THE MODELLING OF ACCELERATING STRUCTURES WITH FINITE-DIFFERENCE TIME-DOMAIN METHOD

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Abstract

The finite-difference time-domain (FDTD) method is very popular for electromagnetic field modeling. The practical interest in the method is the ability to calculate fields in time domain at any point of the accelerating structure. That is to say the FDTD method is able to model transient process, taking into account the peculiarity of RF power input device. A FDTD approach for modeling of alternate phase focusing structure is presented in this paper. The modeling of lossy metals is a problem in classical formulation of FDTD method. This matter is investigated and one of the solutions is presented. There are some problems of signal processing when using time-domain method for resonant structure modeling. The matters of mode determination are also investigated. The simulation results are compared with experimental data.

INTRODUCTION

So called cross-bar accelerating structure with alternate phase focusing has been taken for modeling (Fig. 1).
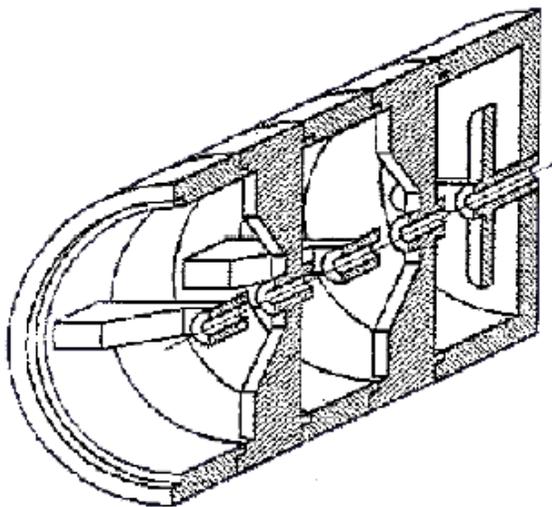
Figure 1: Accelerating structure stereometrical cross-section.

Designing of such cavities is a problem that can't be solved analytically. Therefore, several numerical methods can be applied. The most popular is finite-element method that could be considered today as sufficiently powerful and reliable method, but Finite-Difference Time-Domain (FDTD) method has certain advantages.

While modeling, Maxwell differential equations are solved in finite differences in time domain. The update equations are used in a leap-frog scheme to incrementally render the E and H fields forward in time [1].

Prominent advantage and practical interest of the method is in the ability of transient process simulations. Particularly, such problem arises in the resonant accelerating structures modeling. As shown, FDTD method gives the entire information about transient process. Therefore some problems, such as influence of RF power input device can be resolved with FDTD method [2]. The basic FDTD algorithms and associated Fourie techniques are used to obtain steady-state mode features.

The method presented can be used for accelerator cavity modeling of almost arbitrary configuration.

DESCRIPTION OF ACCELERATING STRUCTURE

For building simulation model, one of manufactured prototypes has been taken. Full-scale structure comprises of metal cylinder of about 1 m long and 200 mm in inner diameter, 39 different lengths drift tubes of 12 mm in inner diameter and 24 mm in outside diameter, and front flanges (Fig. 1).

Acceleration of charged particles takes place on the cylinder axis in the gaps between drift tubes. Each tube is attached to two opposite points via two bars – diametrically oriented holders. Each holder with the drift tube comprises "spar".

To make manufacturing and tuning easier, the structure consists of several "cells", each of them contains one spar. Each cell may be revolved around axis of cylinder independently from other cells. This simplifies the adjustment of accelerating structure – when structure is assembled but not solded, the tuning is made by means of small rotation around their common axis.

Oscillation phases in any two adjacent cells should differ in π when the operation mode is excited. When any other type of oscillation is excited, phases in two or more adjacent cells are the same. Excitation of the structure is performed by the loop placed in one of the end cells.

LOSSY METALS AND Q-FACTOR EVALUATION

For charged particles acceleration, huge RF power is required (up to a few megawatts). Therefore, those

accelerators often work in a pulse mode. Besides, since the accelerator has an extensive structure and Q-factor is of great value, transient process also has long duration. Finite metal conductivity directly influences the transient process nature. Taking metal dissipation into account is a problem that doesn't admit a solution for a classical FDTD method. Prima facie, real metal could be represented as correspondent material, setting correspondent conductivity σ. Then one of Maxwell equations will look like that:

$$\Delta \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

However, the finite difference representation of magnetic field couldn't be implemented in this way. Real metals, even relatively lossy ones, have great conductivity (about 10^7 sm/m), so the field will change very quickly, and, ΔX, ΔY and ΔZ should be taken much less than skin-layer thickness to make model equal to prototype. Such solution of the problem is unacceptable. The way out can be found if the skin-layer thickness is of known value. Then normal derivative of tangent magnetic component could be represented in that way:

$$\frac{\partial H_\tau}{\partial z} = - \frac{H_\tau^{n-1/2}\left(i+1/2, j+1/2, k+1/2\right)}{\delta},$$

where δ is frequency dependent skin-depth. In such representation the model will be equal to real prototype.

Q-factor was computed by means of momentary spectrum computation using discrete Fourier transform (DFT) on different time steps. The results of Q-factor computations evaluated using FDTD method and those, which made using finite element method and obtained from full-scale experiment, are shown in Table. 1. As shown, FDTD method gives enough accurate Q-factor evaluation.

Table 1: Q-factors of the structure, calculated using FDTD, finite element method and measured Q-factor

Frequency, MHz	Q_{FDTD}	Q_{FEM}	Q_{Meas}
905.5	9100	8235	7311
953	10150	8819	5385
1009	6600	9814	5828

MODE ANALYSIS

The very main stage of accelerating structure analysis is eigenfrequencies and calculation of longitudinal axis field distribution. Eigenfrequencies can be obtained when the resonator is excited by a short pulse that excites all modes in the relatively wide band. Gaussian pulse can meet this claim. The pulse bandwidth should be approximately 1…2 GHz. Such band is needed for eigenfrequencies information be obtained. Eigenfrequencies can be retrieved when Discrete Fourier Transform is applied. The greater number of time steps passed and the smaller time step, the higher accuracy of data.

Generally, when determining the mode type, the main interest is to know field amplitude in the middle of gaps and phase relations. Obviously, in standing wave and at arbitrary point there are only cophased and antiphased oscillations. Taking this fact into account, it is enough to obtain Fourier transform in the middle of gaps and then analyze phase relations to determine the mode.

The phase shift is modulo closer to π or to zero. In the operating mode in the adjacent cells there are antiphased and in the spurious mode cophased oscillations. Basing on this and amplitude distribution we could obtain graphs (Fig. 2) that represent amplitude distribution and phase relations. It's necessary for accelerator to work properly that field amplitude in all the cells be the same or, more
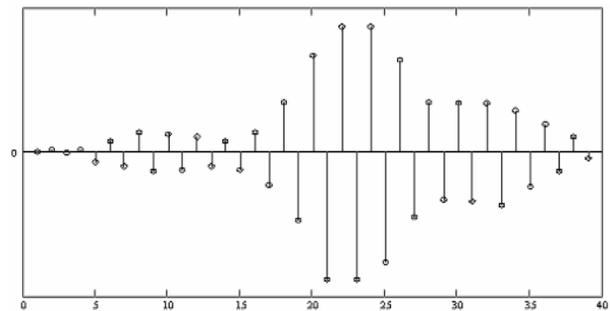
Figure 2: Amplitudes and phase relations in the gaps of operating mode

precisely, lie in a certain range. Today it is obtained on a manufactured prototype by mutual revolving the cells [3]. Such optimization problem has been solved neither by finite element method, nor by FDTD method. Mainly, the reason is in limitation of computational capability of computers available at the moment. FDTD method also has additional constraint concerned with rectangularity of grid. As the result of aliasing, rotation of flat surfaces should invoke heavy geometry distortion.

TRANSIENT PROCESSING

The initial product of FDTD modelling is the transient process at any given point. If the frequency-domain characteristics are needed, the Discrete Fourier Transform (DFT) should be applied. There are some peculiarities of linear accelerator transient signal. Firstly, this signal is digitized with time discrete. Secondly, since the Q-factor is of great value, this signal can be considered closely enough as superposition of several harmonics. DFT implementation of such signals implies the effect of spectrum leakage comes noticeable.

Let's consider the condition when the only harmonic is represented. When its discrete frequency is integer, discrete spectrum consists of only one component (Fig.3). Otherwise, discrete spectrum consists of much more components (Fig. 4). Such effect is called "spectrum leakage". When the discrete frequency of harmonic is integer, it means that number of harmonic periods is integer during the analysis period. Since the cavity eigenfrequencies are not known a priori, the effect of spectrum leakage will also has random nature. This effect
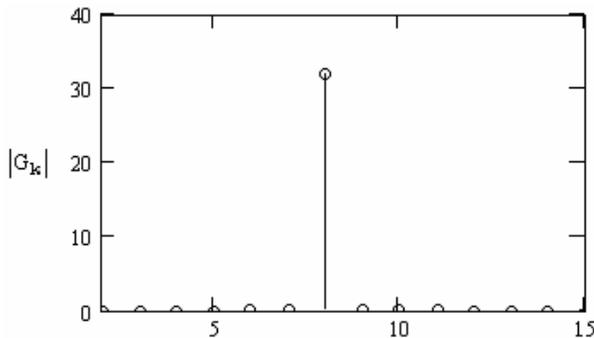
Figure 3: Discrete Fourier transform amplitude sample of harmonic signal with only one component
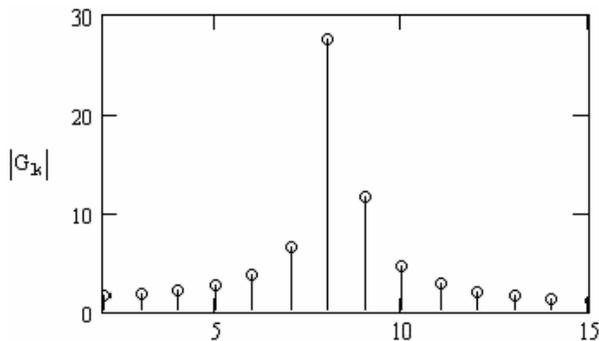
Figure 4: Discrete Fourier transform amplitude sample of harmonic signal with number of component

Figure 5: Discrete spectrum of transient. Solid line is the original discrete spectrum; triangles are the spectrum calculated in the course of optimization.

is considered as parasitic, because the eigenfrequencies of the accelerating structure are very close to each other. The "tails" of leakaged harmonics will interfere with each other, so the parameters of each harmonic will be influenced by other closely situated harmonics.

The lead of this problem is to analyze discrete spectrum of single sinusoid. Since the analytical expression of the spectrum has been known, it becomes possible to consider; the spectrum as a superposition of single harmonics discrete spectrums and to determine the parameters of each harmonic in the course of optimization. In the Fig.5, the program that calculates parameters of harmonics is shown. Solid line is the original discrete spectrum; triangles are the spectrum calculated in the course of optimization. So the improvement of Discrete Fourier transform is developed to precisely determine parameters of each harmonic.

CONCLUSION

The using of the FDTD method for accelerating structure modeling is not discussed in literature. The ability of using of the FDTD method is shown and some problems of implementing this method are solved.

The application of FDTD method to accelerating structures analysis allows to get information about modes, eigenfrequencies, field distribution both along axis and in the whole volume of the structure, while restrictions of the geometry definition accuracy are minimal. The way of taking into account the finite conductivity of metal walls is suggested; Q-factor computation on different modes gives enough accurate evaluation.

The application of FDTD method to accelerating structures analysis produces the signal with complicated spectrum structure. The relative frequency difference between two adjacent modes is about 1%. Discrete Fourier transform of such a signal can result parasitic effect. The values of amplitudes and phases of modes can be distorted or even wrong. One of the ways of harmonic parameter determination is suggested in this paper.

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