# LATTICE DESIGN OF LARGE ACCEPTANCE FFAGS FOR THE PRISM PROJECT 

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## Abstract

On the design on the PRISM-FFAG, dynamics studies and parameters search was performed by using quasirealistic magnetic fields. This method makes quicker and more accurate dynamics studies possible.

## INTRODUCTION

In order to realize a super muon beam that combines high-intensity, low-energy, narrow energy spread and high purity, the PRISM project has been proposed [1]. In this project, a Fixed Field Alternating Gradient (FFAG) ring is used as a phase rotator. A scaling FFAG that consists of radial sector magnets will be adopted. The PRISM-FFAG has to have both of large transverse acceptance and large momentum acceptance to achieve a high intensity muon beam. Furthermore, long straight sections to install RF cavities are required to obtain a high surviving ratio of the muon. Therefore, the PRISM-FFAG requires its magnets to have large aperture and small opening angle. In such magnets, not only non-linear effects but also magnetic fringing field are important to study the beam dynamics of FFAGs. Although 3D tracking using realistic 3D magnetic field maps made with programs such as TOSCA is the best solution to study the dynamics of FFAGs, it takes long time to make such field maps.
On the other hand, the synchrotron optics code such as SAD was used to search initial parameters. It is very useful for rough studies and enable rapid studies, but it is very difficult to estimate tune correctly.
On a design process of the PRISM-FFAG, quasi-realistic 3D magnetic field maps were used to study the beam dynamics. A method of designing the PRISM-FFAG lattice and beam optics parameters will be described in the following sections.

## QUASI-REALISTIC 3D MAGNETIC FIELD

A quasi-realistic 3D magnetic field is calculated applying spline interpolation to POISSON 2D fields. Let me illustrate a procedure to make the field map by a case of triplet radial sector magnet as shown in Fig.1. The triplet magnet consists of a focusing magnet, two defocusing magnets, and two field clumps. The axis of coordinates is defined as in Fig.1. An x-axis is considered as a $\theta$-axis in this method. This approximation can work only for thin magnets.

As the first step, $\mathrm{B}_{\mathrm{z}}$ and $\mathrm{B}_{\theta}$ are calculated in several x-z cross sections using POISSON. Figure 1 has five cross
sections ( $\mathrm{r}_{1} \sim \mathrm{r}_{5}$ ) for example. Current settings of each cross section are decided by a field index ( $k$ value). Figure 2 show a model of the magnet used in POISSON for one cross section. Lines of magnetic force are also plotted.


Figure 1. Top view of a triplet magnet.


Figure 2. A magnet model used in POISSON calculation.

Then, using the Maxwell equations, Br can be derived by:

$$
B_{r}\left(r, \theta, z_{i}\right)=\left(\frac{d B_{z}}{d r}\right)_{\left(r, \theta, z_{i}\right)}\left(z_{i}-z_{i-1}\right)+B_{r}\left(r, \theta, z_{i-1}\right)
$$

, where $z_{i}$ is the ith position of $z$. Thus, we have a rough 3D field map. In particular $R$ direction has very large mesh size.
In order to make a fine mesh field map, 2D spline interpolations are applied to the rough field map. A 3D quasi-realistic magnetic field map shows good agreement with TOSCA field (see Fig. 3). Only several minutes is required to make a quasi-realistic field map.


Figure 3. Comparison between a TOSCA field and a quasi-realistic field. $B_{z}, B_{\theta}$ and $B_{r}$ are plotted as a function of $\theta$.

## BEAM DYNAMICS STUDY

In order to search the best optics parameter set for PRISM-FFAG, the beam dynamics was studied by performing beam tracking using the quasi-realistic field maps with various optics parameter sets. A tracking program based on GEANT3.21 was used for particle tracking. Parameters to be studied are:

- Number of cell
- Combination of magnets: DFD, FDF, FD
- Field index ( $k$ value)
- Ratio of the magnitude of focusing field to that of defocusing field: ( $\mathrm{F} / \mathrm{D}$ ratio)
- Gap size of magnets

Figure 4-(left) and 4-(right) show an example of the acceptance study of horizontal and vertical respectively. Beam trajectories in a phase space are plotted on tune diagrams. The area of each plot indicates the acceptance. Taking resonance lines and the transition energy into account, parameters are selected so as to be the beam acceptance as large as possible. Present parameters of PRISM-FFAG are shown in Table 1.

Table 1: Present parameters of PRISM-FFAG

| Number of sectors | 10 |
| :--- | :--- |
| Magnet | DFD triplet <br> with field clumps |
| Field index $(k$ value) | 4.6 |
| Opening angle of magnets | $\mathrm{F} / 2: 2.2$ deg. <br> $\mathrm{D}: 2.2$ deg. |
| Half gap of magnets | 17 cm |
| Maximum field | Focus $: 0.24$ Tesla <br> Defocus $: 0.026$ Tesla |
| Average radius | 6.5 m for $68 \mathrm{MeV} / \mathrm{c}$ |
| Tune | Horizontal $: 2.69$ <br> Vertical $: 1.30$ |




Figure 4. Beam trajectories in a horizontal (left) and vertical (right) phase space are plotted on tune diagrams. The area of each plot indicates the acceptance. In this study the other emittance was set to zero. Therefore correlation between horizontal and vertical dynamic cannot be seen. Figures beside each plots means F./D ratio in current setting (in BL integration).


Figure 5. Horizontal and vertical phase spaces (top). The other beam emittance was set to zero in this study. The bottoms show the dependence of tune on the amplitude.

Figure 5 shows the phase space and the dependence of tune on the amplitude for horizontal (left) and vertical (right) respectively using the parameters shown in Table 1. The other beam emittance was set to zero in this study. For example, horizontal dynamics was studied with zero vertical emittance. It is found that the PRISM-FFAG of the present design has more than about $140,000 \pi$ $\mathrm{mm} . \mathrm{mrad}$ in the horizontal acceptance and about $3,000 \pi$ $\mathrm{mm} . \mathrm{mrad}$ in the vertical acceptance for 5 turns. The correlation between horizontal and vertical dynamics can reduce an effective horizontal acceptance to $35,000 \pi$ $\mathrm{mm} . \mathrm{mrad}$ according to tracking simulations.

## SUMMARY

For the PRISM-FFAG, beam dynamics studies and optics design were performed with a new method using the quasi-realistic magnetic field, which can treat realistic effects of the fringing fields comparatively. This method enable quick iterations in search for the optics parameters compared with that of using TOSCA fields. This method can be used widely to study and design FFAGs with a complex magnetic field

## REFERENCES

[1] "The PRISM Project -- A Muon Source of the WorldHighest Brightness by Phase Rotation --", LOI for Nuclear and Particle Physics Experiments at the JPARC (2003)

