IMPLEMENTATION OF HIGHER ORDER MOMENTS FOR BEAM DYNAMICS SIMULATION WITH THE V-CODE*

W. Ackermann, T. Weiland, Technische Universität Darmstadt, TEMF,
Schloßgartenstraße 8, D 64289 Darmstadt, Deutschland

Abstract

Based on the moment approach, V-Code is implemented to simulate charged particle beam dynamics in linear accelerators. Its main aim is to perform elementary studies in those cases when the beam can be considered as a whole thus making the motion of individual particles negligible in the overall view. Therefore, an ensemble of particles can be well described by the moments of its phase-space distribution and the regarded order influences consequently the achievable accuracy as well as the computational effort. Since the well known moment equations generally are not closed, a technique to limit the number of involved moments has to be applied. So far all the moments up to the second order have been considered whereas higher order moments were truncated. As a further step towards higher accuracy, the influence of higher order moments has to be investigated. For this reason additional fourth-order equations are implemented into the V-Code and the achieved results are compared with previous second-order-based ones.

INTRODUCTION

Detailed numerical simulations of charged particle beam transport including space charge fields and radiation effects are generally computationally expensive and hence extremely time consuming. In order to perform faster analysis of such significant beam characteristics like averaged particle positions and momenta, a simplified model based on the moments of the underlying charge distribution function can be employed. Programs which utilize the so-called *Ensemble Model* are typically used for fast online simulations or in optimization processes where a lot of parameters have to be regarded for detailed studies.

ENSEMBLE MODEL

The time evolution of a particle density distribution function f is usually carried out in the space of coordinates and momenta but it can also be systematically formulated using FOURIERs theorem. Instead of considering the density function of the individual particles itself, it is possible to apply a multidimensional transformation of the particle distribution function and operate preferably in the FOURIER space. This procedure leads to an approach which introduces different kinds of moments of somehow locally arranged particles and therefore does not need a description of the whole ensemble using lots of discret distributed particles.

Since the principal behaviour of the *Ensemble Model* can still be traced using a simplified description, it is convenient to limit the derivation of the method to the 2D case. All necessary relations can then be obtained for the real model by expanding the corresponding fundamental equations to the 6D space. First of all the 2D-FOURIER transformation

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{j(ux+vy)}\, \mathrm{d}x\mathrm{d}y$$

with the kernel exponential function expanded as a power series in a quite common way is considered. Due to the fact that the infinite expression is uniformly convergent, the order of summation and integration can be interchanged

$$F(u,v) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(ju)^k}{k!}\frac{(jv)^l}{l!}\underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, x^k y^l\, \mathrm{d}x\mathrm{d}y}_{M_{kl}}$$

and the resulting terms are arranged in a proper way. The FOURIER transformed function is therefore given by

$$F(u,v) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{j^{k+l}}{k!\,l!}\, M_{kl}\, u^k v^l$$

$$M_{kl} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^k y^l\, f(x,y)\, \mathrm{d}x\mathrm{d}y\ ,$$

where the coefficients $M_{kl} = < x^k y^l >$ are the moments of the distribution function. In practice the series are not expanded to infinity but are rather truncated at a given maximum order. The approximated FOURIER transformed function

$$\tilde{F}(u,v) = \sum_{k=0}^{M}\sum_{l=0}^{M}\frac{j^{k+l}}{k!\,l!}\, M_{kl}\, u^k v^l$$

is then used in a computer code to describe the characteristics of a real particle beam.

In most cases it is sufficient to consider the different kinds of moments without taking into account the actual particle distribution. This is due to the fact that the moments precisely describe such physical identities like mean values or variances.

Usually, the moments are given in different forms. Together with the already introduced raw moments with weighted monomials as integrands, the centralized moments with shifted coordinate systems are also in use. All

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raw moments of the distribution function depend on the real location of the particles and they are typically used to describe the averaged positions, whereas the centralized moments are commonly translatory invariant and therefore employed to determine the shape of the particle distribution function regardless of the actual position.

The time evolution of either raw or central moments can then be derived by integrating the first order differential equations

$$\frac{\partial <\mu>}{c\,\partial t} = \int \frac{\partial\, \mu\, f}{c\,\partial t}\,\mathrm{d}\vec{r}\mathrm{d}\vec{p} = \int (f\,\frac{\partial\,\mu}{c\,\partial t} + \mu\,\frac{\partial\, f}{c\,\partial t})\,\mathrm{d}\vec{r}\mathrm{d}\vec{p}.$$

The right-hand side of this equations can be rewritten using

$$\frac{\partial\,\mu}{c\,\partial t} = \mathrm{grad}_{<\vec{r}>}(\mu)\cdot\frac{\partial <\vec{r}>}{c\,\partial t} + \mathrm{grad}_{<\vec{p}>}(\mu)\cdot\frac{\partial <\vec{p}>}{c\,\partial t}$$

$$\frac{\partial\, f}{c\,\partial t} = \mathrm{grad}_{\vec{r}}(f)\cdot\frac{\vec{p}}{\gamma} + \mathrm{grad}_{\vec{p}}(f)\cdot\frac{\vec{F}}{m_0c^2} + f\,\mathrm{div}_{\vec{p}}(\frac{\vec{F}}{m_0c^2})$$

with the relativistic factor γ and the applied forces $\vec{F} = \mathrm{d}m\vec{v}/\mathrm{d}t$. Partial integration together with a proper ordering of the different integrals allows then to rewrite the fundamental differential equations in the more compact form

$$\begin{aligned}\frac{\partial <\mu>}{c\,\partial t} &= <\mathrm{grad}_{<\vec{r}>}(\mu)> \cdot <\frac{\vec{p}}{\gamma}> \\ &+ <\mathrm{grad}_{<\vec{p}>}(\mu)> \cdot <\frac{\vec{F}}{m_0c^2}> \\ &+ <\mathrm{grad}_{\vec{r}}(\mu)\cdot\frac{\vec{p}}{\gamma}> + <\mathrm{grad}_{\vec{p}}(\mu)\cdot\frac{\vec{F}}{m_0c^2}>,\end{aligned}$$

which, however, cannot be used directly for a systematic implementation. In order to proceed, it is necessary to express the right-hand side in terms of the time-dependent ensemble parameters only, i.e. the underlying moments. This is achieved by providing a series expansion of $1/\gamma$ as well as by a series expansion of the applied forces $\vec{F}$ in a given operating point and utilizing a truncation which is strongly related to the regarded order of moments.

If the energy spread is small enough compared to the mean energy of the whole particle ensemble

$$\gamma_{\mathrm{rms}} = \sqrt{<\gamma^2>} = \sqrt{1 + <p_x^2> + <p_y^2> + <p_z^2>},$$

it is reasonable to use just a linear approximation of $1/\gamma$; in all other cases higher order approximations are required. The series expansion for the forces can be formally performed by dividing them into two parts

- internal space charge motivated forces and
- forces due to external fields,

the latter being typically generated by RF-cavities or any type of magnet for example.

The external fields vary along the whole beam line, but due to the local structure of the bunched beam in an RF-accelerator the desired series expansion in the vicinity of the particles nevertheless can be carried out.

IMPLEMENTATION

In order to implement an algorithm based on the ensemble model, a proper differential equation for each involved ensemble parameter has to be derived. Starting from the given fundamental differential equations, it is necessary to calculate analytically partial derivatives, to achieve formula expansions and to rearrange them in a proper way. This allows to rewrite the equations in such a form that they can be handled by a more general time integration process.

This important part can be done automatically using a software designed for analytical computations (e.g. Mathematica, Maple). The recently developed Mathematica application is capable to perform all the analytical calculations and subsequently hands over the systematically rearranged equations to the sources of V-Code in a ready-to-use notation.

Based on the ensemble model, the program V-Code was developed under a C++ environment to perform fast and accurate beam dynamics simulations not only for single components but also for complete accelerator structures. The program is designed in such a way that the whole machine can be given as a sequence of various beam line elements whereas the differences arise in the treatment of the applied external fields.

The fundamental preparatory work to be done is to provide a series expansion of all the forces for each element with respect to the center of a locally bounded charge distribution. Starting from a given expansion, the Mathematica script then automatically derives all the desired update equations for V-Code up to an arbitrary order.

SIMULATION

To demonstrate the capabilities of the implemented moment approach, a representative test problem has to be specified. In the following section all performed calculations are related to the PITZ gun which is currently operated at DESY Zeuthen.

The selected test system consists of a 1.5-cell RF-gun and a subsequent TESLA 9-cell-cavity. In Fig. 1, a typical schematic diagram of the examined structure is given to get an impession of the mechanical dimensions and the principal arrangements of all the individual components.

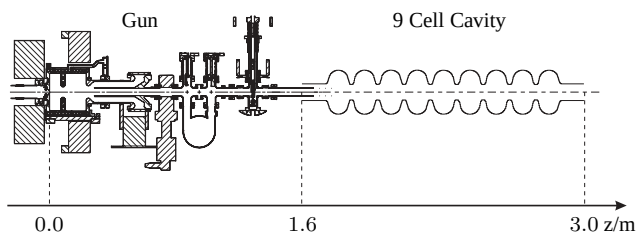

Figure 1: Layout of the examined structure consiting of a gun part and a subsequent TESLA 9-cell-cavity.

The operating conditions and parameters for the simulation were adjusted in such a way that at the end of the gun a

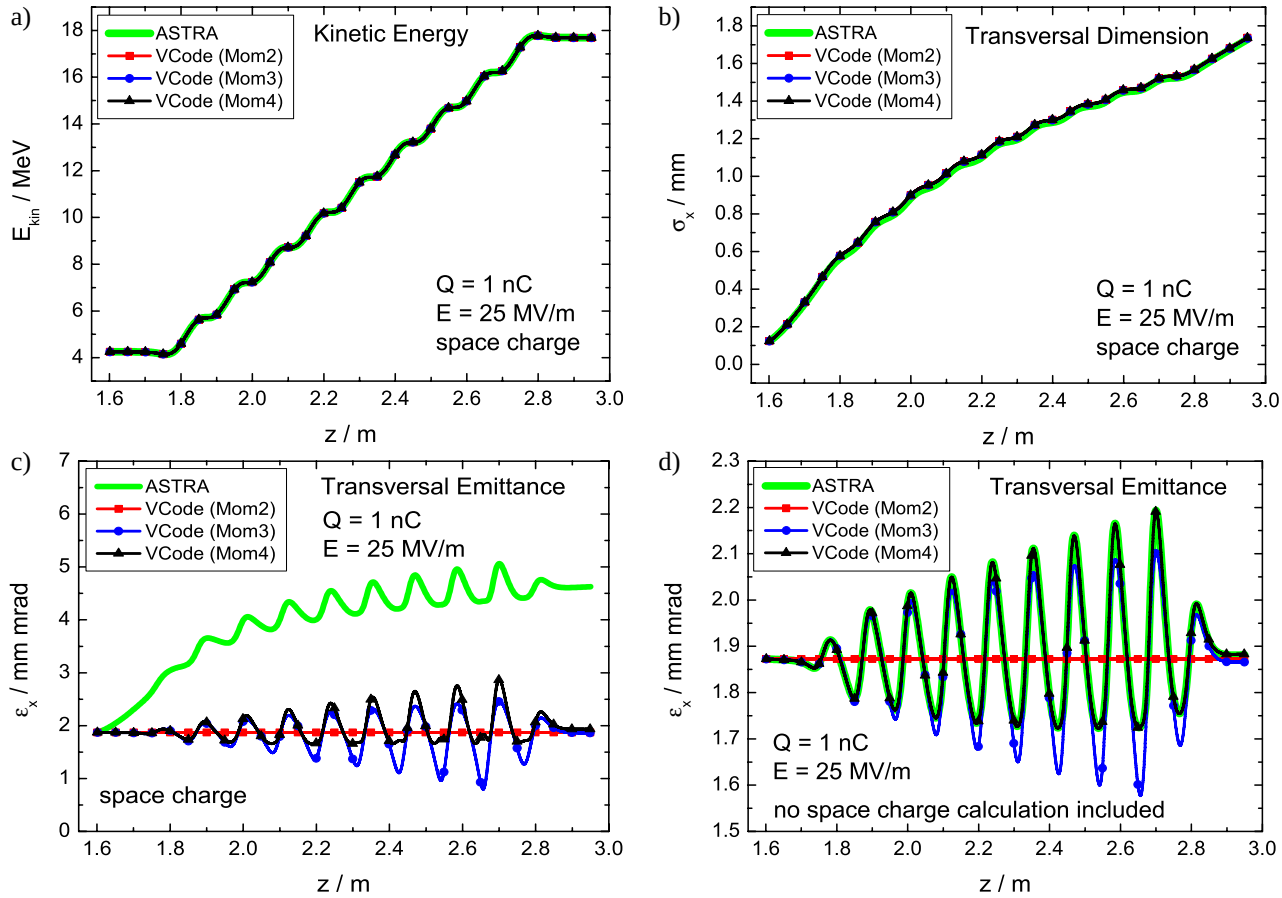

Figure 2: Simulation results for the cavity part of the examined test structure: a)-c) space charge forces are included; d) space charge forces are omitted.

bunched beam with a minimum transversal emittance was obtained. The bunch shape specified by the a priori determined laser profile, and the bunch charge derived from the laser intensity are to be adjusted to approximate the real parameters. The RF-gun and solenoid settings are within the operating range of the real machine.

The following simulations have been performed with different computer programs employing either various orders of momemts (V-Code) or individual macro particles (ASTRA) [6]; both codes do not take into account any interference with the surrounding environment.

In order to show the principal behaviour of the moment approach, the presented results (Fig. 2) focus on the cavity simulation while the gun part is omitted.

CONCLUSION

The illustrated diagrams emphasize the fact that moment based codes are able to correctly simulate the behaviour of charged particle beams in accelerator structures with real parameters even if only lower order moments are considered. In the current version of the presented moment based code, a linear space charge model is implemented . This feature leads to the fact that the corresponding emittance calculations do not agree with those that probably could be obtained using a more accurate approach. Test calculations with omitted space charge forces illustrate that this mismatch is not due to the limited amount of applied moments but are rather caused by the specifics of the used space charge model.

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