INTENSITY DEPENDENT EMITTANCE TRANSFER STUDIES AT THE CERN PROTON SYNCHROTRON

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Abstract

An intensive study has been undertaken since the year 2002 to understand better the various high-intensity bottlenecks of the CERN Proton Synchrotron machine. One of these limitations comes from the so-called Montague resonance. High-intensity proton synchrotrons, having larger horizontal than vertical emittance, may suffer from this fourth-order coupling resonance driven by space charge only. In particular, such resonance may lead to emittance sharing and, possibly, beam loss due to vertical acceptance limitation. Experimental observations made in the 2002, 2003 and 2004 runs on the Montague resonance are presented and compared with theoretical predictions and 3D particle-in-cell simulation results.

INTRODUCTION

The intrinsic space-charge driven fourth-order resonance near the diagonal given by $2Q_x - 2Q_y = 0$ has been first studied on a single-particle basis by Montague [1] who pointed out that it can be avoided by sufficient splitting of tunes. In circular accelerators with un-split tunes, as in the CERN Proton Synchrotron (PS), avoidance of this resonance may be desirable to achieve maximum high-intensity performance. Also, the emittance exchange connected with this resonance was recently observed during foil injection into the KEK synchrotron [2].

The measurements performed from 2002 to 2004 are presented in the first three sections, while comparisons with theory and simulations are discussed in the last one.

MEASUREMENTS IN 2002

The first measurements were performed on a single bunch coming from the CERN Proton Synchrotron Booster (PSB), fast injected into the PS machine at 1.4 GeV kinetic energy, on harmonic $h = 8$. The number of protons per bunch was $N_b \approx 10^{12}$ p/b , the vertical tune was fixed at $Q_y = 6.21$, the horizontal tune Q_x was varied between 6.15 and 6.25, the total bunch length (4 σ) was $\tau_b \approx 200$ ns, the normalized momentum spread (1 σ) was $\sigma_p / p \approx 1.4\times10^{-3}$, the synchrotron period was $T_s \approx 1.5$ ms , the initial (un-coupled) horizontal and vertical emittances were $\varepsilon_{x,2\sigma}^{norm} \approx 25$ µm and $\varepsilon_{y,2\sigma}^{norm} \approx 10$ µm respectively. The e.g. horizontal incoherent small-amplitude space-charge tune shift of a transversally Gaussian bunch, is given by $\Delta Q_{inc,x0} = -2 r_p I_p \beta_x R / [e c \beta^3 \gamma^3 a(a+b)]$, where r_p is the classical proton radius, $I_p = 3e N_b / 2\tau_b$ the bunch peak current considering a longitudinal parabolic line density, with e the elementary charge, $\beta_x \approx R/Q_x$ and $\beta_y \approx R/Q_y$ the average horizontal and vertical betatron functions, $R = 100$ m the average machine radius, c the speed of light, β and γ the relativistic velocity and mass factors, $a = \sqrt{2}\,[\varepsilon_x^{\rm rms} \beta_x + (D_x \sigma_p / p)^2]^{1/2}$ and $b = [\,2\,\varepsilon_y^{\rm rms} \beta_y]^{1/2}$, with $\varepsilon_{x,y}^{\rm rms}$ the rms transverse emittances, and $D_x \approx 2.6$ m the average horizontal dispersion function. This yields $\Delta Q_{inc,x0} \approx -0.06$ and $\Delta Q_{inc,y0} \approx -0.107$. For the KV distribution, which is uniform in all the planes, the space-charge tune shift is $\Delta Q_x^{\rm KV} = \Delta Q_{inc,x0} / 2$, as it is given by the same formula, but with a 2 in the definition of a and b instead of $\sqrt{2}$.

The numerical values have been chosen after careful analysis of the tune diagram in order to avoid the non-compensated resonances in the PS machine. In this first experiment, the horizontal tune was changed from 6.25 to 6.15 in static, i.e. constant value from injection to the measurement point. The measurements were performed on different cycles (1 measurement per cycle), at the same time for each cycle, and the data were averaged over several measurements (see the results on Fig. 1).

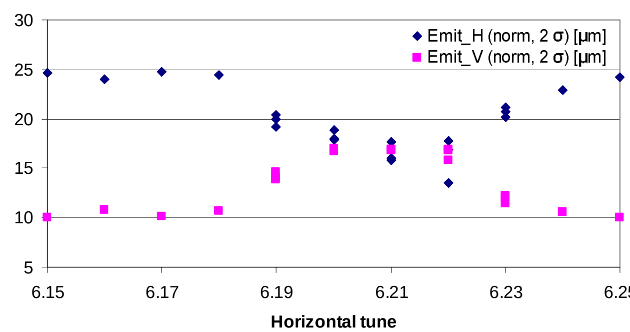

Figure 1: Measured intensity dependent emittance transfer in 2002 in the static case (constant horizontal tune from injection to the measurement point).

MEASUREMENTS IN 2003

The static measurements made in 2002 were repeated in 2003 with a beam slightly bigger in the horizontal plane, $\varepsilon_{x,2\sigma}^{norm} \approx 30$ µm , $\tau_b \approx 180$ ns , and $\sigma_p / p \approx 1.7\times10^{-3}$ (see Fig. 2). The initial emittances were determined by measurements at $Q_x = 6.245$, where exchange is absent. This yields $\Delta Q_{inc,x0} \approx -0.054$ and $\Delta Q_{inc,y0} \approx -0.109$. Other experiments were then performed by changing the horizontal tune dynamically on the injection flat-bottom (see Fig. 3). The horizontal tune was increased linearly from 6.15 to 6.25 in 100 ms. Measurements, both in static and dynamic, were also performed with RF OFF (see Figs. 4 and 5).

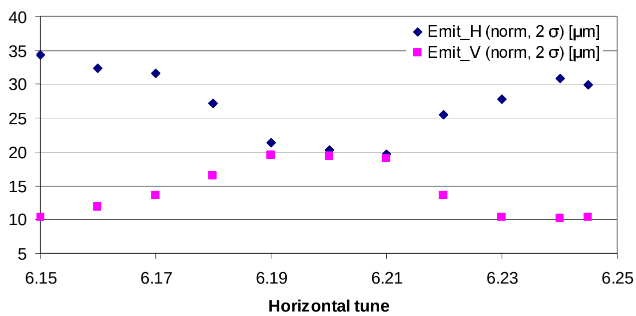

Figure 2: Measured intensity dependent emittance transfer in 2003 in the static case (constant horizontal tune from injection to the measurement point).

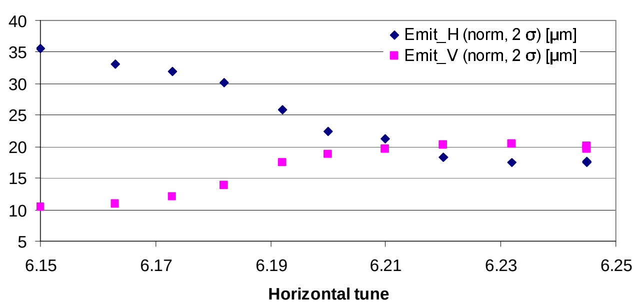

Figure 3: Measured intensity dependent emittance transfer in 2003 in the dynamic case (the horizontal tune was changed linearly from 6.15 to 6.25 in 100 ms).

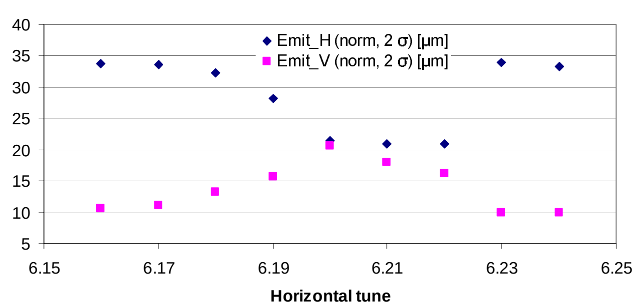

Figure 4: Measured intensity dependent emittance transfer in 2003 in the static case (constant horizontal tune from injection to the measurement point) with RF OFF.

MEASUREMENTS IN 2004

The dynamic measurements made in 2003 were repeated in 2004 with similar beam parameters as in the previous years, $\tau_b \approx 180\,\text{ns}$, and $\sigma_p / p \approx 1.1\times10^{-3}$, but with now larger vertical than horizontal emittance. Both directions for the tune crossing have been investigated. The results of the measurements are shown in Figs. 6 and 7.

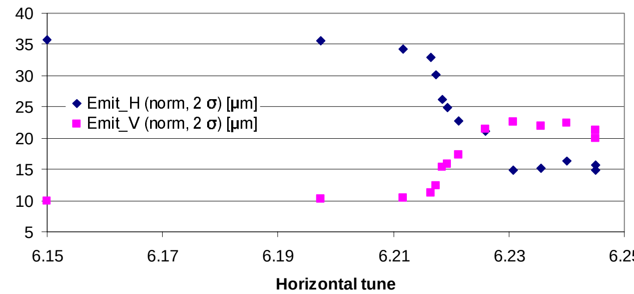

Figure 5: Measured intensity dependent emittance transfer in 2003 in the dynamic case (the horizontal tune was changed linearly from 6.15 to 6.25 in 100 ms) with RF OFF.

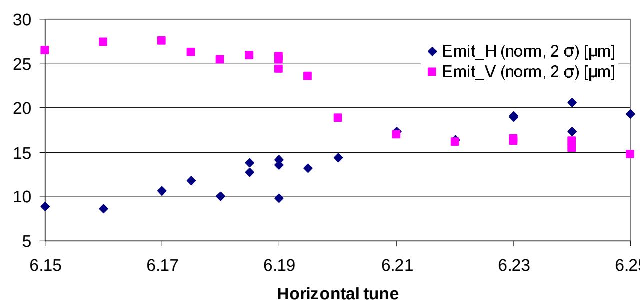

Figure 6: Measured intensity dependent emittance transfer in 2004 in the dynamic case (the horizontal tune was changed linearly from 6.15 to 6.25 in 100 ms).

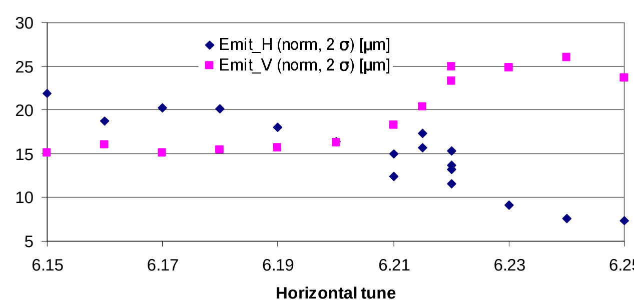

Figure 7: Measured intensity dependent emittance transfer in 2004 in the dynamic case (the horizontal tune was changed linearly from 6.25 to 6.15 in 100 ms).

COMPARISON BETWEEN MEASUREMENTS, THEORY AND SIMULATIONS

Resulting emittance growth factors, normalized to the initial emittances and corresponding to the case of Fig. 2, are shown in Fig. 8. Due to the assumption of fixed tunes and the fast nature of the Montague resonance, the experimental data are not completely independent of the injection process. A dispersion mismatch at PS injection cannot be disentangled from the Montague resonance effect.

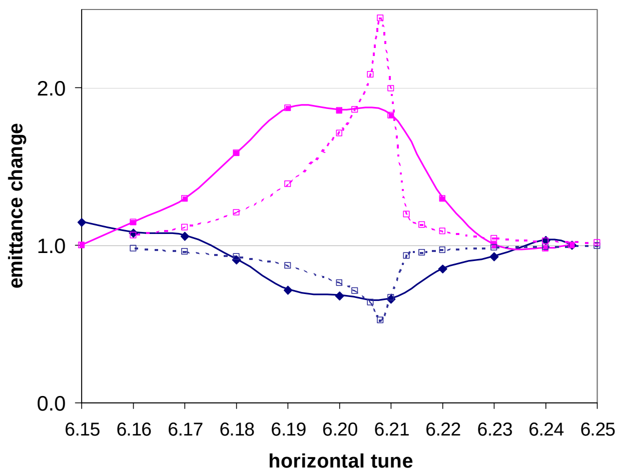

Figure 8: Measured (full line, see Fig. 2) and simulated (dotted line) emittance growth factors for fixed $Q_y = 6.21$ as function of Q_x in static. Vertical: upper (purple) curves; horizontal: lower (blue) curves.

The simulations were performed using the fully 3D particle-in-cell code IMPACT [3] employing a grid of 65×65×257 in *x*, *y*, *z* and 10^6 simulation particles in a constant focusing lattice. Initial distributions are assumed to be Gaussian in all phase planes. The simulations have been run over 1000 turns, which is found sufficient to ensure saturation.

The pronounced asymmetry of the simulation response curve was observed before also in 2D simulations for coasting beams and explained as result of the collective response of the charge distribution [4]. It is noted that for tunes just below 6.21 the emittances shoot over, i.e. the vertical emittance is finally larger than the horizontal one.

The time evolution for the exchange process in the simulation (it is by far too fast to be resolved by the wire-scan measurements limited to ms resolution) is shown in Fig. 9 for the working point $Q_x = 6.208$ in the "overshoot" region. After about 100 turns the emittances have become equalized or "equipartitioned", thereafter a rapid overshoot occurs such that the final vertical emittance is 50% larger than the horizontal one. This leads to the spike in Fig. 8 just below 6.21. We argue that it is related to the spontaneous instability of a skewing (linear coupling) mode as described in Ref. [5]. Obviously, this overshoot and related spike are not confirmed by the experiment.

Using the analytical linearized theory of the fourth order resonance of Ref. [6], the width of the stop-band is to high accuracy proportional to the space-charge tune shift as well as the emittance imbalance; this applies also to the linearized theory maximum growth rate. In Ref. [7] the adequacy of these analytical expressions was fully confirmed by 2D computer simulation. Normalizing the thus obtained widths and rates by $|\Delta Q_x^{\mathrm{KV}}|$, a simple linear relationship for not too large emittance ratio is found, i.e. $\delta Q_{\mathrm{M}} / \Delta Q_x^{\mathrm{KV}} = (\varepsilon_x / \varepsilon_y - 1)\, 2/3$, where δQ_{M} is understood as extent of the stop-band left from the resonant machine tune $Q_x = Q_y$. Applying this formula to the case of Fig. 8 (noting that $\Delta Q_x^{\mathrm{KV}} \approx -0.027$ and $\varepsilon_x / \varepsilon_y \approx 3$), we obtain $\delta Q_{\mathrm{M}} \approx 0.036$, which matches well with the stop-band width of Fig. 2.

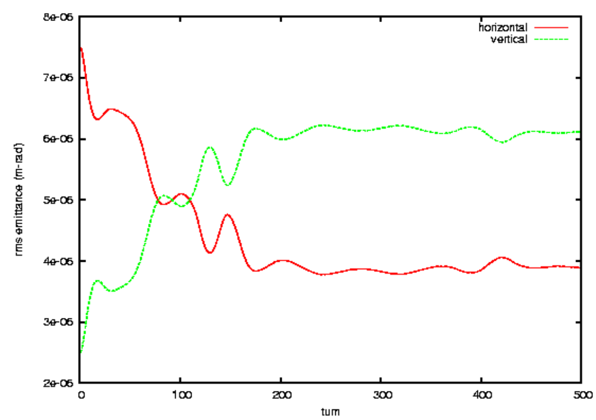

Figure 9: Simulation result for normalized rms emittance evolution for $Q_y = 6.21$ and $Q_x = 6.208$ as function of turns (1 turn in the PS at injection is ~2.3 μs).

Figure 8 reflects a reasonably good agreement between experiment and simulation; in particular the stopband also extends mainly to the left of $Q_x = 6.21$, which would be inverted for the inverse emittance ratio. Simulations for dynamical crossing in bunched beams are in progress. First results are discussed in Ref. [8].

CONCLUSION

The phenomenon of emittance exchange has been found and studied in a series of measurements performed in 2002, 2003, and 2004 in the CERN PS machine, which are confirmed by theoretical predictions and 3D particle-in-cell simulations. Several other studies on the reversibility of the mechanism and on the effect of the tune crossing speed in both directions are ongoing.

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