# SIMULATED EMITTANCE GROWTH DUE TO ELECTRON CLOUD FOR SPS AND LHC

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*Abstract*

The emittance growth caused by an electron cloud is simulated by the HEADTAIL code with conducting boundary conditions. Under some conditions the simulation results may depend on the number of beam-cloud interaction points, the phase advance between them and the number of macroparticles used to represent beam and cloud. Simulations include a transverse feedback system and, optionally, a large chromaticity, as employed in actual SPS operation. Simulation results for the SPS can be compared with observations, and the emittance growth in the LHC is computed as a function of average electron density. An attempt is made to extrapolate to low electron densities. We also compare the initial instability rise times with those obtained for an equivalent broadband resonator.

## INTRODUCTION

Instabilities, beam loss and beam size blow up due to electron cloud are a concern for the future Large Hadron Collider (LHC) at CERN. Simulations of transverse single-bunch instabilities have been performed using the code HEADTAIL [1, 2, 3]. The instability is similar to the regular transverse mode coupling instability (TMCI) and induces both a centroid and an head-tail motion, with a substantial emittance growth.

HEADTAIL is a PIC code which models the interaction of a single bunch with an electron cloud on successive turns, assuming that the cloud is localized at a finite number of positions along the ring, instead of being continuously spread. The Poisson equation defining the transverse electric field could originally be solved only in the open space. Recently, electric conducting boundary conditions (b.c.) have been implemented [4, 5]. The potential is assumed to be zero on the wall and an FFT Poisson Solver for a rectangular pipe is used. In this paper, simulations for LHC and SPS using the new b.c. are shown. We investigate the instability threshold and the emittance growth as a function of chromaticity and electron-cloud density, taking into account that the results may depend on the choice of computing parameters, such as the number of macroparticles and the number and location of the interaction points between the cloud and the beam [5]. In the last section, the possibility to model the electron cloud effect with a broadband impedance [6] is discussed and the results are compared with the PIC simulations.

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## THRESHOLD AND BLOW UP IN LHC

Using the parameters listed in Table 1, we studied the effect of chromaticity and electron cloud density on the development of the instability, for the LHC at injection. The results are obtained assuming 10 Interaction Points (IPs) along the ring, in order to stabilize the behaviour of the simulated growth. In fact, a very irregular dependence on the number of IPs has been seen, if they are less then 5 [5, 7]. Placing IPs at random locations around the ring or concentrating them in over betatron wavelength [8] does not improve the convergence. So a larger number of 10 IPs has been chosen for our simulations, where the result has converged.

Table 1: Parameters used for LHC and SPS

| parameter | LHC | SPS |
|---|---|---|
| cloud density, $\rho_e[\mathrm{m}^{-3}]$ | $6 \times 10^{11}$ | $10^{12}$ |
| bunch population, $N_b$ | $1.1 \times 10^{11}$ | $1.1 \times 10^{11}$ |
| beta function, $\beta_{x,y}[\mathrm{m}]$ | 100 | 40 |
| rms bunch length, $\sigma_z[\mathrm{m}]$ | 0.115 | 0.24 |
| rms beam size, $\sigma_{x,y}[\mathrm{mm}]$ | 0.884 | 0.0021 |
| rms momentum spread, $\delta$ | $4.68 \times 10^{-4}$ | 0.02 |
| synchrotron tune, $Q_s$ | 0.0059 | 0.0059 |
| momentum compact fact, $\alpha_c$ | $3.47 \times 10^{-4}$ | $1.92 \times 10^{-3}$ |
| circumference, $C[\mathrm{m}]$ | 26659 | 6911 |
| nominal tunes, $Q_{x,y}$ | $64.28, 59.31$ | $26.185, 26.13$ |
| chromaticity, $Q'_{x,y}$ | 2, 2 | 4.94, 3.9 |
| space charge | no | - |
| magnetic field | no | yes |
| dispersion, $D$ [m] | 0 | 2.28 |
| relativistic factor, $\gamma$ | 479.6 | 27.728 |
| cavity voltage, $V$ [MV] | 8 | 2 |
| harmonic number, $h$ | 35640 | 4620 |
| # of macro-electrons, NEL | $10^5$ | $10^5$ |
| # of macro-protons, NPR | $3 \times 10^5$ | $3 \times 10^5$ |
| # of slices, NBIN | 70 | 70 |
| # of grid points, N | $128 \times 128$ | $128 \times 128$ |
| size of the grid, $\sigma_g$ | $10\ \sigma_{x,y}$ | $10\ \sigma_{x,y}$ |
| extension of the bunch in z | $\pm 2\ \sigma_z$ | $\pm 2\ \sigma_z$ |
| # of Interaction Points, $nkick$ | 10 | 10 |

We first varied the electron-cloud density in the chamber, from $3 \times 10^{12}$ $\mathrm{m}^{-3}$ down to $2 \times 10^{11}$ $\mathrm{m}^{-3}$. For $\rho_e = 3 \times 10^{11}$ $\mathrm{m}^{-3}$, only a small emittance growth remains (see [5]). This value is consistent with the threshold predicted by the analytical 2-particle model for the TMCI type instability [9], $\rho^{thre} = 2\gamma Q_s/(\pi r_p L\beta)$, which gives $\rho_e = 4 \times 10^{11}$ m$^{-3}$ as a threshold for the fast head-tail instability, for these parameters. Figure 1 shows the emittance growth rise time as a function of electron-cloud density. Daring to extrapolate these 0.1 s simulations to 30 min. operation in LHC at injection conditions, the maximum cloud density for an emittance growth lower than 2.3% is about $3 \times 10^{10}$ m$^{-3}$ which is one order of magnitude below the value permitted by heat-load considerations. But this result has of course to be interpreted with caution and in addition it has been obtained for zero chromaticity.

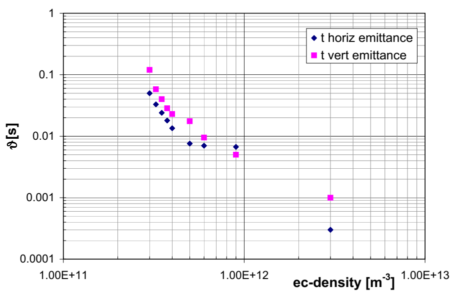


Figure 1: Rise time vs. electron cloud density ($Q' = 2$). $\tau$ is the time during which the emittance increases from $7.82 \times 10^{-9}$ m (initial value) to $8 \times 10^{-9}$ m (+2.3%).

For an electron-cloud density of $6 \times 10^{11}$ m$^{-3}$, increasing the chromaticity helps reducing the blow up (Fig.2), until for high values of $Q' = 30$ we enter into another regime with a slow emittance growth. The threshold value of chromaticity for which the strong head-tail instability is cured depends on the electron cloud density. The relation found in our simulations is almost linear, as predicted by analytical computations for TMCI due to a broadband impedance [10].

It is still to be proven that this slow emittance growth is not an artifact of the code (though similar growth has been seen in some measurements at KEKB [11]). Increasing the number of macroprotons (NPR) helps reducing this linear growth. Figure 3 shows the dependence on $1/\text{NPR}$. The growth does not seem to approach zero in the limit of very large NPR. The dependence on the number of macro- (or beam) particles has also been predicted analytically [12]. Assuming that the observed instability is due to a mechanism similar to the fast beam-ion instability, the (vertical) emittance growth during the time $t$ is:

$$\Delta\epsilon(t) \approx \frac{\tilde{y}^2}{16\pi(t/\tau)^{1/2}\beta} e^{2(t/\tau)^{1/2}} \quad (1)$$

where (for a round proton beam with $\sigma \equiv \sigma_x = \sigma_y$) $\tau = 4\pi\rho_e N_b^{1/2} r_e^{1/2} r_p \sigma_z^{1/2} \beta c/(\gamma 2^{3/2}\sigma)$, depends on the bunch intensity $N_b$, bunch length $\sigma_z$, rms size $\sigma$, average beta function $\beta$. The Schottky-noise amplitude is $\tilde{y} \approx \sigma_y/\sqrt{\text{NPR}}$. The quasi-exponential growth can become linear due to a 'mixing' by synchrotron motion. If the electrons perform $n_{osc}$ oscillations along the bunch the mixing happens after about a time $T_s/(4n_{osc})$. The expected linear growth rate is $\frac{\Delta\epsilon(t)}{\Delta t} \approx \frac{\Delta\epsilon(T_s/(4n_{osc}))}{T_s/(4n_{osc})}$ This gives, for $\text{NPR} = 3 \times 10^5$, a rise time $\Delta\epsilon(t)/\Delta t/\epsilon_0$ of about 0.001 $s^{-1}$, significantly smaller than the simulated growth rate. Further studies are underway to clarify this point.

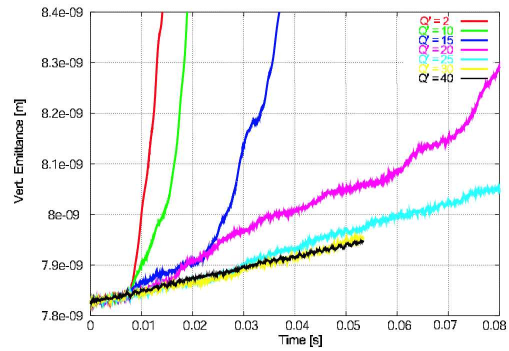


Figure 2: Vertical emittance growth for different chromaticities, $\rho_e = 6 \times 10^{11}$ m$^{-3}$.

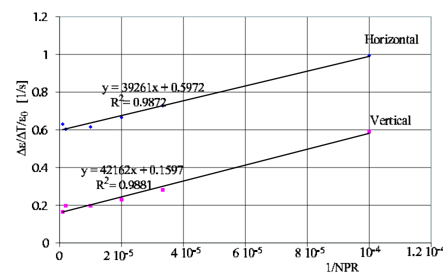


Figure 3: Horizontal and vertical emittance growth rate vs. 1/NPR, in the 'slow term' emittance growth regime, the electron cloud density is $\rho_e = 6 \times 10^{11}$ m$^{-3}$ and the chromaticity is Q'=40.

## HEADTAIL SIMULATION FOR SPS

Simulations have been done for LHC type beam in SPS. The parameters are listed in Tab.1. The aim of these simulations is benchmarking the code with observations. In the SPS, the electron cloud is mainly concentrated in the bending magnets, and so the presence of a constant vertical magnetic field has been assumed, which causes the electron motion to be frozen in the horizontal plane. The feedback system (damping the centroid motion of the bunch) has also been implemented in the code, but it does not help a lot in reducing the single-bunch emittance growth because its main purpose is to cure the coupled-bunch instability and its bandwidth is too low to damp head-tail motion. The scan in chromaticity for $\rho_e = 10^{12}$ m$^{-3}$ (Fig. 4) reveals that increasing the chromaticity only helps up to a certain value of Q'. Including space charge effects in the simulations drastically changes the results, but for a lower level of electron cloud ($\rho_e = 6 \times 10^{11}$ m$^{-3}$) instead, even without space charge, the chromaticity significantly reduces the instability.

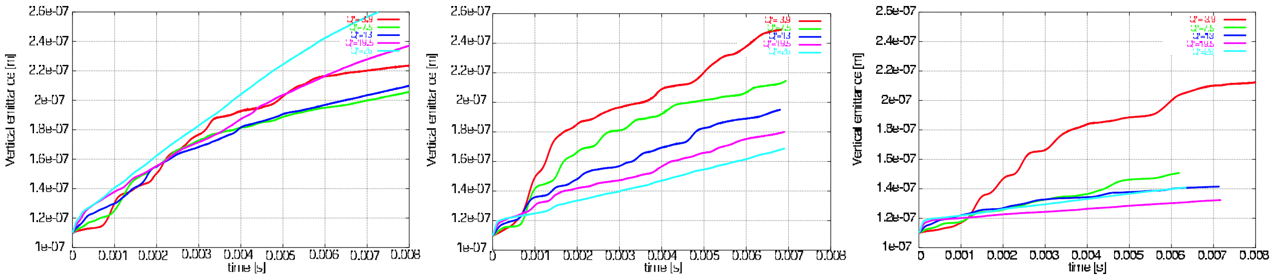


Figure 4: Vertical emittance vs. time for SPS, for different values of chromaticity. Left: $\rho_e = 10^{12}$ m$^{-3}$ without space charge; centre: $\rho_e = 10^{12}$ m$^{-3}$ with space charge; right: $\rho_e = 6 \times 10^{11}$ m$^{-3}$ without space charge.

## BROADBAND IMPEDANCE MODEL

Results obtained by modelling the electron cloud by a resonator [6] with:

$$f_{res} = \frac{1}{2\pi}\sqrt{\frac{2r_e c^2}{2\sigma^2}}\sqrt{\frac{N_b}{\sqrt{2\pi}\sigma_z}}\frac{1}{\sqrt{k}} \quad (2)$$

$$\frac{cR_s}{Q} = H_{emp}\frac{\lambda_c\sqrt{r_e}}{\sigma^3 k^{3/2}\sqrt{\frac{N_b}{\sqrt{2\pi}\sigma_z}}}L \quad (3)$$

have been compared with the PIC simulations in HEADTAIL. The quality factor is assumed to be $Q = 1$, $\lambda_c$ is the cloud line density, $L$ is the ring circonference, $k$ is a coupling parameter and is taken equal to 2, and $H_{emp} = 0.4$ has been obtained empirically by matching against the PIC simulations. In HEADTAIL there is also the possibility to consider the effects of a broadband impedance. So simulations have been performed using the resonator of (2) and (3) and compared with the results obtained using the PIC module. This model gives similar initial growth rates as the full electron-cloud simulation (see Fig.5). For large amplitudes the finite size of the field grid and the non linear force slow down the emittance growth induced by the electron cloud in the case of the PIC calculation.

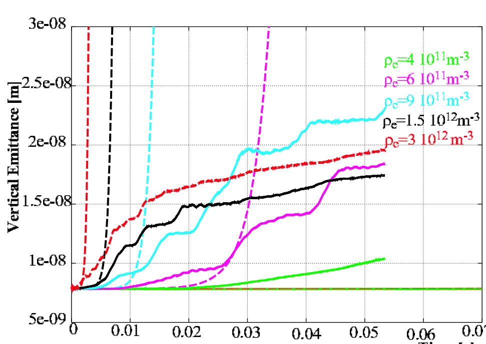


Figure 5: Emittance growth in LHC at injection: resonator model (dotted line) and HEADTAIL PIC module (full line), for different cloud densities.

## CONCLUSIONS

The code HEADTAIL with the new boundary condition of a perfect electric conductor has been used to simulate single-bunch instabilities and emittance growth due to electron cloud. Simulations for LHC at injection show that chromaticity is a cure for the strong head-tail instability, but that it may not be efficient for suppressing a slow, long-term emittance growth. The question is still open whether this incoherent growth is real or is an artifact. By increasing the number of macroprotons, the rise is slower, but it does not go to zero in the limit of infinite NPR. With zero chromaticity and the nominal bunch intensity, an electron density lower then $3 \times 10^{11}$ m$^{-3}$ must be achieved to reduce considerably the blow up. Below this value there is still some emittance growth and we tried to extrapolate the results from $0.1s$ to 30 min. operation in the LHC at injection. The dependence on chromaticity has also been studied for the SPS and here the space-charge effect plays some role. The resonator model for the electron cloud and the PIC simulation seem to agree at the onset of the instability; later the nonlinear effects and the finite size of the cloud and of the grid used for the PIC computation become important leading to a different behaviour at large amplitudes.

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## REFERENCES


[1] G. Rumolo and F. Zimmermann, SL-Note-2002-036 (2002)

[2] G. Rumolo, F. Zimmermann, Proceedings of Chamonix XI, CERN-SL-2000-07 (DI)

[3] G. Rumolo, F. Zimmermann, Proc. PAC'2001 Chicago and CERN-SL-2001-041 (AP) (2001)

[4] E. Benedetto, G. Rumolo, F. Zimmermann, "HEADTAIL journal: updates and modifications", to be published

[5] E. Benedetto, G. Rumolo, D. Schulte, F. Zimmermann, Proc. ECLOUD'04 Napa (2004)

[6] K. Ohmi, F. Zimmermann, E. Perevedentsev, Phys.Rev. E 65, 016502 (2002)

[7] E. Benedetto et al., Proc. PAC'03 Portland (2003)

[8] A.Chao et al., private communication, May'04 (2004)

[9] K. Ohmi, F. Zimmermann, Phys.Rev.Lett. 85, 3831 (2000)

[10] E. Metral, Proc. EPAC'02, CERN/PS 2002-022 (AE) (2002)

[11] H. Fukuma, Proc. ECLOUD'04 Napa (2004)

[12] F. Zimmermann, CERN-SL-Note-2000-004(AP) (2000)