SEPARATRIX FORMALISM APPLIED TO LINACS ACCELERATING PARTICLES WITH DIFFERENT CHARGE TO MASS RATIO

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Abstract

We have developed separatrix formalism for superconducting (SC) linear accelerators [1]. It is shown that behaviour of the quasi-synchronous velocity along a linac can be adjusted by a cavity phasing. This gives a great possibility to accelerate particles with different charge to mass ratio. In this article we present design optimization of the accelerating structure from the longitudinal motion point of view. Starting from the structures for one kind of particles we come to the structures for acceleration more than one kind of particles.

CAVITY PHASING AS A PRINCIPLE OF ACCELERATION IN SC LINACS

Theoretical foundation

SC accelerating structure consists of a sequence of SC cavities with the constant phase velocity. As it is well known, the synchronous particle in such a structure is not involved in acceleration process. In order to make the particle motion accelerated, it is necessary to phase all the cavities independently.

Taking into account the cavity phasing, we can express the phase of some particle with velocity β relative to the accelerating wave with the phase velocity β_{str}:

$$\phi(\varsigma) = \int_0^{\varsigma} \frac{\beta_{str} d\varsigma}{\beta} - \int_0^{\varsigma} d\varsigma + \varphi_{str}(\varsigma),$$

where $d\varsigma = 2\pi \frac{dz}{\beta_{str}\lambda}$ is an independent variable and $\varphi_{str}(\varsigma)$ – the phasing term. Then we get the equations of phase motion:

$$\begin{aligned} \frac{d\phi}{d\varsigma} &= \frac{\beta_{str}}{\beta} - 1 + \frac{d\varphi_{str}(\varsigma)}{d\varsigma} \\ \frac{d\beta}{d\varsigma} &= \frac{eE_{ac}\lambda\beta_{str}}{2\pi\, m_0 c^2 \gamma^3 \beta} \sin\phi \end{aligned}, \qquad (1)$$

where E_{ac} is an amplitude of the accelerating harmonics. Let's denote $\Omega^2 = \frac{eE_{ac}\lambda}{2\pi\, m_0 c^2 \gamma^3} \frac{\beta_{str}^2}{\beta^3}$. Provided that $\frac{d^2\varphi_{str}}{d\varsigma^2} = const \neq 0$ the system (1) describes the stable phase oscillation in the small vicinity of the nonzero constant phase $\overline{\phi_s} = \arcsin\left(\frac{d^2\varphi_{str}}{d\varsigma^2} \Big/ \Omega^2 \right)$. In this case the motion has the accelerated character since the velocity gain per cavity can be expressed as $\Delta\beta/\beta_{str} = 2\Omega \sin\overline{\phi_s} \tan\Phi/2$, where Φ is the phase advance of the longitudinal motion per cavity.

Thus, from the theoretical point of view, the phasing of cavities forces the particles to oscillate around some constant phase $\overline{\phi_s} \neq 0$, which defines the velocity gain per cavity. So the phasing of the cavities ultimately defines the velocity gain of the particle and, consequently, behaviour of the equivalent phase velocity.

The conclusion obtained above is overwhelmingly important because it signifies one of the most distinctive features of SC linacs. Namely the cavity phasing gives an opportunity to accelerate particles with different charge to mass (e/m_0) ratio. Indeed, applying the synchronous condition $\frac{d\phi(\varsigma)}{d\varsigma} = 0$ for the system (1) we can find the phasing function φ_{str}, which realizes the synchronous motion for the particles with any e/m_0 ratio.

However, synchronous acceleration cannot be put into practice. The phasing mechanism can, indeed, be physically envisioned as follows. As a consequence of the nonsynchronism, a particle is sliding relative to the accelerating wave as it passes through the cavity. In ideal situation this sliding must be compensated by the phasing so that to realise the synchronous motion. However, it is clear that phase oscillations inside a cavity cannot be reduced to zero. Therefore, the cavity phasing that can be established in practice allows one to create only a quasi-synchronous motion.

Quasi-synchronous motion

As a quasi-synchronous motion we define the motion of the particle that does not perform the free oscillations. All the particles oscillate around the quasi-synchronous particle, which itself oscillates around some constant phase $\overline{\phi_s}$. Thus, because of the nonsynchronism, the particles perform coherent oscillation, which causes the decreasing in energy spread of the effective separatrix by the value of $\sqrt{J_0(\varphi_a)}$, where φ_a is amplitude of the coherent oscillation and J_0 is the Bessel function. Apparently, the decreasing of the effective separatrix must carefully be taken into account during the optimization procedure. We will see that amplitude of the coherent oscillation serves as a main criterion for optimization of the accelerating structure from the longitudinal motion point of view.

LINAC STRUCTURE OPTIMIZATION

Determinative aspects of the optimization

An efficiency of acceleration changes along a linac. The maximum efficiency occurs only in case of the synchronism. In order to increase efficiency, one uses several families of cavities to cover the whole energy range of a linac. A goal of the optimization is to find the optimal number of cavity families, number of cavities in each family, number of accelerating cells per one cavity. And optimized structure must provide the efficient acceleration along the whole linac. This is the first requirement.

The second requirement ensues from the following consideration. We saw that the nonsynchronism results in the coherent oscillation of the beam, which leads to the decreasing of the effective separatrix. Within each cavity family the amplitude of the coherent oscillation changes slightly from cavity to cavity. The effective separatrix changes its size in the same way. So within one family the size of beam changes adiabatically and there are no particle losses. This is true unless the particles transfer from one family to the next. Indeed, if amplitude of the coherent oscillation in the next family is higher than in previous one, then such a transition entails deterioration of the beam quality or even the particle losses. Thus, amplitude of the coherent oscillation becomes important as the particles transfer to the next family. The most efficient transition occurs if the amplitude of the coherent oscillation in the neighbour families has the close magnitudes. So we get a conclusion: within each cavity family amplitude of the coherent oscillation reflects the efficiency of acceleration. The higher the amplitude, the lower the efficiency. And amplitude of the coherent oscillation acquires the principal importance, when particles transfer between cavity families. Therefore, the second requirement of the optimization is to provide the efficient transition of the particles between families. Certainly, some additional requirements should be taken into account. Among them is a reduction of the family number and cavities in each family. The last requirements arise from the wish to simplify an accelerating structure.

As we saw, in case of ideal phasing, which leads to the synchronism, there is no restriction on range of particles with different e/m_0 ratios that can be accelerated in one linac. But in practice there is. The efficiency of acceleration is being described by the transient time factor (TTF). The TTF value depends on how much the current velocity of particle differs from the phase velocity of the accelerating wave. Let's assume that two kinds of particles have approximately equal energy. If e/m_0 ratios for these particles are considerably different, then their velocities are greatly different as well. This difference might be so much, that it becomes impossible to accelerate effectively these kinds of particles in one structure.

Thus, the optimized structure has to meet following requirements:

- The effective acceleration in the whole energy range of an accelerator for each kind of particles.
- The effective transition of the particles between different cavity families for each kind of particles.

Below we present the algorithm of the optimization. Firstly we discuss the optimization of a linac for one kind of particles. As example of such an accelerator we take the linear accelerator for the European Spallation Source (ESS). After that we turn to the linear injector COSY that is supposed to accelerate protons (P) and deuterons (D). In the last case the linac must provide equally optimal acceleration for both particles. And the choice of parameters in such a case is a kind of compromise.

Optimization of linac for one kind of particle

SC linac in frame of the ESS project [2] must deliver 3.8 mA of average current to energy 1345 MeV. Proposed accelerating structure for the ESS linac is shown at fig. 1.

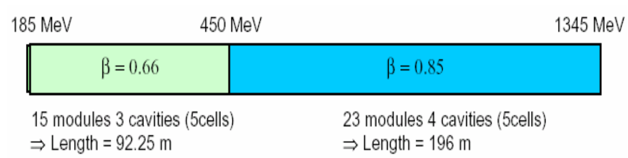

Fig. 1: Accelerating structure for the ESS SC linac.

High energy range, in which particles are being accelerated, makes it possible to use 5-cell cavities. The RF-frequency of cavities in both families is 704 MHz.

We will show that proposed accelerating structure meets both requirements. It provides acceleration with high TTF and efficient transition between families.

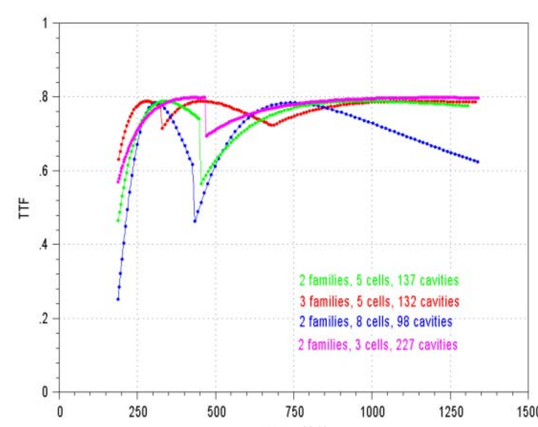

Fig. 2: Transient time factor of the ESS linac.

On the first step we analyse the TTF behaviour, which allows us to estimate the efficiency of acceleration and define at the first approximation the number of cavity families, the number of cavities in each family, the number of cells in a cavity, the value of the phase velocity. At figure 2 we see the TTF of the proposed structure and some comparative structures, which also provide the final energy. But more detailed consideration shows that the most reasonably to use structures based on the 8-cell cavities and two families of 5-cell cavities. The rest options seem not suitable from the economic point of view. 8-cell option allows one to reduce the total number of cavities to 98 in comparison with 137 cavities in 5-cell option.

However, the further analyze reveals that 8-cell option cannot provide the efficient transition to the second family, whereas the 5-cell option can. Figure 3 illustrates a capture of the separatrix bunch by the effective separatrix of the second part in case of the 5-cell option. So this option satisfies both requirements for the optimized structure.

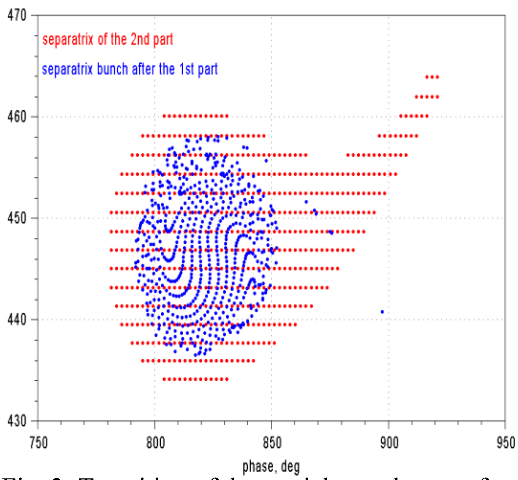

Fig. 3: Transition of the particles to the next family.

However, high intensity of the beam is a feature of the linac for ESS. Therefore, the optimization procedure must include the simulation with space charge.

Optimization of linac for two kinds of particles

COSY-injector linac [3] is supposed to accelerate both P and D beams in two families of 2-cell cavities from 2.5 (5) MeV for P (D) to 50 MeV. Pick current is 2 mA. The 1^{st} family includes 20 cavities with frequency 160 MHz and $\beta_{str,1}$=0.122. The 2^{nd} one includes 24 cavities with 320 MHz and $\beta_{str,2}$=0.245.

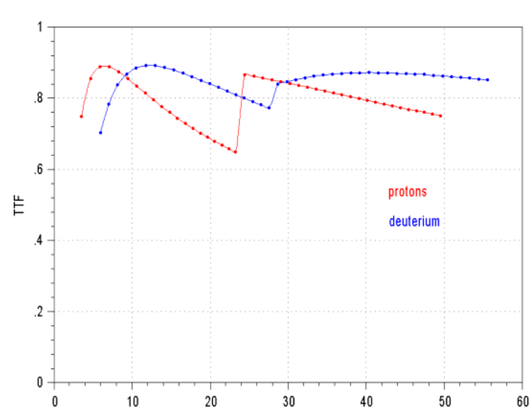

Fig. 4: Transient time factor of the COSY-injector linac.

As one can see from figure 4, the proposed structure ensures the effective acceleration for both P and D beams.

Since the pick current is only 2 mA then it is easier to ensure the efficient transition to the second family. Here is important to take into account the RF-frequency doubling.

Effective separatrises for protons and deuterons together with the initial bunch presented at figure 5. We see that in both cases the effective separatrix large enough to capture the initial bunch into acceleration. So the proposed structure meets both requirements of the optimization procedure.

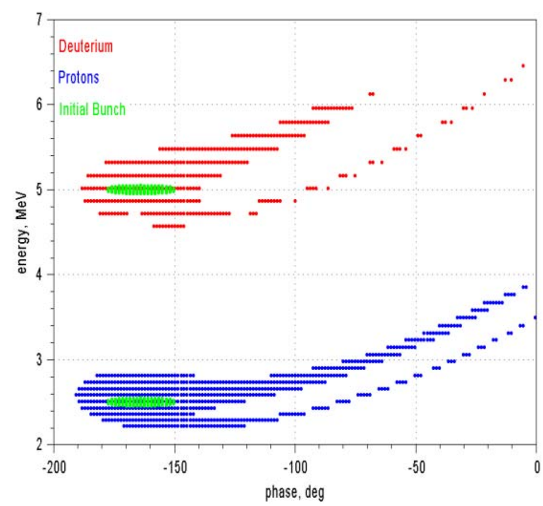

Fig. 5: Effective separatrises for P and D beam of COSY-injector linac.

CONCLUSION AND OUTLOOK

As it was shown, cavity phasing, which serves as the main mechanism of the particle acceleration in SC linacs, allows one to adjust the equivalent phase velocity along a linac. That was the base for development of the procedure that helps to optimize the structure either for acceleration of one kind of particles or for acceleration of particles with different charge to mass ratio. The presented procedure allows one to build the frame of the accelerating structure. It seems to be only the first stage of the full optimization. The further optimization steps should include the analyses of the space charge effects, the transverse motion and the influence of the inter-cryostat drift spaces.

REFERENCES

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[2] The ESS project, vol. III: Technical Report, 2002.
[3] R. Toelle, et al., “A Superconducting Injector LINAC for COSY”, Proc. 2002 EPAC, Paris, France.