

BEAM DYNAMICS STUDY FOR PETRA III INCLUDING DAMPING WIGGLERS

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Abstract

Damping wigglers will be installed in the storage ring PETRA III to control the beam emittance to 1 nrad. These wigglers will produce linear and non-linear perturbations on beam dynamics. The wiggler fields are derived from numerical field calculations. Halbach equations are used to describe the wiggler field analytically. A new expanded transport matrix method is developed to solve linear dynamics, and used to match linear lattice functions. The symplectic method is adapted to track particle through the whole ring including the damping wigglers. According to presently known field quality, the non-linear effects of damping wigglers will not degrade the performance of PETRA III. The dynamic aperture is still larger than the physical aperture.

INTRODUCTION

DESY will rebuild the PETRA storage ring into a dedicated synchrotron radiation light source [1]. The basic idea is to keep the lattice structure on seven eighths of the PETRA II machine whereas one eighth will be completely reconstructed. This new octant consists of 9 double-bend achromat (DBA) cells with zero dispersion straight sections for the installation of eight 5 m undulators and one 20 m undulator. The main contribution to the horizontal storage ring emittance comes from the seven old octants consisting of FODO cells with a phase advance of 72 deg. Together with the new octant including undulators this leads to a horizontal emittance of 4 nrad. For a further reduction of the emittance down to 1 nrad, it is planned to install damping wigglers in long dispersion free straight sections [2]. A total length of about 80m is available for damping wiggler magnets.

The main beam dynamics issues arise from the fact that the wigglers field is varying strongly in the direction of the beam. Most of the effects will cancel over one wiggler period, while some non-linear particle motion arises from the effect that the particle trajectory oscillates in phase with the main wiggler field. This leads to a decrease of dynamic aperture due to the existence of non-linear motion. Simultaneously vertical linear focusing is created by the fringe fields and is mainly a function of the wiggler field and period length, while horizontal linear focusing is influenced by the horizontal field roll-off and thus the pole width. In this paper the study of linear and non-linear dynamics influenced by the damping wigglers and undulators is described.

WIGGLER FIELD AND HAMILTONIAN

Analytical descriptions of wiggler fields are usually given in form of the so-called Halbach equations in Cartesian coordinates [3],

$$\begin{aligned} B_x &= \sum_i \frac{k_{xi}}{k_{yi}} B_i \sinh k_{xi} x \cdot \sinh k_{yi} y \cdot \cos k_i z \\ B_y &= \sum_i B_i \cosh k_{xi} x \cdot \cosh k_{yi} y \cdot \cos k_i z \\ B_z &= -\sum_i \frac{k}{k_{yi}} B_i \cosh k_{xi} x \cdot \sinh k_{yi} y \cdot \sin k_i z \end{aligned} \quad (1)$$

where $k_{xi}^2 + k_{yi}^2 = k_i^2$, $k_i = 2\pi/\lambda_i$ is the wave number of i^{th} harmonic along longitudinal direction, B_i is its peak magnetic field amplitude. Sometimes it is convenient to define the ratio of transverse gradient $k_{xyi}^2 = k_{xi}^2/k_{yi}^2$ to describe wiggler field. In the design phase magnet field solvers calculate the wiggler fields. Given a calculation of the field at a set of points (x,y,z) , the problem becomes how to find a set of coefficients to reconstruct the field using Halbach equations. This is a standard problem in non-linear optimisation, which can be solved by existing mathematical computer tools. Field calculation results and Halbach fit curves of B_y as a function of x and z are shown in Figure 1. Because of symmetry, only odd harmonics are used for the fit. The RMS difference between calculation and fit result is 0.16%. The main parameters of the damping wigglers are listed in Table 1.

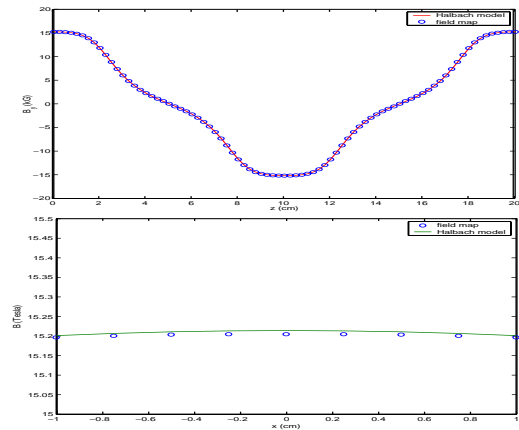


Figure 1: Wiggler field model.

From Halbach equations, a vector potential can be derived to describe this static magnetic field according to Maxwell equation $\nabla \mathbf{A} = \mathbf{B}$. Because there exists one freedom of gauge transformation, one vector potential

component can be chosen arbitrarily. For convenience here A_z is chosen as zero. Then the Hamiltonian of charged particle motion in such a field can be written as

$$H(z) = \frac{1}{2(1+\delta)} \left(x' - \frac{e}{P_0} A_x \right)^2 + \frac{1}{2(1+\delta)} \left(y' - \frac{e}{P_0} A_y \right)^2. \quad (2)$$

Table 1: Damping wiggler field parameters

i	B_i (Tesla)	k_{xyi}^2
1	1.415	0.028
3	0.209	-0.023
5	-0.075	0.011
7	-0.035	0.002
9	0.002	0.017

LINEAR DYNAMICS WITH WIGGLER

In the PETRA III ring, the linear focus, especially in the vertical plane, caused by damping wigglers cannot be ignored. Unlike in other straight elements, there exists non-zero orbit displacement inside the wiggler. It is not very convenient to use this orbit as reference orbit, because it has variable bending radius, and the magnet field becomes complicated in such coordinates.

A new expanded transformation matrix method has been developed to solve the linear dynamics for wigglers in Cartesian coordinates. Usually a 6-dimension vector $\mathbf{X}^T = (x, x', y, y', l, \delta)$ is adopted to describe the particle state in phase space, thus a 6×6 matrix is correspondingly used to describe the transformation of particle in phase space through magnet elements. We form a 7-dimensional state vector $\mathbf{X}^T = (x, x', y, y', l, \delta, 1)$ by adding to \mathbf{X} a 7th component which is always given by unity. We expand the Hamiltonian to second order after substituting the vector potential into the Hamiltonian. The linear term of \mathbf{X} in the Hamiltonian describes bending and orbit displacement in the horizontal plane. Consider a given integration step size from z to $z+dz$, dz is so short that Hamiltonian inside it can be regarded to be independent of z . Using above Hamiltonian, a 7×7 transformation matrix for one step can be obtained.

After concatenating all those matrices piece by piece along one period, the transformation matrix of a complete period is obtained. Because one wiggler period is achromatic, and introduces no orbit displacement, the 7×7 transformation matrix degenerates into a 6×6 one, which can be inserted into an accelerator design program, e.g. MAD for linear lattice calculation.

The validity of the above method has also been verified by particle tracking. Tracking a particle with small initial amplitude through the ring including damping wigglers for certain turns, the non-integer part of the tunes is calculated through spectrum analysis, and agrees with the linear method.

SYMPLECTIC TRACKING METHOD FOR WIGGLER

Tracking particle through the derived field maps can be done in various ways. The direct method is solving the equations of motion with a Runge-Kutta type integration (RK). This method is rather slow and not symplectic, but can be used to verify any other method. The second method is fitting the field map to a set of Halbach equations, which in turn allows symplectic integration (SI) [4] of the Hamiltonian or integration by other methods. Anyway, the above methods require splitting each wiggler period into many pieces. Thus they are relatively slow and time consuming, especially for a ring including long wigglers and undulators, like PETRA III.

In our design, a generating function (GF)[5] in the form

$$F(x_i, xp_f, y_i, yp_f) = \sum_{k+l+m+n=1}^4 x_i^k xp_f^l y_i^m yp_f^n, \text{ where subscript } i \text{ and}$$

f represent the initial and final state of particle passing through wiggler, is built from either form of integration in advance (RK and SI). To build this Taylor map generating function is also time consuming, because numerous particles with different initial states have to be tracked to fit the coefficients, but all those calculation are needed only once. The transformation through the generating function is symplectic, which in turn can simplify the non-symplectic integration, e.g. Runge-Kutta integration. Because the most important components are quadrupole-like and octupole-like multipole, the generating function is truncated into a 4th order power series. Figure 2 gives xp and yp at the end of the damping wiggler as a function of x at the start using the three different tracking methods. Figure 2 shows excellent agreement between RK and SI; the difference of yp between GF and them (RK and SI) is less than 10^{-3} mrad. The GF is chosen for later tracking simulation because of its speed and symplecticity.

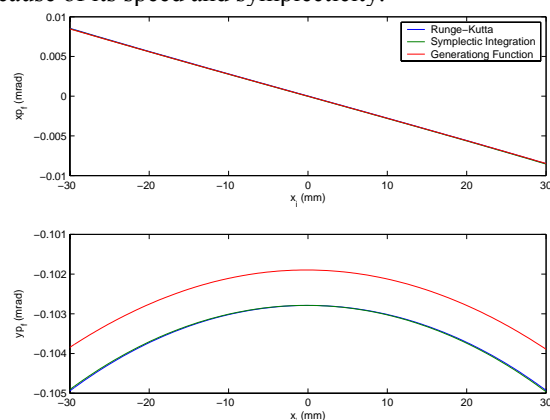


Figure 2: Comparison of different tracking methods.

DYNAMIC APERTURE

For PETRA III, the injected emittance is of the order of 350 nmrad. To reach safely an injection efficiency of close to 100%, an acceptance of 30 mmmrad in the horizontal plane is needed. The required aperture in the

vertical plane should be larger than 2.2 mmmrad, which is limited by the undulator geometric gaps.

The chromaticity correction in PETRA is done with sextupoles, which are located adjacent to the FODO cell quadrupoles [1,2]. Compensation of the chromaticity contribution of the 9 DBA cells within these cells would require large sextupole strengths. The reason is a DBA bending magnet deflection of only 2.5° together with the dispersion section length of 7.6 m, which leads to small dispersion values for all possible sextupole locations. Therefore the chromaticity of the complete ring is corrected only in the seven FODO octants.

The damping wigglers are arranged inside FODO long straight section, eight 5 m long undulators and one 20 m long undulator are available in the DBA octant. In order to minimize the non-linear effects caused by the insertion devices, the β_y functions at those places are designed relatively small, for the reason that the octupole-like component is much larger in the vertical than in the horizontal plane. Figure 3 gives the lattice functions of one 3-cell FODO structure, inside which six damping wigglers are inserted. The linear lattice function calculations and matching include the contribution from wigglers and undulators. The dashed line sections in the beam lines represent damping wigglers or undulators.

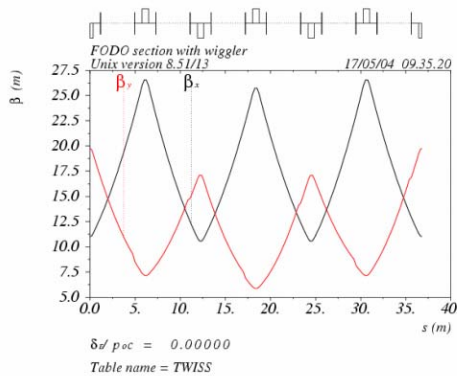


Figure 3: FODO section with 6 damping wigglers.

The dynamic aperture of the lattice without wigglers and undulators has been calculated with the tracking code SIXTRACK [6], which allows for full 6-dimensional particle tracking without radiation damping. To simulate the influence of damping wigglers and undulators on the dynamic aperture, each insertion device is described by their corresponding 4th order Taylor series generating function, which has been obtained in advance. Figure 5 gives an example of the results, in which particles are stable over 2048 turns, which is approximately equal 1/4 damping time in the case without damping wigglers, and 1 damping time with damping wigglers.

Simulation results show that the non-linear effects caused by undulators are also very serious, because their short period length leads to large field gradients. Designing a so-called TME lattice for the 20 m undulator,

where the β_y function is as small as possible, has minimized this effect. Figure 4 gives the lattice functions of a 2-cell TME structure, inside which now two 10 m undulators are inserted.

The dynamic aperture decreases by 50% in the existence of all insertion devices, but it still is large enough for injection (see figure 5).

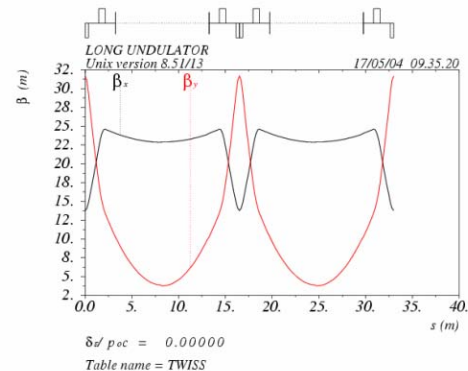


Figure 4: TME section with 2x10m undulators.

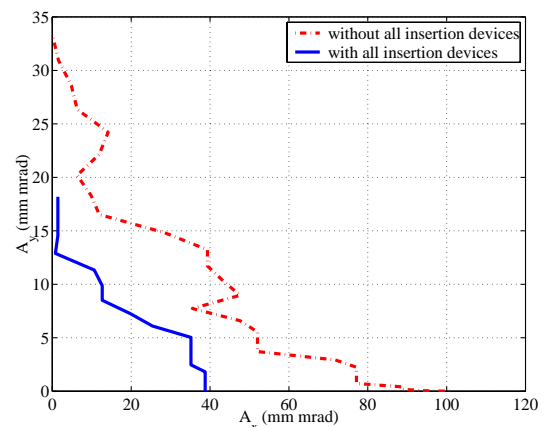


Figure 5: Comparison of dynamic aperture with and without damping wigglers and undulators.

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