energy the velocity gain is high and any field approximation leads to incorrect results. Obviously the accelerating field has a defocusing factor, which depends on the initial phase. This leads to a smearing of the working point in the stability diagram and results in a decrease of stability in the transverse plane, especially in the case of long focusing periods. Besides that, the compensation of space charge linear detuning leads to an additional decreasing of the stable region in the case of a high intensity beam. The particles in the halo, which are affected by non-linear tune shift, are are subject to higher order resonances and can be lost.
- Absence of synchronism. It is desirable to have the super-conducting cavity with constant phase velocity geometry. In contrast to the normal-conducting linac, where the structural velocity is defined by geometry, in the super-conducting linac the equivalent phase velocity is adjusted via phasing of the cavities [2]. Therefore the size of the effective separatrix decreases.
- Relatively long focusing period. In some cases of super-conducting linacs the focusing elements are located between cryomodules only. The transition from the cooled cavities to warm quadrupoles takes some distance. However the space charge effect is proportional to S^2, where S is focusing period. For instance in the COSY injector design [3] the focusing period is about 1,7m.

At low energy these features lead to disagreement with theory, because of adiabatic violation. For instance in the design of a superconducting injector linac suitable for COSY the energy of the proton beam increases from 2.5 MeV up to 7.1 MeV in one period of focusing. In this case it is not correct to assume that the focusing channel is periodical and equation of motion is the equation with periodical coefficients. In practice, numerical simulation is used to determine the relevant parameters. For this purpose a particle-in-cell (PIC) code including space charge forces calculation was developed at FZJ. In this code the equations are integrated by the Runge-Kutta method:

$$\frac{d^2x}{dz^2}=\frac{eE_x(x,y,z,t)}{m_0c^2\beta^2\gamma}+\frac{ecB_y(x,y,z,t)}{m_0c^2\beta\gamma}-\frac{\gamma^2}{\beta}\frac{d\beta}{dz}\frac{dx}{dz}+F_{scx}(x,y,z,t)$$

$$\frac{d^2y}{dz^2}=\frac{eE_y(x,y,z,t)}{m_0c^2\beta^2\gamma}+\frac{ecB_x(x,y,z,t)}{m_0c^2\beta\gamma}-\frac{\gamma^2}{\beta}\frac{d\beta}{dz}\frac{dy}{dz}+F_{scy}(x,y,z,t)$$

$$\frac{d\beta}{dz}=\frac{eE_z(x,y,z,t)}{m_0c^2\gamma^3\beta}+F_{scz}(x,y,z,t)$$

where E and B – electrical and magnetic 3D RF field obtained from CST MicroWave Studio ®. The space charge force is computed in 3D by the FFT method considering a rectangular shape of the conducting walls. Sin-transformation was used in the transverse plane while in longitudinal plane the complex exponential transformation was used. Simulations were completed using 100 000 macroparticles and a 128x128x128-points-mesh for space charge calculation. Gaussian distribution truncated at 3σ was taken as initial distribution. Initial Gaussian distribution provides very optimistic prognoses, but in reality halo was created also in source and RFQ. Also the actual maximum current with a certain level of losses may appear to be much smaller. To get the correct initial particle distribution one should make a simulation right from the source. This is one of the discussed directions of source code modernization.

SIMULATION RESULTS

As an example for a super-conducting accelerator the design of an injector linac suitable for COSY is taken. The focusing period consists of four super-conducting half wave cavities [4] in one cryo-module and a quadrupole doublet. In this design it is possible to accelerate 30mA average current with losses in the order of 5W/m and a duty factor of 100% (neglecting the RF design implications) without significant changes except for quadrupole gradients and initial conditions. In 11 periods the proton beam is accelerated from 2.5MeV up to 50MeV. The first 5 periods operate at 160MHz and the remaining 6 modules operate at 320MHz. Transverse RMS sizes are shown in figure 1.

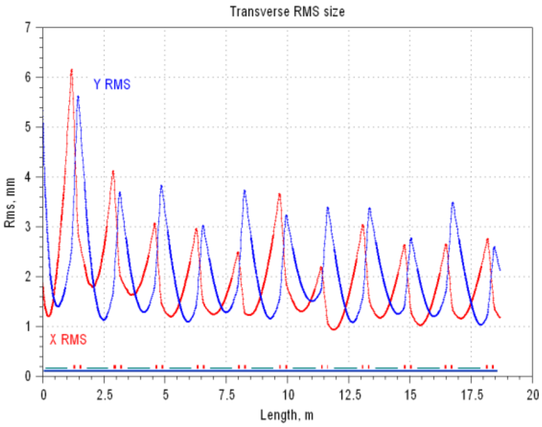

Figure 1: Transverse RMS vs. Linac Length.

In this case it is difficult to make matching because the space charge induced linear tune shift is about 40%. However the particles which are located not in the centre have a different tune shift. Therefore the fixed level of losses was the criterion for optimization. Transverse RMS emittance growth for this case was 32%.

To investigate the importance of the length of the focusing period the simulation has been done for the modified structure with only 2 resonators in one cryo-module and focusing period double shorter. Only the first 20 cavities are considered because most of the problems appear at the beginning. By theory the space charge coefficient is proportional to the squared length of the focusing period. During simulation it was investigated that the maximum current limit raised up to 160mA with the same level of losses. But this is not the only cause of increasing the average current. With the shorter focusing period the parametrical resonance in longitudinal plane is avoided as well. However, from the technical point of view this scenario is more difficult, since the shorter focusing period is possible only with super-conducting quadrupoles or solenoids located inside the cryo-module. Simulation results of this option are shown in figure 2.

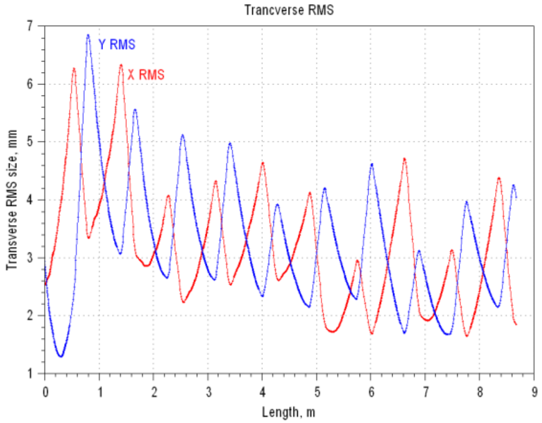

Figure 2: Transverse RMS vs. Linac Length

In order to compare the theory with the numerical model the accelerator at resonant conditions is simulated. The resonance due to mismatching can't be clearly expressed in phase space, because the space charge forces change while the particle distribution changes. In the analytical formula the terms responsible for that should be expressed through the high order term in the particle distribution. Obviously this leads to an insignificant redistribution. Another resonance is the forced resonance caused by the alternate gradient focusing element (q-function). As an example the simulation has been done for the accelerator with resonant conditions (see figure 3), when the phase advance for period is near to $\pi/2$.

It means the lower harmonic of q-function has the frequency by 4 factor higher of eigen frequency, and we see sharply the 4th order resonance. In reality it is desirable to avoid this case.

DISCUSSIONS AND CONCLUSIONS

The high intensity accelerators based on super-conducting structures can be used even for low energy part. However, the specific features have to be taken into account at the design stage. First of all in the case of high current and at low energy it is very important to make 3D matching. The longer focusing period leads to an increase of the space charge coefficient C_{sc}, and this fact causes various resonances. They should be avoided to minimize the problems with the transverse emittance growth.

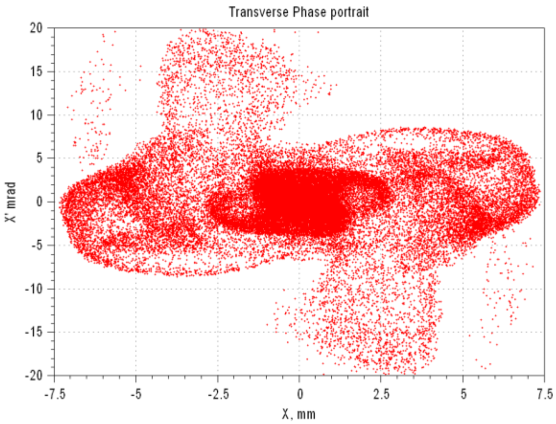

Figure 3: Transverse Phase after linac design with resonant conditions.

There are only few obvious possibilities to rise the maximum current while keeping the same loss level. The first and straightforward way is increasing the average radius of the beam and, hence, decreasing the space charge effect. The second way is more complicated from the technical point of view. It is the shorter focusing period with super-conducting focusing elements inside the cryo-module. Finally, the maximum intensity is much higher for higher initial energy.

ACKNOWLEDGEMENTS

We acknowledge the support of the European Community-Research Infrastructure Activity under the FP6 "Structuring the European Research Area" program (CARE, contract number RII3-CT-2003-506395).

We acknowledge the support of the EPAC organize committee for EPS-IGA student grant.

REFERENCES

[1] Yu. Senichev, Mechanical and thermodynamical approach to halo creation problem in high intense beam, HEACC-98, Dubna, 1998, p. 123-125.

[2] Yu. Senichev, A. Bogdanov, R. Maier, Phys. Rev. ST AB 6, 124001, (2003).

[3] R.Tölle et al., “A SUPERCONDUCTING INJECTOR LINAC FOR COSY” EPAC 02, Paris, 2002, p. 966

[4] http://www.fz-juelich.de/ikp/publications/AR2003/CHAP6/607.pdf