MEASURING LOCAL GRADIENT AND SKEW QUADRUPOLE ERRORS IN RHIC IRS

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Abstract

The measurement of local linear errors at RHIC interaction regions using an ”action and phase” analysis of difference orbits has already been presented [2]. This paper evaluates the accuracy of this technique using difference orbits that were taken when known gradient errors and skew quadrupole errors were intentionally introduced. It also presents action and phase analysis of simulated orbits when controlled errors are intentionally placed in a RHIC simulation model.

INTRODUCTION

It is well known that gradient errors in the optical lattice of a circular accelerator change the beta functions all around the ring. It is precisely this fact what makes so difficult to localize and eventually determine the magnitude of such errors. However, if we think about this problem in terms of particle trajectories it will be clear that it should be possible to study gradient errors in a local way.

In order to clarify the last point let's suppose that a particle makes its first pass through an arbitrary optical lattice with a gradient error at certain point s_1. Using the matrix formalism [1] we can obtained a equation for the betatron oscillations of the particle for $s < s_1$ as:

$$x(s) = A\sqrt{\beta(s)}\sin\left(\phi(s) - \varphi\right), \quad (1)$$

where A and φ are constants that depend on the initial conditions $x(0)$ and $x'(0)$, and $\beta(s)$ and $\phi(s)$ can be taken as the **designed** beta functions which is valid for the first pass of the particle since the lattice without errors is completely equivalent to the lattice with a gradient error for $s < s_1$. The new lattice functions that arises as a consequence of the gradient error can also be used in Eq. 1 to describe the particle motion and this choice will lead to different values of A and φ but equally valid.

Now, let's compare what happens in the lattice without errors and the lattice with a gradient error at $s = s_1$. When the particle goes through s_1, it receives a different kick in each of these lattices. That will make x and x' just after s_1 different in each case. But the two lattices are still the same for $s > s_1$. This means that in both cases I still can use the matrix formalism with the **designed** lattice functions but the initial conditions in each case would be different. Hence, Eq. 1 is still valid to describe betatron motion in the lattice with errors but with different constants A and φ. In contrast, if the new lattice functions (the ones generated by the gradient error at $s = s_1$) are used the values of A and φ will be equal before and after $s = s_1$.

It is possible to see that the previous results are also valid for closed orbits if we remember that they are also possible particle trajectories and the the previous reasoning was done for an arbitrary particle trajectory.

The analysis in which the designed beta functions are used it is obviously ideal to localize magnetic errors. All it has to be done is to obtain plots of A and φ as function of s. Any magnetic error will appear as a jump in the plots of these two “constants”. Such plots can be obtained after applying

$$\begin{aligned} J &= \frac{x_i + x_{i+1} - 2x_i x_{i+1}\cos(\phi_{i+1} - \phi_i)}{\sin^4\left(\phi_{i+1} - \phi_i\right)} \\ \tan\varphi &= \frac{x_i \sin\phi_{i+1} - x_{i+1}\sin\phi_i}{x_i\cos\phi_{i+1} - x_{i+1}\cos\phi_i} \end{aligned} \quad (2)$$

to each pair of adjacent orbit measurements x_i and x_{i+1} where i runs from the orbit measurement done at the beginning of the ring to the measurement done at the end of the ring, ϕ_i and ϕ_{i+1} are the corresponding phase advances and $J = A/2$. The original choice of constants was J and φ rather than A and φ [3]. For that reason, this method was named action and phase analysis.

Action and phase analysis has already proven to very useful to detect and estimate linear errors at RHIC IRs [2] and the same analysis might lead to an accurate method of detecting and measuring non linear errors [4].

This article shows analysis based on simulated orbits when known linear errors are placed on a RHIC model and also we show similar analysis for real RHIC orbits when known linear errors are introduced in the accelerator.

LINEAR ERROR SIMULATIONS USING A RHIC MODEL

RHIC orbits can be easily generated with the MAD program (V. 8.23) providing as an input the RHIC lattice with only linear components activated. After the simulation is run, it generates a Twiss file where all the information about the orbit is recorded. In order to test the action and phase analysis to estimate errors, an orbit with one gradient error was generated with MAD. The action and phase analysis done with the help of Eq. 3 on a particular orbit with a gradient error is shown in Fig. 1. Two jumps in action and phase can be seen in Fig. 1. The first jump at $s = s_1 =$ 614m corresponds to the gradient error introduced in the

RHIC quadrupole bo7-qd1 while the second jump at s = 2584m correspond to the dipole corrector bo2-th2 used to produce a large closed orbit.

It is possible to estimate the value of such gradient error as [5]:

$$\Delta k = \sqrt{\frac{\left(J_x^L + J_x^R - 2\sqrt{J_x^L J_x^R}\cos(\psi_x^L - \psi_x^R)\right)}{\beta_x(s_1)x(s_1)^2}} \quad (3)$$

where J_x^L, J_x^R, ψ_x^L and ψ_x^R correspond to the action and phases for $s < s_1$ (superscript L) and $s > s_1$ (superscript R) respectively.

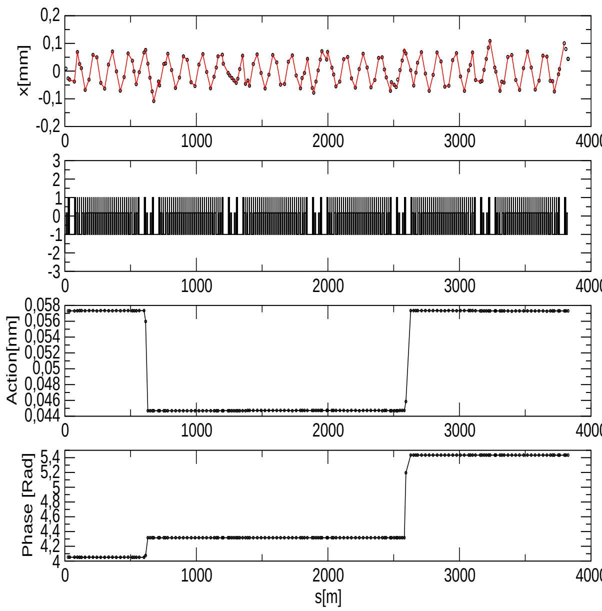

Figure 1: Action and phase analysis on a RHIC simulated orbit. One gradient error as big as 10^{-3} 1/m has been introduced intentionally in the simulation

Orbits with different values of gradient errors were generated with MAD. Action and phase analysis was done on all those orbits and with the help of Eq. 3 the corresponding values were recovered and summarized on Fig. 2. There is a slight difference between the expected values and the recovered values that seems to increase as the gradient error increases pointing to fact that a systematic error might be present either in the simulation or the method to estimate the error. However, the differences are of the order of 1% for a gradient error as big as $7 * 10^{-3}$ 1/m. Similar simulations and analysis were done for skew quadrupole errors with results that can be seen on Fig. 3. As before there is a small difference (3.5 % for the biggest skew quadrupole error used) between the expected values and the recovered ones that increase as the skew quadrupole error is increased.

EXPERIMENTAL TEST

Since the beta functions at RHIC Interaction Regions are significantly bigger than in the arcs, magnetic errors in the

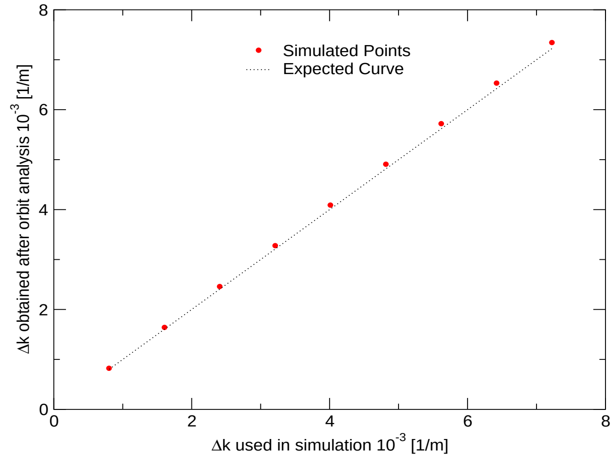

Figure 2: Relation between the gradient errors used to generate the RHIC orbits and the gradients errors recovered using the action and phase analysis on the simulated orbits.

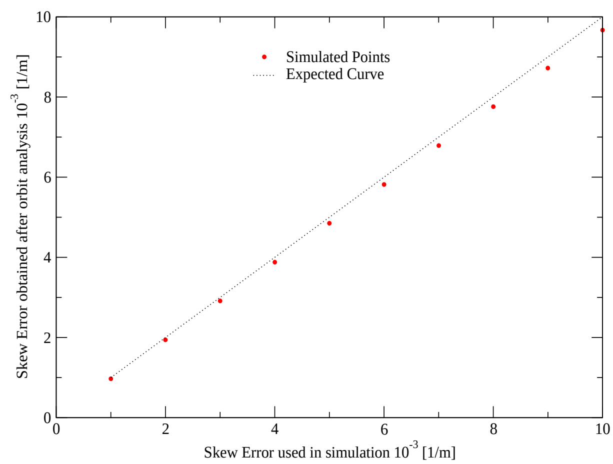

Figure 3: Relation between the skew quadrupole errors used to generate the RHIC orbits and the skew quadrupole errors recovered using the action and phase analysis on the simulated orbits.

IRs have a strong effect in the orbit. Thus, the IRs are an ideal place to test the effect of such errors. In particular the RHIC quadrupole bo7-qd1 located at the 8 o'clock IR was used to perform the experiments related with gradient errors while the skew quadrupole bi8-qs3 was used to perform the experiments related with skew quadrupole errors.

For all the experiments the closed orbit was enlarge significantly after tweaking one of the RHIC dipole corrector. The choice of this corrector was done such that there were a large excursion of the orbit in the region of interest, in this case 8 o'clock.

Before obtaining plots like Fig. 2 and Fig. 3 with the real orbits, it is necessary to do special data processing to reduce noise and isolate the errors that want to be measure. Such procedures have been already described in reference [2] and with more details in reference [5]. One of the differences of the experimental procedure with the simulation

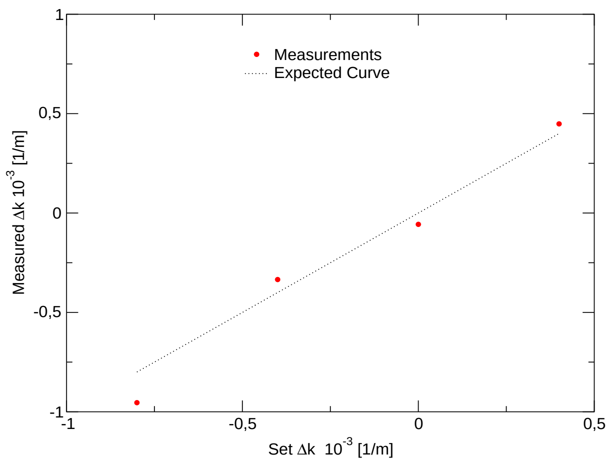

Figure 4: Relation between the gradient errors intentionally placed in the accelerator and the measured ones obtained from action and phase analysis discussed in this paper.

is that more than one orbit is needed to calculate a linear error; several of them in the horizontal plane and several of them in the vertical plane. It is also necessary to discard jumps in action and phase produced by dipole errors different to the dipole corrector used to enlarge the closed orbit. This problem is solved building the difference between the enlarged closed orbit and the closed orbit that results when the dipole corrector is set to its nominal value. The resultant orbit is the so called difference orbit.

Known values of gradient errors were introduced in the RHIC machine and after following the procedure explained in the previous paragraph, the plot in Fig. 4 was obtained.

It is necessary to point out that a correction to Fig. 4 was done to all points in order to discount the gradient error already present in the RHIC IR when the experiments were done.

Fig. 4 shows a dispersion of data around the expected values close to 10%. This means that magnetics gradients could be determined with accuracy of 10^{-4} 1/m or about 0.1% the nominal value of an IR quadrupole gradient.

Also, measurements of intentionally placed skew quadrupole errors were possible with results that can be seen on Fig. 5 . In this case , the accuracy to determine skew errors it is around 15 % which is bigger than the previous case maybe because these orbits were taken during the run 2001 with probably a lower performance of the Beam Position Monitors than the performance of the BPMs when the gradient error measurements were done in the run 2003. Another possible cause is that the orbits in both cases were processed differently: several orbits were needed to find a single gradient error while only one first turn trajectory was used to determine a skew quadrupole error.

As mentioned, the previous experiments were done during the RHIC 2001 run and the RHIC 2003 run. Since then, it has been a significant improvement in the Beam position Monitor system of the accelerator[6] and since the BPMs are at the heart of the present method, a significant improvement in the accuracy it is also expected for future experiments.

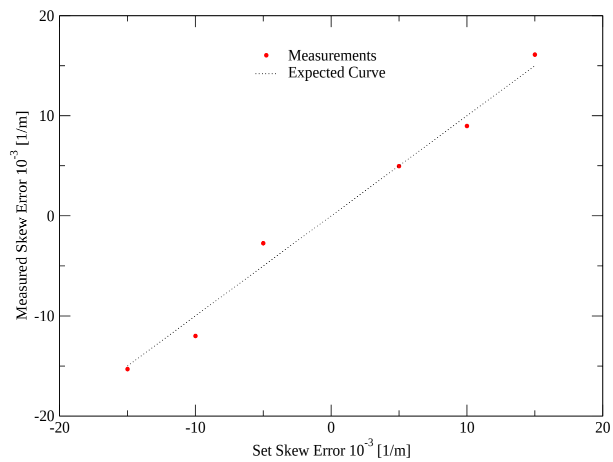

Figure 5: Relation between the skew quadrupole errors intentionally placed in the accelerator and the measured ones obtained from action and phase analysis dicuss ed in this paper

CONCLUSIONS

It was shown that gradient errors and skew errors can be studied in a local way leading to an easy method to locate and estimate these errors in a accurate way. Since BPM performance is vital for the action and phase analysis presented, it is expected than the always improving performance of the RHIC BPM system brings a similar improvement in the accuracy of our method.

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