# NONLINEAR EFFECT STUDIES FOR A LARGE ACCEPTANCE COLLECTOR RING

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## Abstract

A large acceptance collector ring (CR) is designed for fast cooling of rare isotope and antiproton beams, which will be used for nuclear physics experiments in the framework of the new international accelerator facility recently proposed at GSI [1]. This paper describes the linear and non-linear optimisation used to derive a lattice solution with good dynamic behaviour simultaneously meeting the demands for very fast stochastic cooling for two optical modes (for rare isotope and antiproton beams). Effects due to non-linear field contributions of the magnet field in dipoles and quadrupoles are very critical in this ring. Using a single particle dynamics approach, the major magnetic non-linearities of the CR are studied. We discuss the particle dynamics of the dipole and quadrupole fringe fields and their influence on the dynamic aperture and on the tune. Additionally, the CR will be operated at the transition energy (isochronous mode) for time of flight (TOF) mass spectrometery of short-lived radioactive ions. For this mode a specific correction scheme is required to reach a high degree of isochronism over a large acceptance.

## **INTRODUCTION**

At the first stage of the CR design the purpose of linear optimisation was to derive good properties of the ring for fast stochastic cooling at two optical modes [2,3]. Previously, two solutions of the CR lattice were considered: a "symmetric ring" with identical lattice functions in the arcs and the "split ring" with different lattice functions in the arcs. From the point of view of minimising the rf-voltage in the bunch rotation cavities, it is preferable to use a "symmetric ring" for rare isotopes (RI), where the frequency dispersion is relatively low compared with split ring optic. But, to achieve good properties of stochastic cooling for antiproton beams (Pbar) it is better to use the "split ring " optics.

A chromaticity control is a very important ingredient at the large acceptance operation. The side effect of using sextupoles to correct chromaticity is the introduction of additional non-linearities in the ring. In the case of the split ring one requires stronger focussing in one of the two arcs to have a good mixing between kicker and pickup, which is an essential condition of fast stochastic cooling. Stronger focussing results in higher chromaticities and, hence, to correct it stronger sextupoles are required. Therefore, a stronger non-linearity is introduced to the CR.

To have a large acceptance of the CR (200 mm·mrad and  $\Delta p/p$  of 3.5% for RI optic and 240 mm·mrad and

 $\Delta p/p=6\%$  for Pbar optics), large useful magnet apertures are required. Because of the necessarily large dispersion function amplitudes in the arcs for RI as well as for Pbar, horizontal apertures of 380 mm in bending magnets and 400 mm in quadrupole magnets are required. In such magnets the fringing fields effects give additional contribution to the linear and non-linear beam dynamics.

In the following section we discuss the influence of the effects due to non-linear field contributions of the magnet field in dipoles and quadrupoles and how fringe field effects affect the linear and nonlinear dynamics in the CR.

## **COLLECTOR RING CR**

The CR is designed for operation at a maximum magnetic rigidity of 13 Tm. Different quadrupole settings are required to fulfil the conditions of fast stochastic cooling for RI, Pbar and the isochronicity condition  $(\gamma_{tr}=\gamma)$ . In the first-order layout of the CR the required locale phase advances between pick-up and kicker for each quadrupole setting have been achieved. At this stage we tried to avoid large changes in the beta function in order to minimize nonlinear aberrations, furthermore, there should be room for the insertion of the rf-cavities for bunch rotation, injection/extraction devices and stochastic cooling system. The results of those considerations were presented in [2,3].

## NONLINEAR OPTIMIZATION

### Chromatic correction

A well known method to control the natural chromaticity of a synchrotron, while keeping the tunes  $Q_{x,y}$  constant, is to introduce two families of sextupoles placed at the locations of the ring, where the values of the function  $\beta_{x,y}$  and  $D_x$  are high. Two sextupole families however strongly affect the first and second order dependence of the  $\beta_{x,y}$  and  $D_x$  functions as well as of the chromaticity on the momentum spread. The dependence of the functions  $\beta_{xy}$  and  $D_x$  on  $\delta p/p$  introduce strong "beta/dispersion waves", which reduce the dynamic aperture of the ring. In addition, the beta/dispersion waves affect the horizontal and vertical chromaticities  $\xi_{x,y,T}(\delta p)$ through higher order field imperfection. In order to minimize the dependence of the  $\beta_{x,y}$  and  $D_x$  functions and the chromaticity on  $\delta p/p$ , four families of sextupoles are required instead of two. It is also necessary to minimise the higher-order tune shift with respect to momentum. To maximise the dynamic aperture additional harmonic sextupoles are needed, which have to be placed as additional families in the non-dispersive regions of the

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lattice. Calculations with the HARMON module in MAD show that for reaching a reasonably small tune shift with amplitude  $(dQ_{x,y}/d\varepsilon_x, dQ_y/d\varepsilon_y)$ , one needs to use four families of sextupoles placed also in straight sections. After chromaticity correction and minimization of the higher-order tune shift with six sextupole families the residual tune spread of  $\Delta Q_{x,y} \approx 0.018$  still remains for a momentum spread of 6% in the CR for the Pbar optics. For the RI optics ( $\Delta p/p=3.5\%$ ) this value is less than 0.01.

#### Dynamic aperture

Requirements of stochastic cooling for phase advance between pick-up and kicker set a limit for the region of useful tune space. In addition, the limited symmetry of the lattice also gives rise to a high density of resonance lines which further restrict the choice of the working point. The beta functions in long straight sections are not fixed to a precise value, and some variation is permitted when adjusting the tune. The working points of the CR at different modes of operation were chosen to minimise the strength of nearby resonances up to third-order. For each point the dynamic aperture was optimised using sextupole field integrated in quadrupoles placed in the long nondispersive straight sections. For the "symmetric ring" (RI optics) one family of sextupoles is enough to maximise dynamic aperture. In the case of the "split ring" (Pbar optics) two families of such sextupoles are needed. The particle tracking has been done with high order field imperfections (up to 9<sup>th</sup> order) by the MAD code. For the tracking approach, two-dimensional (2D) multipoles were extracted based on 3D field calculations of the dipole magnets and 2D calculations of the large aperture quadrupole magnets [5].

The CR dipoles are 2.2 meters long and the dispersion and beta functions vary rapidly inside. Hence, to be more correct in treating the non-linearity of each thick element under tracking simulations the dipole magnets and quadrupole magnets are split into 20 pieces and 5 pieces respectively. Simulations show that the dynamic aperture is dominated by sextupole components in the quadrupoles of the CR at the Pbar optics (Fig.1). On the other hand, in case of the RI optics the dynamic aperture is restricted mainly by systematic multipoles in the magnets (Fig.2).



Figure 1: Dynamic aperture of the CR for the "symmetric ring" with RI optics ( $\sigma$ =50 mm mrad).



Figure 2: Dynamic aperture of the CR for the "split ring" with Pbar optics ( $\sigma$ =60 mm mrad).

The present tune points were chosen to maximise the dynamic aperture for momentum deviation of up to 6% for Pbar and 3.5% for RI (Fig.3).



Figure 3: Dynamic aperture for on-momentum particles for the "split ring" with Pbar optics and for the "symmetric ring" with RI optics.

## Fringe field effect

Obviously, for a large magnet aperture the sharp cutoff approximation of the magnet field is unrealistic and furthermore the fringe fields affect the linear and nonlinear beam dynamics. Since the non-linear effects are larger for beams with larger emittance, those effects are especially crucial for the CR.

The influence of the fringing field of the quadrupoles on the linear optics and dynamic aperture has been calculated. The real field distribution is approximated by slicing the whole axial quadrupole field distribution in thin segments of varying strength according to the formula of the fringe fields falloffs (Enge function) [4]:

$$F(s) = \left[1 + \exp\left[a_1 + a_2\psi + a_3\psi^2 + a_4\psi^3 + a_5\psi^4 + a_6\psi^5\right]\right]^{-1}$$

 $a_i$  – Enge coefficients,  $\psi = s/b$ , b – full aperture of the magnet. Treating these segments as short hard edge quadrupoles the full transformation matrix is the product of the matrices for all elements. Hence, each quadrupole of the CR was modelled as the total transformation matrix for the real field profile by approximating the actual field distribution by a series of hard edge 'slice' matrices in both planes as a function of the focusing strength  $k_0$ .

Since the sextupole field is integrated in the quadrupole lens, the same distribution of the sextupole strength is simulated.

Calculations show that due to changes of the effective length of the quadrupoles the linear optics are distorted resulting in a tune shift of  $\Delta Q_x$ =-0.293 and  $\Delta Q_y$ =-0.253 (RI optics). To compensate the reduction of the effective length one has to increase the quadrupole strength by 10%. On the other hand, to compensate the risen chromaticity the sextupole strength must be reduced by 5%.

Single particle tracking has been done to study the influence of the fringing field on the dynamic aperture taking into account only chromaticity sextupoles of the CR. In Fig.4 the results of calculations made with the MIRKO code are shown.



Figure 4: The dynamic aperture in the horizontal phase space with chromaticity sextupoles without higher order imperfections for the "symmetric ring" with RI optics.

#### **ISOCHRONOUS MODE**

A special quadrupole setting is required to fulfil the isochronicity condition in the CR ( $\gamma_{tr}=\gamma$ ). The normal value of  $\gamma$  at the injected energy is 1.84. In order to achieve such a value, one has to increase the dispersion function in both arcs. This reduces the ring acceptance. The maximum dispersion function in the CR must be 30 m. At the given magnet aperture of the ring, the momentum acceptance of the CR is 1% and the transverse acceptance is 100 mm mrad in both planes.

For precise mass measurements of short-lived radioactive ions, one has to reach a high degree of isochronism over a momentum acceptance of 1%.

The relation between the revolution frequency and the momentum deviation was calculated for the full momentum acceptance at the optical setting for the isochronous mode operation over  $\Delta p/p$  range of  $\pm 5 \times 10^{-3}$ . In the linear optical setting at the constant value  $\gamma_{tr}=1.84$ , when there is no non-linear field dependence, the relative frequency variation of  $\Delta f/f=4\times 10^{-5}$  arises due to the variation of the Lorentz factor  $\gamma$ . However,  $\gamma_{tr}$  is not constant but reveals a variation that results in an additional frequency variation. Calculations with MAD code taking field non-linearity up to 9<sup>th</sup> order into account show that a closed orbit exists for particles with a maximum relative momentum deviation of  $\Delta p/p=\pm 0.41\%$ . Therefore, the largest momentum spread in the CR is

0.82% and the total variation of  $\Delta f/f$  is  $3 \times 10^{-4}$ . The main contribution to the non-linear dispersion variation is due to the sextupole components of the bending magnets. Obviously, one needs to make pole shims in order to compensate the sextupole components of the bending magnet. The ring setting can be improved by applying the sextupole fields integrated in the quadrupole magnets. The strength of the sextupole fields determines the mean slope of  $\gamma_{tr}(\Delta_{B\rho})$ . Simulations with the HARMON option in the MAD code show that the mean gradient of  $\gamma_{tr}(\Delta_{B\rho})$ can be controlled by four families of sextupole field components integrated in quadrupole magnets. Under such conditions the frequency variation can be decreased to about  $3 \times 10^{-5}$  over a momentum range of  $\pm 4.1 \times 10^{-3}$  as shown in Fig.5



Figure 5: Momentum compaction in the operation area of the momentum deviation for the CR isochronous mode.

## **CONCLUSIONS**

To correct the chromaticity and at the same time to reduce the first and second order dependence on the momentum spread of the the  $\beta_{x,y}$  and  $D_x$  functions as well as of the chromaticity, one has to apply at least six sextupole families in the arcs of the CR for both RI and Pbar optics. For improvements of the dynamic aperture additional two sextupole families are required and should be placed in the non-dispersive straight sections. With such a number of sextupole families one can reach a frequency spread of RI in the isochronous mode of the CR better than  $3 \times 10^{-5}$ .

### REFERENCES

- [1] W.Henning, "An International Accelerator Facility for Research with Ions and Antiprotons", Proc. of this conference.
- [2] A.Dolinskii, et al, EPAC'02, Paris, June 2002, p.572.
- [3] A.Dolinskii, et al, "Optimized lattice for the Collector Ring CR", to be published in NIMA, 2004.
- [4] M. Berz, B. Erdelyi, K Makino, "Fringe field effects in small rings of large acceptance". Physical Review ST-AB, Vol 3, 2000, p.124001.
- [5] A.Kalimov, G.Moritz, "Design of a superferric dipole magnet with high field quality in the aperture", IEEE, Trans. Appl. Superconductivity, Vol.14, No.2, June, 2004.