# BETATRON RESONANCE STUDIES AT THE CERN PS BOOSTER BY HARMONIC ANALYSIS OF TURN-BY-TURN BEAM POSITION DATA 

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#### Abstract

\section*{Abstract}

High brightness and high intensity beams are required from the PS Booster for LHC, CNGS and ISOLDE operation. The large space charge tune spreads associated with these beams, especially at injection, require an optimised resonance compensation scheme to avoid beam blow-up and subsequent beam losses. For this a detailed knowledge on strength and phase of resonance driving terms is needed. A new measurement system has been installed to determine resonance driving terms from turn-by-turn beam position data using fast Fourier transform. The multi-turn acquisition system as well as the specific measurement conditions at the PS Booster are discussed. As an example, the measurement and compensation of the linear coupling resonance driving term is presented. Excellent agreement between measurement and simulation for resonance phase and strength was found.


## INTRODUCTION

The presently used resonance compensation scheme at the PS Booster (PSB) was established 25 years ago with trial and error measurements by minimising beam losses [1]. In view of the increasing demand of high intensity proton cycles a revisit of the resonance compensation scheme and a resonance analysis was required. For this series of measurements were performed in 2003 to determine resonance driving terms.
Modern methods allow to extract resonance driving terms from turn-by-turn position data of a beam executing coherent oscillations. Perturbation theory [2] and more recently Normal Form techniques [3] have been used to analyse the nonlinear motion of single particles in accelerators. Applying a fast Fourier transform (FFT) to a turn-by-turn complex signal (normalised phase space $\hat{z}+i \hat{z}^{\prime 1}$ ) gives information about the strength and phase of betatron resonances. The frequencies of the spectral lines are the fundamental tunes $Q_{x}$ and $Q_{y}$ and their linear combinations, representing the resonance lines. Amplitudes and phases of these lines are directly related to strength and phase of the corresponding resonance driving terms.

## ACQUISITION SYSTEM

To record transverse turn-by-turn beam position over typically 1000 consecutive turns in the PSB, a new acquisition system was installed [5]. During the shutdown 2002/03 the standard closed orbit pick-ups in sections 5 and 6 of rings 1 and 2 (of the four PSB rings) were equipped

[^0]with new head amplifiers to increase the bandwidth, thus leading to proper beam position signals in both planes. The electrode signals from each of the two pick-ups are amplified and passed to a passive hybrid circuit to match the impedances and to build the horizontal and vertical delta signals. The signals are transfered to the fast digitiser on separate channels. The digitiser simultaneously samples all selected input channels following a trigger coming from the control system. Data were recorded with two 8-bit 4 channel Acqiris digitiser modules. The use of a memory extension of $2 \mathrm{MS} /$ channel enabled data taking over 4 ms at a sampling frequency of $500 \mathrm{MS} / \mathrm{s}$. Hence approximately 2500 turns could be stored at a revolution time $\tau_{\text {rev }} \approx 1.6 \mu s$ at injection. The input signals were further processed with the so called Control and Processing Program ( CaP ) [6]. This program controls the digitiser, provides a graphical user interface and converts the digital data into real beam position.

## MEASUREMENT SET-UP

Two basic measurement conditions are necessary for the analysis of transverse particle motion:

- The oscillation amplitude of the beam has to be reasonably large (some mm).
- The decoherence of the signal has to be avoided.

The former is normally achieved with a kicker, the latter with chromaticity correction in both transverse planes.
In the case of the PSB, no proper kicker is available, hence injection mis-steering had to be used to excite the beam. As a consequence only the first few thousand turns after injection into the machine could be analysed, limiting the studies to injection energy ( 50 MeV ).
The fact that the PSB is equipped with only one sextupole family for chromaticity correction is a further important restriction for the measurement of resonance driving terms. Decoherence of the oscillations in one of the transverse planes is unavoidable. To obtain proper signals, despite these limitations, the following adjustments on a special 50 MeV flat MD-cycle were done:

1. Only one third of the ring was filled to obtain a quasibunched beam containing 1 to $2 \cdot 10^{11}$ protons.
2. In order to conserve the bunch and to avoid longitudinal debunching, the RF was already switched on at injection and the beam was injected into the "waiting" bucket.
3. Injection mis-steering was used to obtain a sufficiently large oscillation amplitude.
4. The chromaticity was adjusted either to zero in one plane or to reasonably low values in both planes when coupling resonances were considered.

## MEASUREMENTS AND RESULTS

The PSB is standardly operated in the tune area, $4.0<Q_{x}<4.3$ and $5.0<Q_{y}<5.6$ [9]. The linear coupling difference resonance $Q_{x}-Q_{y}=-1$ is not compensated. Instead, it is deliberately excited during the multiturn injection to transfer some of the horizontal oscillation to vertical to avoid beam losses. Therefore, the knowledge of the bare machine driving term is of importance to adjust the skew quadrupoles for the emittance exchange properly. This resonance is strong and hence offered a good opportunity to gain experience with the whole acquisition system. For the determination of the linear coupling difference resonance driving term $h_{1001}$ for the bare (uncorrected) machine, the following procedure was applied:

1. The tunes were set close to the resonance condition: $Q_{x} \approx 4.20, Q_{y} \approx 5.14$.
2. The chromaticities were adjusted to be equal ( $Q_{x}^{\prime} \approx Q_{y}^{\prime} \approx-5.3$ ) to obtain reasonably large decoherence times in both planes.
3. Measurements for the bare machine were performed.
4. For calibration reasons, measurements with a defined skew quadrupole were done to deduce the right strength of the bare machine excitation and subsequently the proper compensation currents.
5. To compensate the resonance, two independent skew quadrupoles were used. The resonance phases of these compensation elements were obtained via simulations.
6. New measurements were carried out to confirm the calculated compensation currents.

According to theory [3, 4] strength and phase of the $h_{1001}$ resonance driving term are given by:

$$
\begin{align*}
\left|h_{1001}\right| & =\sqrt{\frac{\operatorname{line}(0 / 1)_{H}}{\text { line }(1 / 0)_{H}} \frac{\operatorname{line}(1 / 0)_{V}}{\operatorname{line}(0 / 1)_{V}}} \cdot \sin (|\hat{\phi}|)  \tag{1}\\
\psi_{1001} & =\phi_{x 1}-\psi_{y_{0}}+\frac{\pi}{2}-\operatorname{sgn}(\hat{\phi})\left(\frac{\pi}{2}-|\hat{\phi}|\right) \tag{2}
\end{align*}
$$

where $\phi_{x 1}$ is the phase of the resonance line, $\psi_{y_{0}}$ the phase of the vertical tune line and $\hat{\phi}$ a measure for the distance to the resonance.

## Bare machine excitation

With the mentioned set-up, measurements for the bare machine excitation were performed in ring 1. Figs. 1 and 2 show the horizontal and vertical Fourier spectra obtained from the transverse beam positions.


Figure 1: Horizontal Fourier spectrum for bare machine.


Figure 2: Vertical Fourier spectrum for bare machine.

The spectra are normalised to the amplitudes of the tune lines (denoted with $(1,0)$ and $(0,1)$ in the horizontal and vertical spectrum respectively). In each Fourier spectra two additional spectral lines are indicated. The spectral line $(0,1)$ in the horizontal spectrum and the line $(1,0)$ in the vertical correspond to the driving term $h_{1001}$. The lines $(0,-1)$ and $(-1,0)$ are related to the linear coupling sum resonance or the $h_{1010}$ driving term.
With the results from the measurements, Eqs. 1 and 2 yield for the bare machine excitation:

- Strength: $\left|h_{1001}\right|=0.71 \pm 0.01 \cdot 10^{-2}$
- Phase: $\psi_{1001}=282.8^{\circ} \pm 5.2^{\circ}$


## Defined excitation

One skew quadrupole family was powered with $I_{Q S K 210 L 3}=35 \mathrm{~A}$ for calibration measurements.

The bare machine contribution was then subtracted from the measurement results, to obtain the resonance phase, which has to agree with simulations [7, 8]. The measurement of resonance strength is influenced by decoherence processes due to amplitude detuning and chromaticity. A comparison of the measured and simulated resonance strength gives then a calibration factor. The obtained results were:

- Measurements: $\left|h_{1001}\right|=9.00 \pm 0.02 \cdot 10^{-2}$, $\psi_{1001}=122.0^{\circ} \pm 1.1^{\circ}$
- Simulation: $\left|h_{1001}\right|=10.0 \cdot 10^{-2}, \psi_{1001}=302.7^{\circ}$

Taking into account the effects of signal decoherence, which are not considered in the simulations, the agreement in resonance strength is excellent. As expected the simulated strength is larger than the measured one. The measured resonance phase is exactly opposite to expectations, indicating an inversed polarity of the magnet. During the shut down period, the polarity was checked and the observations from the measurements were confirmed.

## Compensation of the resonance

With the knowledge of the bare machine excitation, the resonance phases of the compensation elements and the calibration factor, the compensation currents are calculated to:

- $I_{Q S K 210 L 3}=+3.6 \mathrm{~A}$ (including the opposite polarity in the machine)
- $I_{Q S K 614 L 3}=+1.2 \mathrm{~A}$

Figs. 3 and 4 show the horizontal and vertical Fourier spectrum in case of compensation. The resonance strength could be reduced down to $\left|h_{1001}\right|=1.7 \pm 0.1 \cdot 10^{-3}$ or $24 \%$ of the initial bare machine excitation.


Figure 3: Horizontal Fourier spectrum for compensation.

## CONCLUSIONS

It has been shown that the new multi-turn acquisition system in the PS Booster allows efficient determination and


Figure 4: Vertical Fourier spectrum for compensation.
compensation of the linear coupling difference resonance. Excellent agreement between measurements and simulation for a deliberate excitation of skew quadrupoles was found.

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[^0]:    ${ }^{1} \hat{z}$ stands for horizontal ( x ) or vertical (y) phase space respectively

