TECHNIQUES TO EXTRACT PHYSICAL MODES IN MODEL-INDEPENDENT ANALYSIS OF RINGS*

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Abstract

A basic goal of Model-Independent Analysis is to extract the physical modes underlying the beam histories collected at a large number of beam position monitors so that beam dynamics and machine properties can be deduced independent of specific machine models. Here we discuss techniques to achieve this goal, especially the Principal Component Analysis and the Independent Component Analysis.

INTRODUCTION

In Model-Independent Analysis (MIA) [1, 2], one collects turn-by-turn beam histories at a large number of beam position monitors (BPMs) and attempts to deduce machine and beam properties from the data without fiddling with a specific machine model. The beam motion recorded at a BPM is a superposition of machine responses to the physical signals due to various beam excitations, i.e.,

$$b_p = b_0 + \sum_s \Delta q_p^s \partial_s b + \cdots, \tag{1}$$

where Δq_p^s is the s-th signal at the p-th turn and $\partial_s b$ is the response function that characterizes machine response to Δq^s. For example, the slope x' (due to a kick or whatever) at a certain location could be the signal, and the transport function R_{12} from that location to each BPM would be the response function. Usually the average orbit is subtracted and Eq. (1) can be casted into a matrix form as

$$B = UV^T, \tag{2}$$

where the data matrix B contains beam histories at all BPMs in its columns, U contains the temporal signals with the s-th column vector $u_s = \Delta q^s$, and V contains the spatial response functions with the s-th column vector $v_s = \partial_s b$. For convenience, here we assume the signals are normalized by their rms values and the response functions are scaled accordingly. The decomposition in Eq. (2) is referred to as the physical-mode decomposition. Each pair of temporal and spatial vectors (u_s, v_s) depicts a physical mode. It is a difficult but important goal of MIA to extract individual physical modes from the measured beam histories, which are superpositions of signals from all physical modes. Depending upon the knowledge we have about the signals and the responses, various techniques can be used to accomplish this goal.

When either the signals U or the responses V are known, it is straightforward to extract the corresponding unknown components by least-square fitting, i.e., $V^T = U^\dagger B$ or $U^T = V^\dagger B^T$, where the † indicates the pseudo-inverse of the matrix. In storage rings, it is common to have sinusoidal excitations (for example, in resonant excitation at certain tune). Knowing this characteristic of such signals, the corresponding response functions can be obtained using harmonic analysis (projection).

When there is no prior knowledge of either the signals or the responses, it is impossible, in principle, to reconstruct the physical modes from the BPM data only. However, there are major statistical techniques that can at least partially accomplish this goal based on reasonable assumptions on the statistical properties of the signals. Here we discuss the Principal Component Analysis (PCA) and the Independent Component Analysis (ICA). PCA is a century-old and widely-used multivariate statistical analysis method and has been instrumental in the development of MIA (where it is better known as SVD mode analysis because it was developed before we learned about PCA [2]). ICA is an exciting advance in statistical data analysis in the last decade or so [3] and has been explored recently for MIA applications [4]. Here we briefly review the basics of these techniques in order to appreciate their potentials in extracting the physical modes from beam histories. BPM noise will be ignored since we are interested in the basic principles rather than technical details.

PCA / SVD MODES

Principal Component Analysis finds a small number of uncorrelated principal components that can account for the maximum amount of observed variances and covariances in the data. Each principal component is a linear combination of the observed signals and retains the maximum variance along its direction. Principal Component Analysis can be achieved by an SVD of the data matrix B as

$$B = \hat{U} S \hat{V}^T = \sum_{\text{modes}} \sigma_i \hat{u}_i \hat{v}_i^T, \tag{3}$$

where $\hat{U}$ and $\hat{V}$ are orthonormal matrices whose columns contain the temporal and spatial singular vectors $\hat{u}_i$ and $\hat{v}_i$, and S is an upper-diagonal matrix contains the singular values σ_i. The i-th principal component is given by $B\hat{v}_i = \sigma_i \hat{u}_i$. The uncorrelatedness of the principal components is assured by the orthogonality of the $\hat{U}$ matrix since the covariance matrix $(BV)^T(BV) = S\hat{U}^T\hat{U}S = S^2$ is diagonal. The maximum variance is assured by the orthogonality of the $\hat{V}$ matrix since, according to the Courant-Fischer minimax theorem, the variance $(B\hat{v}_i)^T(B\hat{v}_i) = \hat{v}_i^T B^T B \hat{v}_i$ is maximized when $\hat{v}_i$ is an eigenvector of the sample covariance matrix $B^T B$, and the eigenvectors of a symmetric matrix are mutually orthogonal (assuming no degeneracy).

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Principal Component Analysis provides a faithful representation (in the sense of minimum mean-square errors) of the measured data with a small number of PCA/SVD modes. It is well known for its ability to suppress random noise and reduce dimensionality of the data. In MIA beam-history analysis, it has been shown to be very informative regarding the underlying beam dynamics and machine properties as well as the performance of the BPM system [5]. However, instead of getting into the details of existing applications, here we examine the connection of the PCA/SVD modes to the physical modes.

As mentioned above, all the principal components are uncorrelated (thus $\hat{U}$ is orthogonal). Therefore the PCA/SVD modes may be identifiable with the physical modes only when the original physical signals are uncorrelated (i.e., U is orthogonal). Fortunately, this is often the case. For example, the common betatron and synchrotron oscillations in phase space are usually harmonic (or pseudoharmonic) oscillations, which are generally uncorrelated in the normal coordinates. This is the underlying reason why the PCA/SVD modes seems rather physical. However, in addition to their uncorrelatedness, PCA/SVD modes require orthogonality of the spatial vectors in $\hat{V}$, but the spatial components of the physical modes are not necessarily orthogonal (although they are often approximately orthogonal when the number of BPMs is large). This contradiction means that the PCA/SVD modes may not be as physical as we want. Instead, they can be mixtures of several physical modes. For example, a horizontal betatron mode (which has a dominant horizontal tune in the temporal vector) could be mixed with vertical betatron modes or synchrotron mode (identifiable through the mixed-in vertical betatron tune or synchrotron tune). The PCA's deficiency in resolving the physical modes is due to the lack of information. After all, the only information used so far for the physical signals are their mutual uncorrelatedness, which by itself is insufficient to recover the physical modes. Despite this potential deficiency, since PCA/SVD modes provide a faithful, compressed, and noise-suppressed representation of the data, they are very useful for a wide variety of applications and are an important first step in many other techniques (including those discussed below) that use additional information about the physical signals to help reconstruct them.

Untangling Mixed PCA/SVD Modes Using Spectral Characteristics

To resolve the mixing of physical modes in the PCA/SVD modes, one has to rely on extra information about the physical modes in addition to the mutual uncorrelatedness. One simple idea is to use the often available spectral information about the physical signals. Most of the physical signals, especially in rings, have identifiable characteristics in their frequency spectra. By using such information, it is possible to untangle the mixed PCA/SVD modes so that the reconstructed modes have no mixed characteristics of different physical modes. The idea is to find a rotation O that rotates the principal components to a new set of components whose spectra have the expected characteristics [6]. A rotation is used to preserve the mutual uncorrelatedness since $(\hat{U}O)^T(\hat{U}O) = O^T\hat{U}^T\hat{U}O = O^TO = I$. Also, because there are sign ambiguities in both the principal components and the physical signals, it seems unnecessary to use a more general orthogonal matrix other than a rotation.

One way to determine the rotation O is to minimize the mixed-in frequencies in $\hat{U}O$. This is easy to do when the mixing is small so that the rotation can be linearized as $O = I + \sum \theta_{ij}L_{ij}$, where θ_{ij} is the rotation angle in the subspace spanned by the i-th and j-th principal components, and $L_{ij} = (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})$ is the generator of the corresponding infinitesimal rotation. Let $F_j[u]$ be the j-th Fourier component of the temporal vector u, then the j-th component of the k-th rotated vector $\tilde{u}_k$ is

$$F_j[\tilde{u}_k] = F_j[u_k] + \sum_{\{\theta_{\alpha\beta}\}} F_j[u_l]\,(\delta_{\alpha l}\delta_{\beta k} - \delta_{\alpha k}\delta_{\beta l})\,\theta_{\alpha\beta}. \quad (4)$$

The coefficients $F_j[u_k]$ and $F_j[u_l]$ are known from the singular vectors. Collecting $F_j[\tilde{u}_k] = 0$ for all j's and k's that $F_j[\tilde{u}_k]$ needs to be minimized sets up a set of equations to solve for the rotation angles.

After finding a rotation O that allows $\hat{U}O$ to have the expected spectral characteristics, the spatial part of the physical modes is then given by $O^TS\hat{V}^T$. Now note that the spatial vectors are no longer orthogonal since $(O^TS\hat{V}^T)(O^TS\hat{V}^T)^T = O^TS^2O$ is not diagonal in general. In a word, by reducing the mixing of spectral characteristics of different physical modes, a rotation O may be determined such that it rotates the principal components to a more meaningful decomposition

$$B = (\hat{U}O)(O^TS\hat{V}^T), \quad (5)$$

which better represents the physical modes in Eq. (2). Simulations have shown that this simple technique works well in extracting the coupled betatron modes [6, 7].

In the following section, we will see many different techniques that amount to the determination of O based on certain statistical assumptions. For example, O could be obtained as the matrix that diagonalizes the time-delayed correlation matrix $\langle \hat{U}^T(t)\hat{U}(t+\tau)\rangle$ for nonwhite signals.

ICA MODES

Independent Component Analysis attempts to reconstruct the source signals based on the assumption that they are mutually independent and non-Gaussian. Like PCA, ICA assumes mutual uncorrelatedness since it is a necessary condition for independence. For non-Gaussian signals, independence requires more than uncorrelatedness and thus provides extra conditions for determining the source signals. Extensive research has been devoted to the development of ICA in the last decade in many fields, which has resulted in many different approaches/algorithms for computing the independent components. A large number of documents and many free codes are available. A recent study indicates that ICA is a useful technique for MIA [4]. Here we take a brief look at the basics of ICA in order to appreciate its principles and potential for MIA applications.

The independence of any two source signals u_i and u_j implies that the expectations of all cross moments of order m and n satisfy

$$E[u_i^m(t)u_j^n(t+\tau)] = E[u_i^m(t)]E[u_j^n(t+\tau)]. \quad (6)$$

From this, a sufficient number of conditions can be set up to solve for independent source signals that are either nonstationary, nonwhite, or non-Gaussian [8, 9]. For nonstationary signals, both the covariance of the physical sources $C_s(t) = \langle u_i(t)u_j(t)\rangle$ and the covariance of the observations $C(t) = \langle b_i(t)b_j(t)\rangle$ vary with the time t. Since the spatial responses (the mixing channels) stay the same, the relation

$$C(t) = VC_s(t)V^T \quad (7)$$

holds for the covariance matrices $C(t)$ and $C_s(t)$ at different times. Since $C_s(t)$ must be diagonal all the time for independent sources, these relations provide sufficient conditions to determine V and extract the physical modes. For stationary signals, both $C(t)$ and $C_s(t)$ do not change with time, thus covariances at different times provide no extra information. However, if the signals are nonwhite (with nonzero autocorrelation), the time-delayed correlation matrix $C_s(\tau) = \langle u_i(t)u_j(t+\tau)\rangle$ will be different for different delay time τ, thus providing the extra information for determining V through Eq. (7), which holds for the time-delayed correlation matrices as well [10, 11]. We see that, for nonstationary or nonwhite signals, second-order statistics is sufficient for determining the sources. One way to carry this out is to find a matrix V that simultaneously diagonalizes two or more different sample covariance matrices. This can be done with many widely-available algorithms. Algorithms that (approximately) jointly diagonalize several matrices (e.g., [11]) are usually more robust against errors in the sample covariances.

Instead of using second-order statistics, many techniques use higher-order statistics to formulate a certain objective function that measures the degree of independence. By optimizing such an objective function, independent components can be obtained. For example, according to the central limit theorem, the sum of independent random variables tends to be closer to a Gaussian random variable. Thus independent components can be obtained by optimizing functions that measure the non-gaussianity, such as the fourth-order cumulants. A nice survey of various techniques/algorithms are available in [12].

To preliminarily examine ICA for MIA applications, we made a few simple tests on two data sets with several readily available programs. One data set is the experimental data from the Advanced Photon Source, whose PCA/SVD modes are described in [13]. The other is tracking data containing coupled betatron modes, which has been used in [6]. We used the SOBI algorithm in ICALAB [14] and the program in [4] for second-order statistics and FastICA [15] for higher-order statistics. For the tracking data without noise, all programs work in terms of extracting unmixed coupling modes. For the experimental data, second-order programs work quite well, but FastICA fails probably because higher-order statistics are more sensitive to noise and needs much larger signal samples. Further studies are needed to explore various algorithms for MIA applications.

REMARKS

Many techniques are available to extract the physical modes from beam histories based on often realistic assumptions. They are important for achieving the goal of MIA. Physical modes of linear beam dynamics (betatron modes, coupling modes, synchrotron modes) are well understood and utilized, but much work is needed to understand other modes in order to deduce beam and machine properties from them. The statistical assumptions for various physical modes also need to be examined more rigorously.

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