A MORE ACCURATE APPROACH TO CALCULATING PROTON BUNCH EVOLUTION UNDER INFLUENCE OF INTRA-BEAM SCATTERING IN A STORAGE RING.

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Abstract

Some perturbations of discrete nature are known to influence the performance of a proton storage ring, contributing to parasitic background, decay of beam currents and bunch tail buildup. Such are, for example, intra-beam scattering and residual gas scattering. These processes are to a large extent described by existing analytical theory. The latter, employing a large amount of averaging, usually neglects effects arising from system nonlinearity. So, the motion of tail particles in the presence of a sufficiently nonlinear RF voltage under influence of intra-beam scattering strongly deviates from the average across the bunch and the analytical approach seems inadequate for it. To overcome this situation we have developed more accurate numerical methods for calculations of bunch evolution under influence of a rather broad class of jump-like perturbations. Here we present the computational algorithms and their application to assessment of coasting beam and proton background in HERA-p.

INTRODUCTION

Particles in a bunch are subject to Coulomb interactions.These interactions become weak at high energies. However, microscopic incoherent interactions, or collisions, still cannot be neglected completely. Such collisions lead to chaotic particle momentum change. A transition of momentum from the transverse to the longitudinal direction is enlarged by the relativistic factor γ and can lead to bunch size growth and particle losses. For proton accelerators like HERA, the intrabeam scattering leads mostly to longitudinal bunch growth. The momentum transfer from the transverse to the longitudinal direction in a single scattering event is approximately [4]

$$\Delta p = \gamma p \cos\psi \tag{1}$$

where ψ is the azimuthal angle to which the particle is deflected in the c.o.m. system. One can either calculate the rate of events leading to direct particle loss (the Touschek effect, [8] [4]) or the average change of the synchrotron invariant (the intra-beam scattering, [7], [2]). To find the density evolution one would need to solve an equation of Boltzmann type. Such an equation is usually too complex and further simplifications are employed. The most common one is the Fokker-Planck or diffusion approximation. It is extensively used, for example, to study the influence of intrabeam scattering on the electron beams where due to synchrotron radiation stationary bunch distrtibution is established. We expect that for the problem of proton (or ion) escape from the stable RF bucket this approach may be inappropriate since it does not take large momentum jumps into account. So, the Touschek losses are not described by a diffusion process. Therefore, the diffusion coefficients depend on the bunch density and such processes cannot be described by a Fokker-Planck equation as soon as the beam distribution changes noticeably on the time interval of interest. To analyse the beam tail distribution and particle losses more accurate computations may be required. We propose a computational procedure that is essentially a method of solution of the Boltzmann equation.

THE CHAIN METHOD

The proposed method is aimed at calculating particle distribution evolution in electromagnetic fields under influence of various types of small discrete perturbations (jumps), perhaps depending on the bunch distribution itself. It rests upon following assumptions.

1. Jumps happen rarely in comparison to typical oscillation periods in the media. Then the statistical information about the bunch is concentrated in the probability distribution of some slowly varying parameters or adiabatic invariants. It is possible to deduce from them the entire information needed about the statistical behavior of the bunch. for example free path distributions and so on.

2. The distribution changes slowly under the perturbations.

3. There is no correlation between motion of individual particles, and of the motion of individual particles and the perturbation so that the motion is approximately a Markov process (process without aftereffect) on small time scales.

4. The motion is not chaotic.

Suppose the perturbation depends on the state of the system, but in such a way, that when we fix the system state it turns into a Markov process. This is the case for intra-beam scattering - under fixed bunch density the scattering can be considered to be a Markov process and the density changes slowly. Then techniques practically identical with those of Markov processes can be applied. Suppose the we are given a Markov process of jump type. Then we can either consider it to have a continuous phase space , write down an appropriate (Chapman-Kolmogorov) integro-differential equation for the density evolution and then develop a discrete numerical scheme. Or we can divide the phase space in cells and assume that the jumps can happen only between such cells.Then the collision process can be modelled by a Markov chain with the modification that the self-consistency is taken into account.

A Markov chain is given by a discrete space of states Ω and transition probabilities from state i to state j in time t $p_{ij}(t)$. The role of the infinitesimal generator of such Markov process is played by

$$a_{ij} = \lim_{t\downarrow 0} \frac{p_{ij}(h) - \delta_{ij}}{h} \tag{2}$$

Therefore, the transition probabilities satisfy the Kolmogorov's system of equations

$$\frac{d}{dt} p_{ij}(t) = \sum_k a_{ik} p_{kj}(t) \tag{3}$$

and the solution satisfies the initial conditions $p_{ik}(0+) = \delta_{ik}$ (see [5]).

Let the transition probabilities $p_{ij}(h)$ depend on the current state of the chain and be sufficiently regular at $h = 0$,

$$p_{ii}(h) = 1 - a_{ii}h + O(h^2)$$
$$p_{ij}(h) = a_{ij}h + O(h^2), \quad i \neq j$$

denote

$$x_1 = p_{10} \ldots x_{n+1} = p_{1n}, x_{n+2} = p_{20} \ldots x_{n(n+1)} = p_{nn} \tag{4}$$

then

$$\dot{x}(t) = \mathcal{A}x(t)$$

where

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}_{11} & \ldots & \mathcal{A}_{1n} \\ \ldots & \ldots & \ldots \\ \mathcal{A}_{n1} & \ldots & \mathcal{A}_{nn} \end{pmatrix} \tag{5}$$

and $\mathcal{A}_{ij} = diag\{a_{ij}\} \in \mathbb{R}^n$ is a diagonal matrix with diagonal elements all equal.The solution is

$$x(t) = \exp\{At\}x(0)$$

The transition matrix $P(t) = \{p_{ij}(t)\}$ is thus found and the probability distribution at time t is $p(t) = p_0^T P(t)$ where p_0 is the initial distribution.

Applying this formalism in small steps and recalculating A after each step, we have a tracking procedure for the distribution.

$$\begin{array}{ccccc} x_\tau & = & x_0^T P(x_0, \tau) & ,0 \leqslant \tau \leqslant h_1 \\ x_\tau & = & x_{h_1}^T P(x_{h_1}, \tau) & ,h_1 \leqslant \tau \leqslant h_2 \\ \ldots & \ldots & \ldots & \ldots \\ x_\tau & = & x_{h_i}^T P(x_{h_i}, \tau) & ,h_i \leqslant \tau \leqslant h_{i+1} \\ \ldots & \ldots & \ldots & \ldots \end{array} \tag{6}$$

In [1] convergence estimates are given.Due to the special structure of the Kolmogorov system it admits a fast solution algorithm:

$$x(t) = e^{At}x_0 = (E + (At) + \frac{1}{2}(At)^2 + \ldots)x_0$$

$$e^{At} = \sum_{i=0}^{\infty} C_i$$
$$C_i = C_{i-1}\frac{1}{i}(At)$$
$$C_0 = E$$

and because of the sparse structure of A the number of operations required to calculate e^{At} with precision ε is estimated as $O(n^4)(\log \varepsilon)^{-1}$. In practice n is no more than 40 and the time step is sufficiently large so that the most time-consuming task is the calculation of infinitesimal generator A.

In the case of intrabeam scattering collisions happen rarely and thus their probabilities do not depend on local bunch density but on the average density around a test particles over a large time interval. In other words by the averaged density

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T f(g_t(\phi, p))dt = \hat{f}(\phi, p)$$

where g_t is the transformation associated with the synchrotron motion. One can show that in case this motion is Hamiltonian the averaged density has the form $\hat{f}(q, p) = |J|\rho(H(q, p))$ where J is the Jacobian of the action-angle transformation, ρ an arbitrary function and H the Hamiltonian function. This shows that the statistical information needed for description of the scattering process is detemined by the distribution of the invariant.

An average of a function $u(q, p)$ over the invariant curves with respect to the ergodic density $\hat{f}$ is

$$\langle u \rangle |_h = \int_{H(q,p)=h} \hat{f}(p, q)dqdp \tag{7}$$

All possible values of the invariant are divided into states $\{\Omega_i\}_{i=1}^N$. In the longitudinal plane we have to distinguish between various regions of the phase space which may have the same value of the invariant. So we first divide the phase space into domains not containing separatrices and then enumerate them as before.

The distribution is then given by the vectors $\{\rho_i\}_{i=1}^N$. When a collision occurs , the collided particle either stays in the same state or jumps to another. Transition probabilities $p_{ij}(t)$ (depending on time) denote the probabilities that a particle starting in an arbitrary point lying in state i lands in state j after time τ. To apply the chain formalism one needs to know the infinitesimal generator. To obtain it one inserts some finite time τ into expression 2 for which $p_{ij,t}(\tau) - \delta_{ij}$ is still small. This can be, for example, the time of one turn in the ring, one second or any other time for which the probability of two or more collisions is negligibly small compared to the probability of just one collision. Let the transition probabilities between the states resulting in a single collision be given by the matrix $T = \{T_{ij}\}$ and the probabilities of (one) collision be given by $\hat{\gamma}_i(\tau) = \gamma_i\tau + O(\tau^2)$, then for a sufficiently small τ the transition matrix $p_{ij}(\tau)$ is given by

$$\begin{pmatrix} T_{11}\gamma_1\tau + (1-\gamma_1\tau) & \dots & T_{1n}\gamma_1 \\ T_{12}\gamma_2\tau & \dots & T_{2n}\gamma_2 \\ \dots & \dots & \dots \\ T_{n1}\gamma_n\tau & \dots & T_{nn}\gamma_n + (1-\gamma_n\tau) \end{pmatrix} \tag{8}$$

Then

$$\{a_{ij}\} = \lim_{\tau\to 0}\frac{p_{ij}-\delta_{ij}}{\tau} = \Gamma(T-E) \tag{9}$$

where $\Gamma = diag\{\gamma_i\}$ and E is the unit matrix. T_{ij} is obtained by averaging the collision cross-section over the ergodic density and the collision probability $\gamma(t)$ is proportional to the average bunch density on the particle path. We finally get the expression for the infinitesimal generator of te form

$$a_{ij} = r_0^2\pi N_p diag\{\langle v\rangle \langle f(q)\rangle_{H\in\Omega_i}\}(T-E) \tag{10}$$

with the cutoff impact parameter r_0.

APPLICATION TO THE STUFY OF THE COASTING BEAM PRODUCTION

The coasting (unbunched) beam has been a serious problem in the operation of the HERA proton ring. After about 10 hours the proton bunched beam of 100mA produced 1-2mA of the coasting beam current (see figure 1).

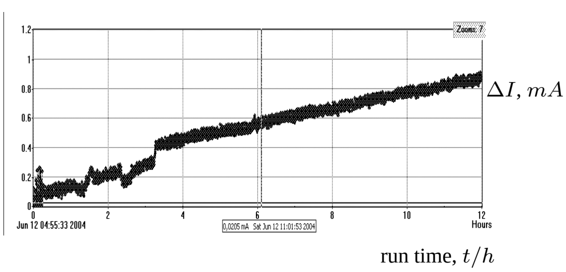

Figure 1: The observed coasting beam current at HERA-p

The suspected sources have been the Rf noise and the intrabeam scattering. We applied the described technique to assess the portion of the coasting beam produced through intrabeam scattering. The simulations showed that the bunch bahavior is expected to be diffusive. The expected rates of escape from the Rf buckets caused by the intrabeam scattering are below those required to accumulate the observed coasting beam current.The bunch density evolution and the loss rates from the stable Rf bucktes are shown in figures 2, 3. Recent studies of influence of the Rf noise [6] indicate that it is potentially the major cause of the problem [1]. We think that the proposed simulation technique may appear to be useful for the assessment of the impact of intrabeam scattering on the beams of heavy ions for which this effect is stronger and higher loss rates are expected.

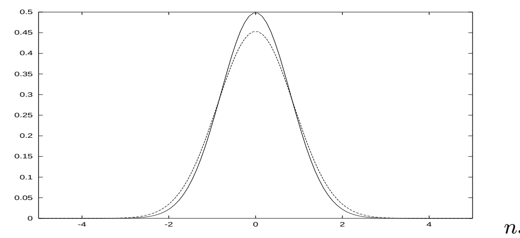

Figure 2: Distribution function diffusion over 10 hours

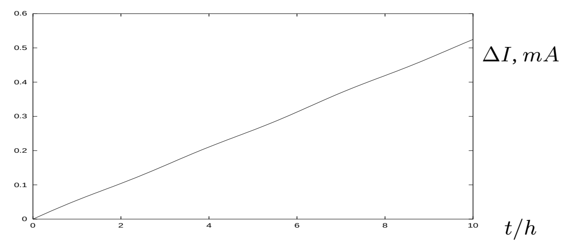

Figure 3: Loss rate from the Rf bucket

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[1] in [1] it is proposed that the usual assumptions about the mixing properties of the synchrotron motion used to derive the Fokker-Planck equations for the synchrotron motion with Rf noise [3] may not be true and a correction to the diffusion coefficient is introduced. With this correction the diffusion in the bunch centre is weaker. This leads to a possibility that the Rf noise has a strong impact on the tail particle stability while exhibiting smaller influence on the bunch core. This propositions seems to be in agreement with observations at HERA.