NONLINEAR BEAM DYNAMICS STUDY WITH MATLAB

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Abstract

The importance of nonlinear beam dynamics effects in synchrotron light sources is caused by strong demands on beam quality to obtain long lifetime, small emittance and stability. In this paper, the non-linear particle trajectory equation modifications are given, which simplify the single particle tracking simulation in storage ring using the Matlab environment [1]. The radiation damping times in electron storage rings are of the order of few ms ($10^2 - 10^4$ turns) and the beam short-term stability is determinant. Therefore the tracking method symplecticity condition for the electron machines is not as important as in hadron machines. Applying recently introduced modern tools for post process analyzing, such as interpolated FFT, early indicators for determination of the onset of chaotic behavior etc, the dynamic aperture, amplitude dependent tunes, phase space distortions, nonlinear resonances and other beam dynamics aspects in the ring can be evaluated. The developed code is applied for nonlinear beam dynamics study in CANDLE [2] storage ring.

1 INTRODUCTION

Particle tracking simulation remains the basic tool to study the single and collective phenomena in accelerators that allow a very explicit and visual analysis of obtained results. Its drawbacks are computational expense and difficulty of interpreting the results. Tracking of single particles by symplectic numerical integration of particle trajectory equations provides direct information on the orbit stability

Number of powerful computer codes like COSY INFINITY [3], MARYLIE [4] and LEGO [5] are developed based on the sophisticated methods of differential or Lie algebra technique. These codes are indispensable in design and study of large hadron machines.

In the meantime, during the last decade Matlab environment becomes widely used at many laboratories for accelerator optics and orbit calculations and control [6,7]. This is caused by the favourable Matlab environment for the matrix simulations and the exhaustive set of library routines in mathematics, statistics, optimisation and graphical outputs. An important Matlab feature is also its support of external routines written in C/C++. In addition, Matlab provides an on-line modification of the optical lattice performance.

In this paper, a simple approach to the non-linear beam dynamics study based on direct integration method using commercially available integrator is presented. The equation of particle trajectory in natural system of coordinates is solved by direct integration method using

Matlab ODE23 solvers.

For the CANDLE storage ring lattice it takes about 50 minutes of CPU time for 2048 turn tracking over the ring on the AMD Athlon XP 2800 processor. Using Matlab compiler to C++ the CPU time of the simulation can be reduced by factor of ten.

2 PHYSICAL MODEL

The non-linear beam dynamics study is based on the Euler-Lagrange equations for the particle trajectory in coordinates (x, y, s) related to the designed orbit:

$$x'' = h_0^2 x + h_0 + \frac{h_0' x x' + 2 h_0 x'^2}{1 + h_0 x} + \frac{e}{pc}\sqrt{1 + \frac{x'^2 + y'^2}{(1 + h_0 x)^2}} \times$$

$$\{x' y' B_x - [(1 + h_0 x)^2 + x'^2] B_y + (1 + h_0 x) y' B_s\};$$

(1)

$$y'' = \frac{h_0' x y' + 2 h_0 x' y'}{1 + h_0 x} + \frac{e}{pc}\sqrt{1 + \frac{x'^2 + y'^2}{(1 + h_0 x)^2}} *$$

$$\{[(1 + h_0 x)^2 + y'^2] B_x - x' y' B_y - (1 + h_0 x) x' B_s\};$$

where h_0 is the designed orbit curvature, x, y are the horizontal and vertical coordinates, s is the distance along the designed orbit and B is the magnetic field. The independent variable is s and the primes denote the derivations with respect to s. Note, that as an alternative way of presentation the Hamiltonian formalism can be used. Magnetic field components in (1) can be either expressed through fitting function obtained from the magnets field measurements or by using an analytic expression for the magnetic field expansion. Note, that the inclusion of high order non-linear terms in these equations is not time consuming from numerical integration point of view.

The high order terms in right hand side of equation (1) describe the curvature effects (third term for x-motion and first term for y-motion) and the kinematic corrections (square root terms). Such a presentation allows the study of the processes where conventional symplectic tracking via kick approximations is not possible because the Hamiltonian cannot be decomposed into two explicitly solvable parts, as in all split-operator approaches.

To study the nonlinear chromaticity and kinematic effects the non-linear terms in equation of motion are expanded into Taylor series with keeping the linear terms in expansion.

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3 MATLAB ENVIRONMENT

3.1 Solver Description

As it mentioned above, the equation of motion (1) is solved by direct integration method using Matlab ODE23s solver [8] that is based on modified Rosenbrock method adopted for stiff systems. For moderate stiff problems one can apply ODE23tb that is based on trapezoidal rule of numerical integration and belongs to family of symplectic integrators [9].

To start, one has to introduce conjugated momenta $x' = p_x, y' = p_y$ and present (1) as a system of four first order differential equations for the vector $z = [x, p_x, y, p_y]$:

$$z' = F(z, s) \qquad (2)$$

where functions $F(z, s) = (F_1, F_2, F_3, F_4)$ describe the right hand side of equation (1).

The Matlab environment allows the numerical solution of (2) by using built-in, or external solvers for the system of ordinary differential equations (ODE's).

The calling syntax for the problem can be written as

```
tspan=[s0 sf]; % integration interval;
z0=[x0;px0;y0;py0]; % column vector of initial conditions;
options=[];
[s,z]=ode23s('Lattice',tspan,z0,options);
```

The string '*Lattice*' contains a lattice description and can be written as a Matlab M-file that operates as a Matlab built-in function:

```
function dz = Lattice(s,z);
dz = zeros(4,1); % preallocate column vector dz;
dz = [F1;F2;F3:F4];
```

The last release of Matlab 6 provides Symbolic Math Toolbox that allows one using differentiation and other operations to present lattice description in very compact form. For the more complicated analytical tasks Matlab provides access to Maple [10] powerful kernel of symbolic algebra system with its vast library of built-in functions and operations.

The ODE23 solvers contain as a routine the option '*numjac*' that allows an automatic numerical calculation of Jacobian matrix for the integrator, which we use for verification of symplectic condition (up to order of simulation precision) of integration method. In order to enhance the reliability and efficiency of integration process, one can introduce the Jacobian matrix by analytical way via solver property '*Jacobian*':

```
options = odeset('Jacobian',@J);
function dFdz = J(s,z)
```

$$dFdz = [\partial F_1/\partial z_1 \ \partial F_1/\partial z_2 \ \partial F_1/\partial z_3 \ \partial F_1/\partial z_4$$
$$\partial F_2/\partial z_1 \ \partial F_2/\partial z_2 \ \partial F_2/\partial z_3 \ \partial F_2/\partial z_4$$
$$\partial F_3/\partial z_1 \ \partial F_3/\partial z_2 \ \partial F_3/\partial z_3 \ \partial F_3/\partial z_4$$
$$\partial F_4/\partial z_1 \ \partial F_4/\partial z_2 \ \partial F_4/\partial z_3 \ \partial F_4/\partial z_4];$$

Similarly, putting '*Events*' command in options argument one can check the position along the orbit where some function of trajectory variables reach to maximum, minimum or passes through zero. This property has been used to obtain survival plots for particle loss on vacuum chamber walls.

Another very useful feature of ODE23 solvers is the possibility to control the simulation accuracy by appropriate choice of absolute and relative tolerances via '*AbsTol*' and '*RelTol*' options, respectively. Finally, one can use a graphical and visualization tools available in Matlab to obtain the necessary plots and diagrams, animations and movies of the processes.

In some cases, the usage of Matlab '*vectorized*' algorithms instead of '*for... end*' loops can be very effective in execution fastening. The comparatively slow computation speed is the only disadvantage of the proposed approach, which can be overcome by using modern high-speed processors and/or parallel computing technique.

3.2 Post-processing Tools

In case of non-linear motion one can determine the tunes of given particle from particle tracking data $Z(n) = \{Z(1), Z(2), ..., Z(N)\}$. For this purpose we are using an interpolation of the FFT plus Hanning filter of order two (see [11] for details). In this case the Fourier coefficients of the orbit are read as

$$\phi(\nu_j) = \frac{1}{N} \sum_{n=1}^{N} Z(n)\chi(n)\exp(-i2\pi n\nu_j) \qquad (3)$$

where $\nu_j = \dfrac{j}{N}$ and Hanning window filter

$\chi(n) = 2\sin^2\left(\dfrac{\pi n}{N}\right)$ is used as a weight function. The

tunes then are calculated as an abscissa of the interpolating function maximum. This method provides the precision of the order of $1/N^4$ for regular trajectories. This technique allows more precise tune determination using particle tracking data up to tens of turns over the ring and construct tune footprint, survival plots etc. Matlab can handle this operation just through one simple command '*fft*' or '*ifft*' that fulfil fast Fourier transformation operations.

We used the Lyapunov exponent approach to define the regular trajectories region as a dynamic aperture.

4 NUMERICAL RESULTS

In this section the numerical results for the 3 GeV CANDLE synchrotron light source project are presented. The CANDLE storage ring consists of 16 superperiod of DBA lattice. The main characteristics of ring design are listed in Table I.

TABLE 1. CANDLE storage ring parameters.

Energy (GeV)	3
Number of periods	16
Bending radius (m)	7.485
Tunes Q_x, Q_y	13.22/4.26
β_x, β_y, η in straight (m)	7.9/4.8/0.18
Horiz. Emittance (nm)	8.4
Dynamic aperture (mm), $\pm 3\%$	15

The corresponding working point for the betatron tunes in linear approximation is (13.22; 4.26). In [12] we have reported the results of fringe field induced tune shifts, phase space distortion, excitation of high order resonances using Matlab based code. Particularly, it was found that in order to reach design values of tunes, the strengths of quadrupole magnets have to be increased by 0.39% for the 0.5m length central focusing quadrupole, by 0.67% for the 0.38m length focusing quadrupole and by 1.48% for the 0.16m length defocusing quadrupole.

As an example, Fig.1 presents the tunes footprint for the particle motion in CANDLE storage ring. The simulations are based on the frequency map analysis method [13].
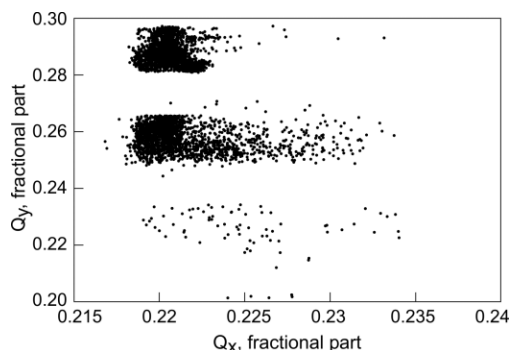

Figure 1.The frequency map of CANDLE ideal optics.

The study is performed for the tracking of on-momentum particles in perfectly aligned lattice (16-fold symmetry). Integration has been performed over 32 turns neglecting the particle radiation damping effects.

The initial condition at the centre of straight section is the grid of 2000 particle transverse positions with zero slopes that covering the physical aperture of vacuum chamber: x=(-7x7) mm and y=(-3x3) mm. The 90% particles with the initial coordinates within the rectangular limits x=(-3x3)mm and y=(-1x1)mm have the tunes around the design tunes value (13.22;4.26). Particles with the large amplitudes beyond this rectangle are entering the tune region around the tune value (13.22; 4.29). One can

identify the empty spaces between the stability regions on the tune footprint with the resonance lines (1; -5) and (2; -9), respectively.

5 CONCLUSION

We have successfully applied Matlab based computer code for nonlinear beam dynamics study in CANDLE storage ring. The proposed approach allows simple and straightforward calculation of complicated beam dynamics effects without any simplification of trajectory equations and magnetic field distribution over the lattice. An application of the code for the real machine with misalignments is in progress.

Authors hope that the proposed approach after some improvements to speed up simulations process can be integrated with other Matlab based accelerator codes and can serve as an essential tool to study various linear and non-linear beam dynamics processes in accelerators.

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