

CALCULATING LHC TUNING KNOBS USING VARIOUS METHODS.

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Abstract

By measuring and adjusting the β -functions at the IP the luminosity is being optimized. In LEP this was done with the two closest doublet magnets. This approach is not applicable for the LHC due to the asymmetric lattice and common beam pipe through the triplet magnets. To control and change the β -functions quadrupole groups situated on both sides further away from the IP have to be used where the two beams are already separated. The quadrupoles are excited in specific linear combinations, forming the so-called “tuning knobs” for the IP β functions. We compare the performance of such knobs calculated by different methods: (1) matching in MAD, (2) inversion of the response matrix and singular value decomposition inversion and conditioning and (3) conditioning the response matrix by multidimensional minimization using an Adapted Moore Penrose Method.

INTRODUCTION

In accelerator physics the term tuning knob is commonly used for one (or several) magnet(s) which is (are) used to tune one variable. In this paper a tuning knob is a group of quadrupoles, situated left and right of the IP. In the case of LHC these are situated between the end of the arc and the triplet magnets on either side of the IP (see Fig.1). These

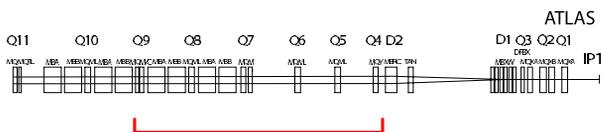


Figure 1: Lattice design of one half of the interaction (IR) region of IP1 and IP5. The red line indicates the region where one half of the tuning quads are situated. The second half of the IR is mirrored at the IP.

tuning quadrupoles are powered in series with fixed ratios of excitation strength and form the linear knob vector. To create a specific change of the β -function a common multiplier is used (see Fig.2). The vector is normalized such that if the multiplier equals one the vector increases the β -function by one meter.

MATCHING β^* WITH MAD

An obstacle is the choice of constraints for the matching. Analytically the number of constraints (variables, which are actively used in the matching process) is limited to be the same as the number of degrees of freedom in order to have a unique solution. The degree of freedom is given by the number of available tuning quadrupoles. In the case

$$m \cdot \begin{pmatrix} \Delta K_1 \\ \Delta K_2 \\ \Delta K_3 \\ \vdots \\ \Delta K_n \end{pmatrix} \rightarrow \Delta\beta_{x,y}$$

$$m = 1 \iff \Delta\beta_{x,y} = 1m$$

Figure 2: Principle of the composition of the tuning knob. The ΔK values are the fixed increments of change of quadrupole gradient, which are assigned to the different quadrupoles. To create a specific change a common multiplier m is used.

of LHC the degree of freedom is smaller than the number of constraints for an optimum solution. Also not all tuning quadrupoles act on the different constraints in the same way. This is a characteristic of the lattice. One now has to find an optimum combination of applicable tuning quadrupoles and an equal number of constraints, which not only optimizes the chosen constraints, but also other variables, which are not actively used in the matching process. These will be referred to as observables. Which variable will be chosen as constraint or observables has to be analyzed independently for each lattice. A variable is a constraint if it is absolutely necessary to minimize its change and it is possible to do so. This procedure is not straightforward and there are no analytical procedure available, which predicts the result. To find the optimum solution, all possible combinations have to be simulated. Fig.3 shows the flow of this process.

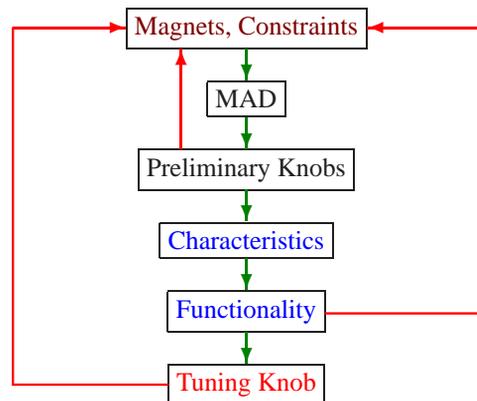


Figure 3: Work flow diagram for the procedure to match the tuning knobs with MAD.

The components of the knob vector are computed as follows. The IP β -function is matched in small steps from the minimum to the maximum of the range of application. The increments of the tuning quadrupoles (ΔK_n) are plots as a function of the β -function. By fitting the slope of these curves at the nominal value of the β -function the components are determined. For the matching the constraints must be kept constant and the changes of the observables must be minimized.

RESPONSE MATRIX

The technique of response matrix analysis is a standard method to measure and correct the closed orbit. LOCO (Linear Optics from Closed Orbit)[1]-[3] is an algorithm for debugging the optics of storage rings. It uses the orbit response measured at the beam position monitors (BPM's) caused by dipole and quadrupole excitations.

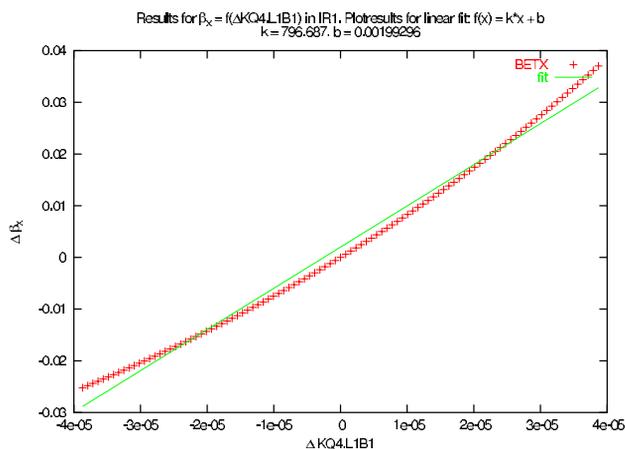


Figure 4: Response function for $\Delta\beta_x$ in IP1 in LHC when exciting the fourth quadrupole (KQ4.L1B1) left from the IP.

The same mechanism is used to simulate the “quadrupole response matrix”. In this case, instead of the orbit changes at the BPM's, the changes of the constraints and observables (global and local) as a function of the gradient change at the tuning quadrupoles are recorded. This technique allows us to simulate the behavior of the individual quadrupoles and is a vital tool in the selection process for the composition of the knob vector. By varying ΔK in several steps the change of the investigated variable can be plotted as a function of ΔK as shown in Fig.4. By fitting the gradient to a straight line around $\Delta K = 0$ the linear matrix element $\frac{dv}{dK_n}$ is calculated. Repeating this for all quadrupoles and variables (constraints and observables) the response matrix is calculated.

The range, for which the calculated matrix is valid, depends on the behavior of the function (see Fig.4). If the gradient is not constant the matrix is strictly valid only in an infinitely small range around $\Delta K = 0$. Using it in a finite range will introduce errors in the calculations. To

minimize the error, one has to calculate the response matrix, set a small increment and then recalculate again. The amount of data to be handled demands an automated processing and can be rather time consuming depending on the dimension of the matrix.

Calculating Response Matrix Using MAD

To calculate the response matrix with MAD all in principle adaptable quadrupoles were analyzed. This was done by varying the integrated quadrupole strength ΔK and recording the α -, β -function, dispersion in the IP of interest and at a second point of the ring separated from the IP by phase advance of $\phi = \frac{\pi}{2}$ to minimize the β - and dispersion beat, and the tune change. For the LHC the matrix was calculated once with the dimension 20×20 . 400 plots had to be generated and edited. For this reason scripts were written for automation. For the fitting gnuplot is used.

CALCULATING TUNING KNOBS USING RESPONSE MATRIX

By inverting the response matrix the elements of the knob vector can be directly calculated:

$$\Delta \vec{K} = \mathbf{R}^{-1} \Delta \vec{P}$$

$\Delta \vec{P}$ is a vector composed of constraints, observables and the variable which is supposed to be changed. The components of this vector are all zero (constraints) except for the one corresponding to the value $(\beta_{x,y})$ which is to be changed. This is only possible if the dimension of \mathbf{R} is quadratic and if it is not singular or poorly conditioned (close to singular). If inversion is possible the validity of the solution depends on the behavior of the response functions. A calculated tuning knob might not work in practice if its range of application is too small.

Pseudo Inverting the Response Matrix by SVD

In case of non square matrix and poor condition there exists a mathematical formalism, called Singular Value Decomposition (SVD)[4], which “pseudo inverts” a matrix. The solution is also not mathematically rigorous and has to be analyzed. The formalism is based on a mathematical theorem, that all matrices can be decomposed into a product of three matrices,

$$\mathbf{R} = \mathbf{U}\mathbf{S}\mathbf{V}$$

of which the first and the third are orthonormal and the second is diagonal (only the values on the diagonal are non zero). Due to these special attributes the matrix can be pseudo inverted as follows:

$$\mathbf{R}^{-1} = \mathbf{V}^T \mathbf{S}^{-1} \mathbf{U}^T$$

The diagonal matrix contains information about the singularity. Codes performing this calculation usually arrange

the matrices in a way, that the entry in this matrix is ordered from the largest to the smallest singular value. To invert this matrix the inverses of the diagonal elements are taken. In case of a too small singular value the matrix becomes nearly singular and the inversion would introduce large quadrupole (corrector) changes. To avoid this for small values of s_i , the large quantities $1/s_i$ in the inverse matrix are set to zero. Doing this makes the matrix pseudo invertible but loses some information. The resulting matrix has to be analyzed for its validity.

To characterize different combinations of degrees of freedom and constraints one can make use of the diagonal matrix. The condition of the matrix is characterized by the ratio between the largest and the smallest element. The smaller this ratio is the better the system.

NON LINEAR OPTIMIZATION USING AN ADAPTED MOORE PENROSE METHOD

The range of the calculated tuning knob depends on the response function of the tuning quadrupoles. The linearity of the slope of these functions is different for different quadrupoles and constraints. The range of the knobs may be maximized by selecting quadrupoles and constraints with optimum linear response. In the minimization algorithms the error function χ is defined as [5]:

$$\chi^2 = (\vec{c} - \hat{c})^T \mathbf{W} (\vec{c} - \hat{c}) \quad (1)$$

where \vec{c} is the expected constraint vector, \hat{c} is the simulated constraint vector and \mathbf{W} the weight matrix consisting of the weight factors w_i . With

$$\Delta \vec{c} = \vec{c} - \hat{c}$$

Eq.(1) becomes

$$\chi^2 = (\Delta \vec{c})^T \mathbf{W} (\Delta \vec{c}).$$

This does not take into account the non linear behavior which can lead to limited ranges of applicability. To include this, one has to add a further term to the error function, so that the changes of the quadrupoles are also considered in the minimization process.

The change of the constraints is usually not exactly zero using a perturbative approach for computing the tuning knobs. Therefore, an allowed range of change is assigned to each constraint. The range of the tuning knob is defined by the limit of the constraint that is reached first. This can be balanced and adjusted by using weight functions for the constraints when computing the knobs. To deal with the nonlinear response of the quadrupoles penalty-(weight-) functions can be introduced to minimize changes of quadrupoles, depending on the nonlinearity they introduce. Together with weight functions for the constraints and quadrupoles, this system is minimized.

The minimization condition for the change of quadrupoles is added such that the system is dimensionally consistent. To do so the response matrix is included.

$$\chi^2 = (\mathbf{R}\Delta\vec{K})^T \mathbf{W}_1 (\mathbf{R}\Delta\vec{K}) + (\Delta\vec{K})^T \mathbf{W}_2 (\Delta\vec{K})$$

For minimizing the error function the derivative with respect to $\Delta\vec{K}$ is set to zero.

$$\begin{aligned} \vec{\nabla}\chi^2 &= 2\mathbf{R}^T \mathbf{W}_1 \Delta\vec{c}_0 + 2\mathbf{R}^T \mathbf{W}_1 \mathbf{R} \Delta\vec{K} \\ &+ 2\mathbf{W}_2 \Delta\vec{K} + 2\mathbf{W}_2 \Delta\vec{K}_0 \\ &= 0 \end{aligned}$$

Simplifying the system gives the new condition:

$$\begin{aligned} \mathbf{A} \Delta\vec{K} &= -\mathbf{R}^T \mathbf{W}_1 \Delta\vec{c}_0 - \mathbf{W}_2 \Delta\vec{K}_0 \\ \mathbf{A} &= \mathbf{R}^T \mathbf{W}_1 \mathbf{R} + \mathbf{W}_2 \end{aligned}$$

The minimization of the change of quadrupole strength happens at the cost of the change of the constraints. But due to the nonlinear behavior which is reduced by minimizing quadrupole changes, the overall gain can be that the range of application is extended by minimizing the sum of the weighted change of constraints and quadrupoles in the error function.

RESULTS AND CONCLUSIONS

With each of the above discussed method sets of tuning knobs were computed. The different knob vectors and their performance do not differ significantly from each other. This is due to the fact that the response of the constraints to the changes by the used tuning quadrupoles is very linear. A combination of the methods using the response matrix has been used to calculate tuning knobs for a real machine and has been successfully applied to RHIC [6].

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