Comparison of Rate Equation Models for Equilibrium Beam Parameters

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Abstract

We calculate equilibrium beam parameters from the counteraction of intrabeam scattering (IBS), electron cooling (EC) and, possibly, target interaction for typical beams in the GSI cooler storage ring ESR and in the proposed HESR. This work is complementary to kinetic modeling efforts at GSI. As IBS models we employ various approximations to the exact Coulomb scattering model. We developed an easy to use simulation tool that includes various models for the EC rates and the IBS rates, averaged over the detailed ring lattices. The obtained scaling laws of the equilibrium parameters with beam current are compared with existing experimental data from the ESR. In addition, we predict equilibria for the HESR.

INTRODUCTION

Important quantities for the quality of newly designed storage rings are the minumum reachable horizontal and vertical emittances $\epsilon_{\rm z}$ (z=h,v) and the longitudinal momentum spread $\epsilon_{\rm s} = \delta p/p$. Emittance growth due to intrabeam scattering (IBS) is a major limitation for high energy high density proton and heavy ion machines, presently existing or proposed like the high intensity upgrade of the SIS18 or the proposed antiproton storage ring HESR [1]. In addition, the reachable maximum particle density depends strongly on how IBS can be compensated for by electron cooling (EC).

Hence calculations of the growth rates of the horizontal and vertical emittances $\dot\epsilon_{\rm z}/\epsilon_{\rm z}$ and of the (longitudinal) momentum spread $\dot\epsilon_{\rm s}/\epsilon_{\rm s}$ due to IBS are necessary for quick estimates of the final beam quality. These rates were put into a Python script and quilibrium is found by make the total rates stationary. In the spirit of Sørensen [2] we have derived with this method scaling laws with current of the emittances, see [3].

RATES

Apart from Struckmeiers [4] method, the existing theories of IBS are based on the theory of Coulomb scattering by Piwinski [5] and Bjorken-Mtingawa [6, 7]. Struckmeier [4], on the other hand, obtained IBS rates by treating intrabeam scattering as a stochastic process based on equilibration of the entropy, however, not in a relativistically invariant way and without bending magnets. Bhat *et al.* [8] have applied the standard IBS theory with the approximation of Piwinski [9] to beams in the Fermilab Antiproton Accumulator and Fischer *et al.* [10] to RHIC beams. The approximations employed in the different formulae are as follows:

- Wei [11] employs a simplified lattice,
- Rao and Katayama [12] just rewrite the remaining integrals,
- Piwinski [9] uses linear coupling between vertical and horizontal betatron oscillations,
- Parzen [13] employs a high energy approximation.

All these approximations result in rate expressions like

$$\frac{\dot\epsilon_{\rm z}}{\epsilon_{\rm z}} \propto \frac{I}{\epsilon_{\rm h}\epsilon_{\rm v}\epsilon_{\rm s}} \times f(\epsilon),$$

where $f(\epsilon)$ is a function which depends only weakly on the three emittances and on the Twiss parameters.

The cooling rates of electron coolers obtained by various groups, on the other hand, are still debatable. The Novosibirsk group, for example, assumes the vertical cooling force [14]

$$F_z \propto \frac{v_{\rm z}}{(v_0^2+v^2)^{3/2}},$$

where $\mathbf{v}$ is the particle velocity and v_0 is a fitting parameter. Integrating over the Gaussian particle density they obtain

$$\frac{\dot\epsilon_{\rm z}}{\epsilon_{\rm z}} \propto \int_0^\infty \frac{{\rm d}u\sqrt{u}\,{\rm e}^{-a^2u^2/2}}{(1+\epsilon_{\rm z}u/\beta_{\rm z})^2\sqrt{1+\delta_{\rm z}^2u/\gamma^2}},$$

where $\delta_{\rm z}, \beta_{\rm z}$ are Twiss parameters and γ is the relativistic factor. This expression has two limits: for large v_0 it turns into a constant which is independent of the emittances, whereas for small v_0 it becomes proportional to $(\epsilon_{\rm z}\beta_{\rm z})^{-3/2}$. The longitudinal cooling rate is based on similar reasoning. The Dubna group, on the other hand [15], uses diferent frictional force which is fitted at existing experimental data. As shown in ref. [16], experimental data of equilibrium momentum spreads are reasonably well reproduced by constant electron cooling rates.

Internal target effects have been treated, again with the approxiomation of small angle Coulomb scattering, by Hinterberger and Prasuhn [17]. The resulting heating rates are incorporated in our analyses.

RESULTS

The procedure to obtain equilibrium emittances and momentum spreads is as follows: First, MAD X calculates the Twiss parameters from the lattice under consideration. Then the IBS rate is averaged over one turn and the sum

Table 1: Equilibria of IBS with different approximations and constant electron cooling in the ESR for an U^{92+} beam

	Wei [11]	Rao [12]	Piwinski [9]	Struckmeier [4]
$\delta p/p$ $[10^{-4}]$	2.0	2.2	2.9	2.9
$\epsilon_{\rm h}$[mm mrad]	0.69	1.6	1.8	3.0
$\epsilon_{\rm v}$[mm mrad]	0.92	1.7	0.8	3.0

of (positive) IBS rate and (negative) EC rate plus, possibly, the (positive) target heating rate is minimized to yield the stationary values.

Some results for the ESR (without internal target) are shown in Fig. 1: The upper line shows the full equilibrium solution of the IBS rates with the Piwinski approximation [9] and constant cooling with cooling times set equal to 100 sec. The line with slope 0.3 is obtained when the vertical emittance remains fixed because in the simulations it comes out to be negaive. Note that the experimental slope is around 0.27. On the other hand, the Novosibirsk EC cooling rates [14] mentioned above become proportional to $\epsilon_{\rm h,v}^{-3/2}$, yielding about twice as big slopes, not in agreement with the experiment.

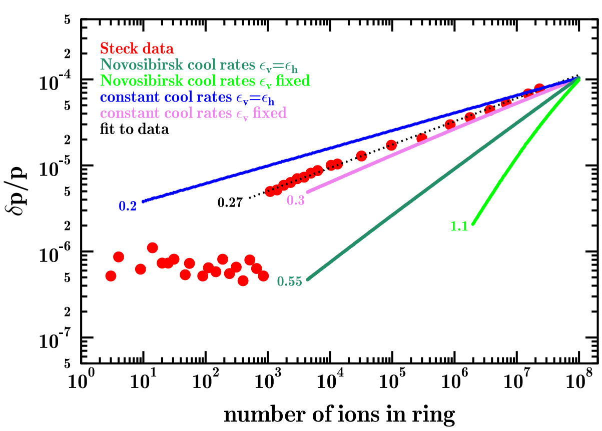

Figure 1: Measured (dots [16]) and calculated (with the Piwinski approximation [9]) equilibrium momentum spreads in the ESR for U^{92+} at 360 MeV/u. The lower left dots are ultracold values. Numbers indicate the slopes with current.

The full solution including the dependence in the function depending weakly on the Twiss parameters yields a slope of 0.55 in Fig. 1 and, setting $\epsilon_{\rm h} = \epsilon_{\rm v}$ a slope of 1.1. The best fit to the data, excluding the ultracold ones [16] is obtained with a slope around 0.27, closer to the results with constant cooling rather than cooling with the Novosibirsk rates. The same behaviour is extracted with all the approximations for the IBS scattering models. In Fig. 2 all slopes of the equilibrium emittances and the momentum spreads evidently are the same, their absolute magnitude, however, varies at most by a factor of two. Calculations for the ESR yield differences up to factors of four, see Table 1.

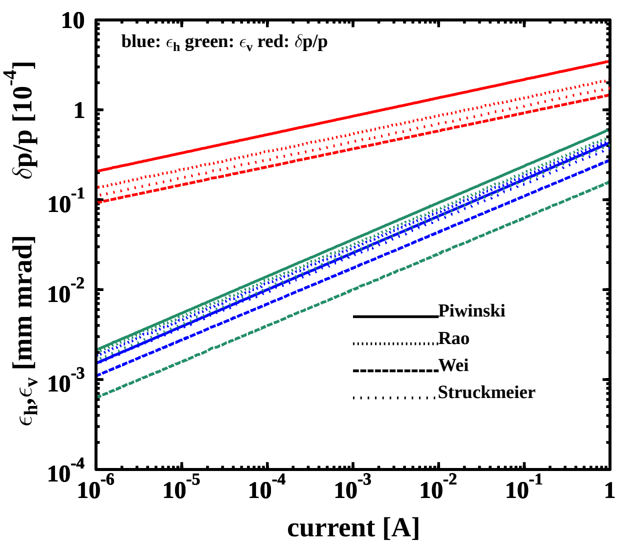

Figure 2: Equilibrium momentum spreads (upper bunches in red) and emittances (lower bunches in blue and green) calculated with different approximations for IBS and constant cooling times of 100 sec. (HESR, 1GeV)

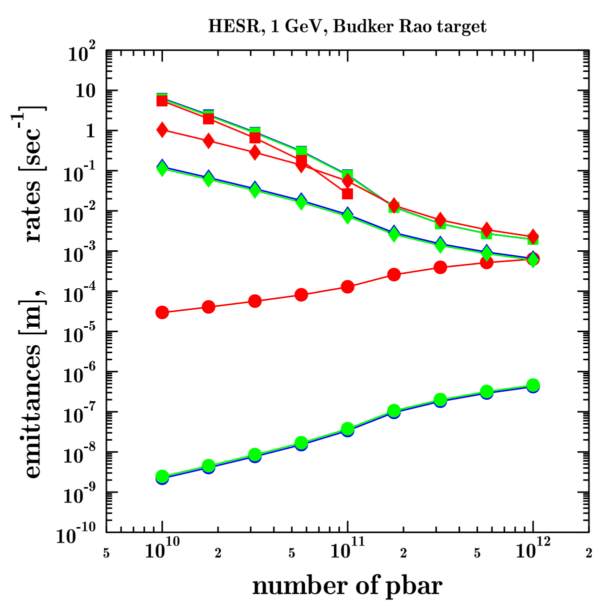

Figure 3: Rates and emittances for antiprotons in the HESR at 1GeV (Rao IBS approximation, Novosibirsk cooling rates). squares: rates, diamonds: target heating rates, circles: emittances and momentum spreads, red: momentum spread, blue: horizontal, and green: vertical emittance

Employing the electron cooling model of the Novosibirsk group [14] instead of constant cooling times, on the other hand, manifests itself in larger slopes as mentioned above. This can also be observed in Fig. 3 for the case of antiproton beams at 1 GeV in the proposed HESR [1]. here the total rates and, hence, also the IBS and EC rates decrease with higher beam density, about by one order of magnitude with one order of magnitude increase of the total number of $\bar{p}$, whereas the emittances and the momentum spread increase by one order of magnitude.

CONCLUSION

The rates obtained with different approximations to the theory of IBS by Coulomb scattering differ roughly by a factor of two to four. In addition, the rates obtained from theories of electron cooling give rather different results as concerns the dependences on the Twiss parameters. This stems from the fact that the analytic form of the cooling forces are still under discussion. As a result, theoretical verification of existing data and, in particular, predictions for new rings are still subject to variations up to a factor of four.

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