

# MEASUREMENT OF THE VERTICAL QUADROPOLAR TUNE SHIFT IN THE PHOTON FACTORY STORAGE RING

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## Abstract

To make an advanced study on the collective beam behavior, we measured the vertical quadrupolar tunes in the 2.5-GeV Photon Factory (PF) storage ring at KEK. We observed that the vertical quadrupolar tunes decreased as the bunch current. This contrasted to our previous measurement for the horizontal quadrupolar tunes, which showed a positive tune slope. The above results can be explained by an effect of the quadrupolar short-range wakefields which were induced in asymmetric structures.

## INTRODUCTION

Betatron tune shifts (i.e. dependence of the betatron tunes on the bunch current) for dipolar oscillations have been measured in many storage rings [1]. These tune shifts give us information on the transverse wake forces due to beam-line components. We expect that additional information may be obtained by measuring the tune shifts for the quadrupolar oscillations since such oscillations cannot be affected by the dipolar wakefields.

In a previous paper [2], we reported a measurement of the horizontal quadrupolar tune shift, which showed a positive tune slope as the bunch current. Another preliminary measurement for the vertical quadrupolar oscillations suggested a negative tune slope. This paper reports our further measurement of the vertical quadrupolar tunes, which is more accurate than the previous one. An explanation for the measurement results is also presented.

## EXPERIMENT

### Method

The measurement was carried out under single-bunch operation of the PF electron storage ring. Storage ring parameters under the measurement are given in Table 1. The setup for the measurement is shown in Fig. 1. To

excite the vertical quadrupolar oscillations, we applied an oscillating quadrupole field to the beam at close frequencies to a frequency of  $(3+2\Delta\nu_y)f_r$ , where  $\Delta\nu_y$  is the fractional vertical tune and  $f_r$  is the revolution frequency. An applied tune modulation was approximately  $7.4 \times 10^{-5}$  (peak) which gave a growth rate of about  $370 \text{ s}^{-1}$  for the quadrupolar oscillations. To detect the quadrupolar oscillations, we focused an optical beam image on a slit, and detected central part of the light using a photo multiplier. Then, the quadrupolar oscillations could be detected as an intensity modulation of the visible synchrotron light.

To avoid considerable noise due to the excitation field, we detected the signal at close frequencies to  $(2+2\Delta\nu_y)f_r$ , which was lower by the revolution frequency. During the measurement, we checked using a streak camera that the quadrupolar oscillations were excited.

Table 1: Storage ring parameters under the measurement

Beam energy	$E$	2.5 GeV
RF frequency	$f_{rf}$	500.09942 MHz
Revolution frequency	$f_r$	1.60288 MHz
Horizontal tune*	$\nu_x$	9.5988
Vertical tune*	$\nu_y$	4.2892
Synchrotron tune	$\nu_s$	0.015
Natural r.m.s. bunch length	$\sigma_z$	10 mm
Transverse radiation damping time		7.8 ms

\* At low currents.

### Result

The frequency responses of the quadrupolar oscillations were measured using a tracking generator under different bunch currents. We noticed that the amplitudes of the vertical quadrupolar oscillations fluctuated even when we fixed the excitation frequency. This resulted in a slightly different response curve in every sweep of the tracking measurement. Then, we recorded ten traces at every

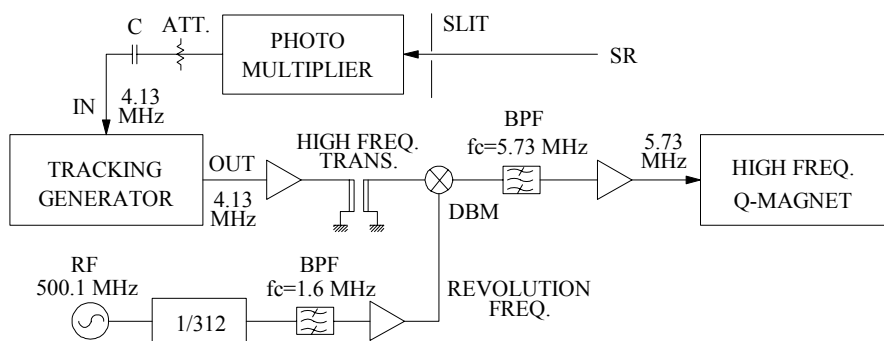


Figure 1: Setup for measuring the vertical quadrupolar tunes.

current, and estimated the uncertainty of the measurement from the scattering of the measured responses.

The result of the measurements is shown in Fig. 2. Each of the nine traces in Fig. 2 indicates a superposition of ten sweeps which were measured at the same bunch current. The traces for different currents are separated by adding an offset. Every peak in each trace in Fig. 2 can be interpreted as the frequency of the coherent vertical quadrupolar oscillation. It can be seen that the peak frequency shifted down as the bunch current.

Figure 3 shows the fractional vertical quadrupolar tunes as a function of the bunch current. The error bars indicate the uncertainty in the measurement due to the above mentioned fluctuations. Fitting the data below 35 mA, the vertical quadrupolar tune shift of  $-0.076 \text{ A}^{-1}$  was obtained.

At the same time to the above measurement, the tune shifts for the dipolar oscillations were also measured by an rf knockout method. The results for the measurement are shown in Figs. 4 and 5. Fitting the data below 40 mA, we obtained the dipolar tune shifts of  $-0.11 \text{ A}^{-1}$  (vertical) and  $-0.013 \text{ A}^{-1}$  (horizontal), respectively. Thus obtained tune shifts, as well as the horizontal quadrupolar tune shift obtained in our previous measurement [2], are summarized in Table 1.

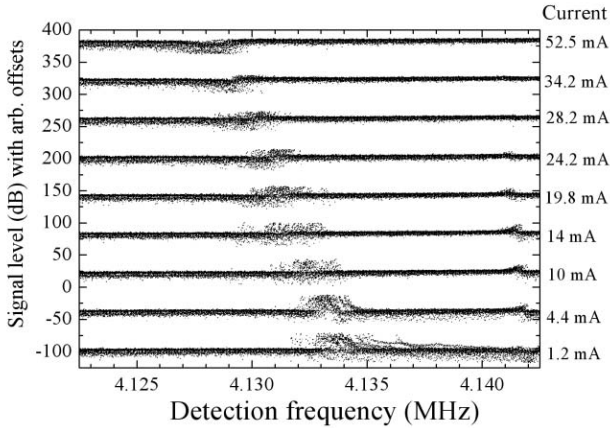


Figure 2: Responses of the vertical quadrupolar oscillations at different bunch currents. Measured ten traces at every current were superposed.

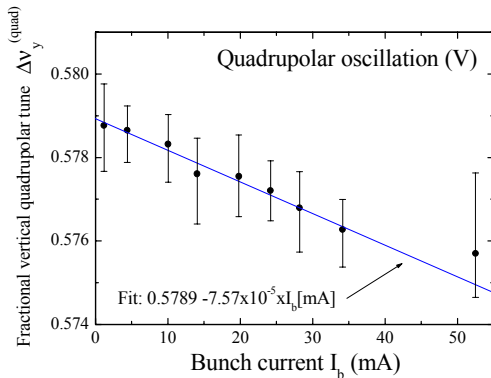


Figure 3: Measured fractional quadrupolar tunes in the vertical direction as a function of the beam current per bunch.

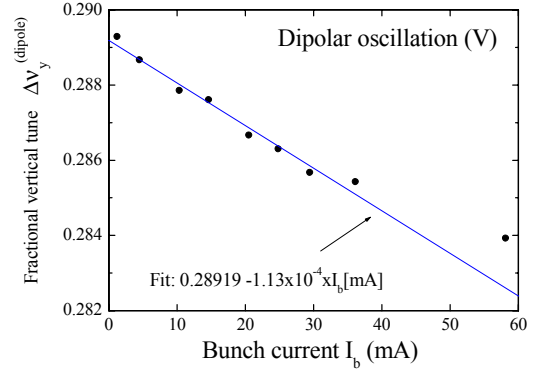


Figure 4: Measured fractional vertical tunes for the dipolar oscillations.

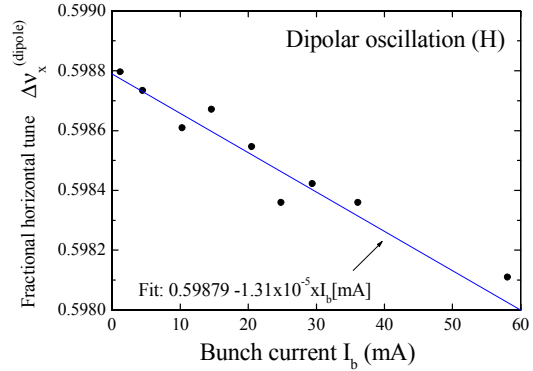


Figure 5: Measured fractional horizontal tunes for the dipolar oscillations.

Table 1: Summary of the measured tune shifts under the single bunch operation.

	Dipolar	Quadrupolar
Horizontal	$-0.013 \text{ A}^{-1}$	$0.048 \text{ A}^{-1}$ [2]
Vertical	$-0.11 \text{ A}^{-1}$	$-0.076 \text{ A}^{-1}$

## DISCUSSIONS

### Simple Formalism

When the structures of the beam line are not axially symmetric, the transverse wake potentials due to such structures can be represented by a superposition of dipolar, quadrupolar and higher-order components [3]. It has been pointed out [3-6] that the quadrupolar component significantly affects the motion of particles as well as the dipolar component.

Let us consider the vertical ( $y$ ) motion of a single particle under the wake forces. The transverse wake potential  $\mathbf{W}_\perp$ , which is induced by an imaginary bunch having infinitesimal transverse sizes, is defined by

$$\mathbf{W}_\perp(\mathbf{r}_b, \mathbf{r}, z) = \frac{1}{q_b} \int ds \cdot \left[ \mathbf{E}\left(\mathbf{r}_b, \mathbf{r}, s, t = \frac{s-z}{c}\right) + \mathbf{v} \times \mathbf{B}\left(\mathbf{r}_b, \mathbf{r}, s, t = \frac{s-z}{c}\right) \right]_\perp,$$

where  $\mathbf{r}_b$  is the transverse offset of the thin driving bunch,  $\mathbf{r}$  is the transverse position of a test particle,  $z$  is the relative longitudinal position of the test particle, and  $q_b$  is the bunch charge. For a structure having the symmetry about the horizontal and vertical planes, we can

approximate that the vertical component  $W_y$  depends on the vertical positions  $(y_b, y)$  and  $z$  [3].

To describe the vertical motion of the particle among the bunch having finite transverse sizes, we approximate the wake force by its average over the longitudinal particle distribution. The equation of motion is given by

$$\frac{d^2 y}{ds^2} + K_y(s) \cdot y = \frac{q_b e}{E} \delta_p(s - s_1) \int \lambda(z) dz \int W_y(y_b, y, z) f(y_b) dy_b,$$

where  $K_y$  is the focusing force,  $e$  is the electron charge,  $\delta_p(s)$  is the periodic delta function with a period of the ring circumference,  $\lambda(z)$  and  $f(y_b)$  are the normalized particle distributions in the longitudinal and the vertical directions, respectively. We have assumed that there is a single wake source at a location of  $s_1$ . Expanding the  $W_y$  by its Taylor series, we can approximate the equation by

$$\frac{d^2 y}{ds^2} + K_y(s) \cdot y = \frac{q_b e}{E} \delta_p(s - s_1) [k_y^{(1)} \cdot \langle y \rangle + k_y^{(2)} \cdot y],$$

where  $\langle y \rangle$  denotes the vertical position of the bunch center, and  $k_y^{(1)}$  and  $k_y^{(2)}$  are defined by

$$k_y^{(1)} = \int \left( \frac{\partial W_y}{\partial y_b} \right)_{(0,0,z)} \lambda(z) dz, \quad k_y^{(2)} = \int \left( \frac{\partial^2 W_y}{\partial y^2} \right)_{(0,0,z)} \lambda(z) dz,$$

respectively. The above  $k_y^{(1)}$  and  $k_y^{(2)}$  can be interpreted as the kick factors for the dipolar and the quadrupolar components, respectively.

When there are many structures in the ring, the dipolar tune shift is given by

$$\delta V_y^{(dipole)} = -\frac{q_b e}{4\pi E} \sum_j \beta_y(s_j) [k_{y,j}^{(1)} + k_{y,j}^{(2)}],$$

where  $j$  denotes the index of the structure, and  $\beta_y(s_j)$  is the betatron function at the location of the  $j$ -th structure. Note that both the dipolar and quadrupolar wakes contribute to the tune shift. On the other hand, the quadrupolar tune shift should be given by twice the incoherent tune shift:

$$\delta V_y^{(quad)} \approx 2\delta V_y^{(inc)} = -\frac{q_b e}{2\pi E} \sum_j \beta_y(s_j) k_{y,j}^{(2)}.$$

Since the center of the bunch does not oscillate, only the quadrupolar wake contributes to the tune shift. The other horizontal tune shifts can be represented similarly.

The above analysis indicates that by measuring both the dipolar and quadrupolar tune shifts, we can estimate the dipolar and the quadrupolar kick factors separately.

### Analysis of the Result

Applying the above-mentioned analysis to our measurement results in Table 1, we estimated the kick factors for the PF storage ring as shown in Table 2. In the present analysis, we did not take account of the bunch lengthening effect, and assumed that the kick factors were constants.

Assuming average betatron functions ( $\beta_x \sim 3.1$  m,  $\beta_y \sim 6.9$  m), the dipolar kick factors are roughly estimated to be 610 V/pC/m (horizontal) and 520 V/pC/m (vertical), respectively. The other quadrupolar kick factors are roughly estimated to be -390 V/pC/m (horizontal) and 280 V/pC/m (vertical), respectively. These results indicate that the quadrupolar wake force is focusing in the horizontal direction and defocusing in the vertical direction. This is

consistent with our calculation of the wake force for a typical flat structure. It is worth noting that the above  $k_x^{(2)}$  is not far from the  $-k_y^{(2)}$ , which is consistent with a theoretical relation of  $k_x^{(2)} = -k_y^{(2)}$  [3].

The above measurements were carried out under the single bunch operation. We also measured the dipolar tune shifts under a multibunch operation with 280 bunches. In a preliminary measurement, we observed the tune shifts of 0.002 A<sup>-1</sup> (horizontal) and -0.003 A<sup>-1</sup> (vertical) as a function of the total beam current. Since the tune dependence on the total current was much smaller than that on the bunch current, we can consider that the contribution of long-range quadrupolar wakes due to resistive vacuum chambers [7] should be small and that the tune shifts under the single bunch operations should mainly be caused by the short-range wake forces.

Table 2: Estimated products of the kick factors and the betatron functions.

$\sum \beta_x(s_j) \cdot k_{x,j}^{(1)}$	$1.9 \times 10^{15}$ V/C
$\sum \beta_x(s_j) \cdot k_{x,j}^{(2)}$	$-1.2 \times 10^{15}$ V/C
$\sum \beta_y(s_j) \cdot k_{y,j}^{(1)}$	$3.6 \times 10^{15}$ V/C
$\sum \beta_y(s_j) \cdot k_{y,j}^{(2)}$	$1.9 \times 10^{15}$ V/C

## CONCLUSION

We have shown that by measuring both the dipolar and the quadrupolar tune shifts, we can estimate the kick factors for the dipolar and the quadrupolar wakes independently. The present measurement indicated the negative slope for the vertical quadrupolar tunes, while our previous measurement revealed the positive slope for the horizontal quadrupolar tunes. Using the measured tune shifts, we could estimate both the dipolar and the quadrupolar kick factors.

The measured quadrupolar tune shifts give us evidence that the short-range quadrupolar wakefield is similarly important as the dipolar one. It also provides evidence for the explanation why the horizontal dipolar tune shift is much smaller than the vertical one [5]: the horizontal tune shift can be partly canceled by the quadrupolar wakes while the vertical shift can be enhanced.

## REFERENCES

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