SIMULATION TECHNIQUE FOR STUDY OF TRANSIENT SELF-CONSISTENT BEAM DYNAMICS IN RF LINACS

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Abstract

The report describes a simulation technique for study of unsteady self-consistent dynamics of charged particles in resonant linacs that consist of cavities and travelling wave sections. The proposed approach is based on unsteady theories of excitation of resonant cavities and waveguides by a beam of charged particles and RF feeders. The theory of waveguide excitation [1] is extended to the case of spatially inhomogeneous travelling wave structures. The SUPERFISH code [2] is used to evaluate characteristics of the axially symmetrical travelling wave sections. The PARMELA code [3] is applied to simulate motion of the particles at each time step of the integration.

INTRODUCTION

Acceleration of intense charged beams in the RF linacs with the short pulse regime, when the filling time of standing wave (SW) bunchers and travelling wave (TW) sections cannot be neglected, requires detail learning of transients to diminish their influence on beam parameters.

Although the greatest information about unsteady acceleration can be got using the particle-in-cell (PIC) codes, for calculating the beam dynamics in the long RF structures these PIC codes need rather large computing resources. At the same time it is often necessary to study slow varying phenomena of narrow frequency spectrum in working pass band of the RF linacs [4, 5, 6, 7]. It was shown in [8] that in this case such well-known code as PARMELA, which usually is applied to simulation of steady dynamics, can be used to simulate unsteady self-consistent particle dynamics in SW cavities. The present work is aimed at developing the simulation technique of unsteady particle dynamics in inhomogeneous TW structures and integrating this technique in the unified algorithm of self-consistent unsteady beam dynamics simulation in RF linacs containing both SW and TW structures.

METHOD

Extending the theory of excitation of waveguides [1] to inhomogeneous accelerating structures, we suppose that the eigen orthogonal waves $\vec{E}'_{\pm s}$, $\vec{H}'_{\pm s}$ (electrical and magnetic field respectively) with the norm

$$N'_s = \frac{c}{4\pi}\int\limits_{S(z)}\left(\vec{E}'_s\times\vec{H}'_{-s} - \vec{E}'_{-s}\times\vec{H}'_s\right)\vec{e}_z \mathrm{d}S = const, \quad (1)$$

can propagate along the longitudinal direction ±z of an inhomogeneous waveguide. Here $\vec{e}_z$ is the unit vector along OZ axis, $S(z)$ is the waveguide cross-section at z coordinate. Then, according to [1] the Fourier harmonics of fields can be presented as follows:

$$\vec{E}_\omega = \sum_s C'_s\vec{E}'_s + C'_{-s}\vec{E}'_{-s} + \frac{4\pi}{i\omega}\vec{j}_{\omega,z},\quad \vec{H}_\omega = \sum_s C'_s\vec{H}'_s + C'_{-s}\vec{H}'_{-s} \quad (2)$$

where $\vec{j}_{\omega,z}$ is the longitudinal component of current density; $C'_{\pm s}(z)$ is the function of *z,* that satisfies the equation:

$$\frac{dC'_{\pm s}}{dz} = \frac{1}{N'_{\mp s}}\int\limits_{S(z)}\vec{j}_\omega\vec{E}'_{\mp s}dS, \quad (3)$$

If the geometry of a structure varies slowly in a longitudinal direction, the eigen waves in cross-section z can be expressed through the eigen waves $\vec{E}_{\pm s} = e^{\pm ih_s z}\vec{E}^{(0)}_{\pm s}$, $\vec{H}_{\pm s} = e^{\pm ih_s z}\vec{H}^{(0)}_{\pm s}$ that correspond to the homogeneous waveguide with cross section S(z), as follows:

$$\vec{E}'_{\pm s} = \xi_s(z)e^{\pm i\psi_s(z)}\vec{E}^{(0)}_{\pm s},\ \vec{H}'_{\pm s} = \xi_s(z)e^{\pm i\psi_s(z)}\vec{H}^{(0)}_{\pm s}, \quad (4)$$

where: $d\psi_s/dz = h_s(z)$, ($\mathrm{Re}\{h_s(z)\}$ is a propagation constant, $\mathrm{Im}\{h_s(z)\}=\alpha_s(z)$ is an attenuation constant; $\xi_s(z)$ is a real function of z coordinate. The condition (4) provides adiabatic invariance of wave power flow. In a periodic waveguide functions $\vec{E}^{(0)}_{\pm s}, \vec{H}^{(0)}_{\pm s}$ can be expressed as Floquet's series:

$$\vec{E}^{(0)}_{\pm s} = \sum_{n=-\infty}^{\infty}\vec{E}_{\pm s,n}e^{i\frac{2\pi n}{D}z},\ \vec{H}^{(0)}_{\pm s} = \sum_{n=-\infty}^{\infty}\vec{H}_{\pm s,n}e^{i\frac{2\pi n}{D}z} \quad (5)$$

Substituting Eq.(4) in Eq.(1) one can obtain connection between norms of waves ($\vec{E}'_{\pm s}, \vec{H}'_{\pm s}$) and ($\vec{E}_{\pm s}, \vec{H}_{\pm s}$)

$$N'_{\pm s} = \xi_s(z)^2 N_{\pm s}(z) = const \quad (6)$$

Differentiating Eq.(6) with respect to *z* and substituting Eq.(4) in Eq.(2) and Eq.(3), the Fourier harmonics of a field can be found in the following form:

$$\vec{E}_\omega = \sum_s C_s\vec{E}^{(0)}_s + C_{-s}\vec{E}^{(0)}_{-s} + \frac{4\pi}{i\omega}\vec{j}_{\omega,z},\ \vec{H}_\omega = \sum_s C_s\vec{H}^{(0)}_s + C_{-s}\vec{H}^{(0)}_{-s} \quad (7)$$

where $C_{\pm s}(z)$ is the new amplitudes of expansion ($C_{\pm s} = \xi_s(z)e^{\pm i\psi_s(z)}C'_{\pm s}$) that satisfy the equation

$$\frac{dC_{\pm s}}{dz} + \left(\frac{1}{2N_{\pm s}}\frac{dN_{\pm s}}{dz} \pm ih_s\right)C_{\pm s} = \frac{1}{N_{\mp s}}\int\limits_{S(z)}\vec{j}_\omega\vec{E}^{(0)}_{\mp s}dS. \quad (8)$$

In the case of acceleration of a train of bunches, fields and beam current are specified by narrow frequency spreads around working frequency ω_0. Thus, accomplishing the inverse Fourier transformation in Eqs. (4) and (5) one can obtain expression for fields of forward wave in the lower band in a time dependent form:

$$\vec{E}(t,\vec{r}) = \mathrm{Re}\left\{C_{+0}(t,z)\vec{E}^{(0)}_{+0}(\vec{r})\exp\left(i\int_0^z h_0(z)dz - i\omega_0 t\right)\right\},$$
$$\vec{H}(t,\vec{r}) = \mathrm{Re}\left\{C_{+0}(t,z)\vec{H}^{(0)}_{+0}(\vec{r})\exp\left(i\int_0^z h_0(z)dz - i\omega_0 t\right)\right\} \quad (9)$$

The slow varying amplitude $C_{+0}(t,z)$ obeys the equation

$$\frac{\partial C_{+0}}{\partial z}-\frac{1}{2R_0}\frac{dR_0}{dz}C_{+0}+\frac{1}{\mathrm{v}_g(z)}\frac{\partial C_{+0}}{\partial t}=\frac{R_0}{2}e^{-i\int_0^z h_0(z)dz}\int_{S(z)}\vec{j}_{\omega_0}(t,\vec{r})\vec{E}_{+0}^{(0)*}(\vec{r})dS \quad (10)$$

where $R_0(z)=-4\left|E_{z,+0}^{(0)}(0)\right|^2/N_0(z)$ is the serial impedance of the synchronous space harmonic of field, $\mathrm{v}_g(z)$ is the group velocity,

$\vec{j}_{\omega_0}(t,\vec{r})=\frac{\omega_0}{2\pi}\int_{t-\pi/\omega_0}^{t-\pi/\omega_0}dt\left\{e^{i\omega_0 t}q\sum_{k=1}^{K}\delta\left[\vec{r}_\perp-\vec{r}_{\perp,k}\right]\delta\left[t-\tau_k(t,z)\right]\right\}$ is the harmonic of current density expressed through the Lagrange coordinates $\vec{r}_k,\tau_k(t,z)$ of particles that represent a beam, q is the charge of the particles. The coordinates depend on the field amplitude $C_{+0}(t,z)$, so they are slow-varying functions of time.

It should be noted that the field of the backward wave could be found in the same way.

For numerical solution of Eq. (10) we will use the difference equation of the first-order of approximation [8]

$$\frac{\overline{C}^{(n,m)}-\overline{C}^{(n,m-1)}}{\Delta z}+\frac{\overline{C}^{(n+1,m)}-\overline{C}^{(n,m)}}{\mathrm{v}_g^{(m)}\Delta t}=-f^{(m)}I^{(n,m)} \quad (11)$$

where the indexes n and m – correspond to the discrete time $t_n=n\Delta t$ and longitudinal coordinate $z_m=m\Delta z$, accordingly; the amplitudes $\overline{C}^{(n,m)}$ are defined as $\overline{C}^{(n,m)}=C_{+0}(n\Delta t,m\Delta z)\sqrt{R_0(0)/R_0(m\Delta z)}$; the factor $f^{(m)}$ is $f^{(m)}=\frac{1}{2}\sqrt{R_0(m\Delta z)R_0(0)}e^{i\int_0^{m\Delta z}h_0(z)dz}$; the Fourier harmonic of a current $I^{(n,m)}$ with the mean value $I(n\Delta t)$ over the period $2\pi/\omega_0$ is given in the form

$$I^{(n,m)}=I(n\Delta t)\sum_k e^{i\omega_0\tau_k(n\Delta t,m\Delta z)}. \quad (12)$$

To obtain a self-consistent set of the equations, it is necessary to supplement Eq. (11) by a set of equations of motion of the particles in the fields Eq. (9). For solution of the equations of motion we will use the PARMELA code.

SIMULATION TECHNIQUE

The developed simulation algorithm was aimed at using the PARMELA v.3.22 code. The code evaluates beam loading in SW cells. Processing the output file of the code with coordinates of particles it is possible to calculate the Fourier harmonic of a beam current using the Eq. (12) at the end of each TW cell that represents period of a dick-loaded waveguide (DLW). Obviously, in this case Δz must be multiple to cell length and Δt - must be an integer number of periods of an accelerating field. Then $\overline{C}^{(n,m)}$ determines a field in the m^{th} cell on the n^{th} step of time Δt. Knowing the initial amplitudes in cells $\overline{C}^{(0,m)}$ and the amplitude in the first cell $\overline{C}^{(n,0)}$ as the function of time the Eq. (11) can be solved evaluating increments of amplitude in each TW cell at each time step.

Slow varying complex amplitudes C_r of fields $\vec{E}(t,\vec{r})\approx\mathrm{Re}\{C_r(t)\vec{E}_r(\vec{r})e^{i\omega t}\},\vec{H}(t,\vec{r})\approx\mathrm{Re}\{C_r(t)\vec{H}_r(\vec{r})e^{i\omega t}\}$ in SW cells (except cells of the input and output couplers) can be evaluated in a similar way. The increment ΔC_r of the amplitude can be calculated according to the technique [8]

$$\Delta C_r=\Delta t\left\{-iC_r^{(n)}\left(\omega_0-\omega_r\left(1+\frac{i}{2Q_r}\right)\right)-\right. \quad (13)$$

$$\left.\frac{Z_{sh}\omega_r q}{2Q_r dE_0^{\,2}}\sum_{k=1}\overline{\vec{\mathrm{v}}_k(n\Delta t)\vec{E}_r(\vec{r}_k(n\Delta t))e^{i\omega t}}-\frac{\omega_r}{Q_r}\left(\frac{\beta C_r^{(n)}}{2}-\frac{e^{i(\varphi_r+\pi/2)}}{E_0}\sqrt{\frac{Z_{sh}\beta P_{r0}^{(n)}}{d}}\right)\right\}$$

where ω_r is the resonant frequency of the cavity, Q_r is the unloaded quality factor, Z_{sh} is the shunt impedance per unit length, d is cavity length, E_0 is the mean amplitude of the on-axis electric field, $\vec{\mathrm{v}}_k$ is the particle velocity, β is the coupling coefficient of the cavity with the feeder, φ_r is phase shift, $P_{r0}^{(n)}$ is incident RF power at the given time step. The superscript line in Eq. (13) means time averaging. This procedure is discussed in Ref. [8].

To simulate motion of particles the PARMELA code needs the field distributions in SW cells and relative amplitudes of the spatial harmonics (see Eq.(5)) that have to be normalized according to Ref. [9] to restore fields of the eigen waves in TW cells (see Eqs. 4 and 5). Besides, values of ω_r, Z_{sh}, Q_r, and values of α_0, R_0, v_g as functions of z have to be specified. Evaluation of the values can be done with the SUPERFISH group of codes. For the standing wave cells the procedure is obvious. The characteristics of an inhomogeneous DLW are presented as a set of the characteristics of a homogenous DLW with iris radii that are equal to each iris radius in the simulated DLW (as it follows from Eq. 4). Characteristics of the homogenous DLWs are evaluated using technique [10] from field patterns in the cavity stacks. The relative amplitudes of the spatial harmonics can be obtained from the on-axis field distribution of the stacks.

Thereby the self-consistent simulation of beam dynamics can be fulfilled in linacs with both SW bunchers and TW sections.

Let's consider the features of the designed algorithm. If the beam current at the given temporary step is not equal to zero the running values of the amplitudes and phases of fields in the cells are written into the input file of the PARMELA code and the simulation of particle motions starts. After the PARMELA code completes the task, its output files are processing to get necessary data to evaluate the increments of the amplitudes. Then, the process repeats. Therefore, to keep a physical sense of the obtained results, the step Δt should be longer than the time-of-flight of particles through the simulated segment. On the other hand, Eq. (11) gives correct results only at $v_g\Delta t<\Delta z$, so quantity of TW cells M should be less then c/v_g, were c is the velocity of light. Besides, there is an artificial tailing of a wave front along the section if $v_g\Delta t\neq\Delta z$ because of the amplitude averaging over the cell length. Therefore edges of current and RF power pulses should be at least longer than $M\Delta t$. The typical values of

Δt and M are 10 periods of RF oscillations and 40 cells accordingly. The long linac can be broken in several segments that contain acceptable amount of cells. The beam from the previous segment is injected into the subsequent one by creation of files with coordinates of particles in the six-dimensional phase space at each time step. The time steps are the same through the simulation of the whole linac. At creation of the files the Lagrangian time of particles is decreased by the integer number of periods of the RF field. At simulation of the subsequent segment the PARMELA code accepts the file that corresponds to the running time step. The segments of DLW are bounded by half-cells. The field of the last half-cell is the boundary condition to simulate the subsequent segment.

The testing of the algorithm was carried out in several ways. First of all the results of simulation of excitation of pillbox cells and homogenous DLW by short ultra relativistic bunches were compared with the analytical results for a steady state mode (the corresponding formulas can be found, for example, in [4]). The simulated results and the analytical prediction coincide each other. For example, the simulated field distribution along the DLW containing 40 cells agreed within 0.1 % with an analytically derived one for all the regular cells. Then, inhomogeneous DLW was tested. As an example DLW of the KUT linac was chosen [11]. The DLW contains four homogenous pieces conceded with matching cells, altogether there are 35 cells; phase advance is 120° per a cell; v_g drops from 0.024% to 0.009% of c. An analytical steady state solution for accelerating field distribution was evaluated from the equation of power diffusion [12]. The test has shown that results agreed within 3.5%. It is a good result taking into account so steeply changing of v_g.

To check capabilities of the designed algorithm, the full-scale simulation of the KUT linac from an entrance of the injector, which consists of bunching and accelerating cavities, up to an exit of the DLW was carried out. In Fig. 1 the time-space distribution of the self-consistent field in the DLW excited by a RF source and the simulated beam is shown.

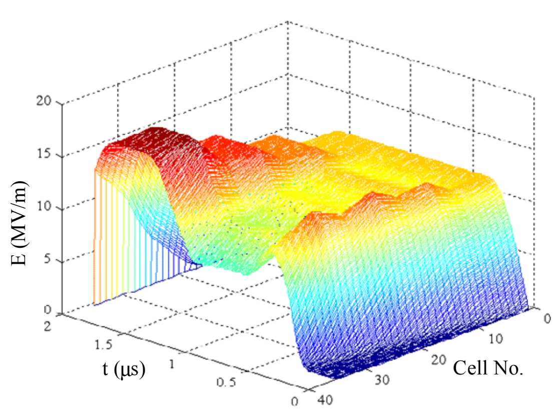

Figure 1: Distribution of the self-consistent field.

The data were obtained at a beam current and RF power that are characteristic for KUT operation (accelerated beam current is about 0.8 A, power of a RF source is 12 MW). The duration of a beam current pulse was chosen shorter than a RF pulse to observe influence of a beam loading.

CONCLUSION

The designed technique of simulation of transients in RF linacs allows obtaining the data on time dependent accelerating fields and characteristics of a beam. The carried out testing has shown, that the model adequately describes physical processes. Accuracy of simulation of the characteristics of fields and a beam corresponds to the approximations that were made while developing the technique. The technique can be useful for design of linacs as well as at researches of beam dynamics.

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