SIMULATION OF MULTI-BUNCH MULTI-TURN INSTABILITIES IN HIGH ENERGY PROTON RINGS: ALGORITHMS AND RESULTS

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Abstract

A simulation code to study collective effects in multi-bunch proton machines has been developed and applied to the CERN SPS and LHC. The 3D simulation program allows the exploration of long-range effects due to resistive-wall and HOMs in circular, elliptic and rectangular vacuum chambers also for uneven filling schemes. The code has been benchmarked with measurements in the SPS. Preliminary beam stability results obtained for LHC are presented as well.

INTRODUCTION

High energy proton accelerators and storage rings are more and more operated with multiple bunches of high intensity to achieve maximum possible luminosity for the physics experiments. In this case the unavoidable finite impedance of the machine will cause bunch-to-bunch coupling, giving rise to multi-bunch instabilities.

Concerning the upcoming operation of LHC, lots of effort has been invested to keep the machine impedance as low as possible. However, instabilities could still become an issue as far as nominal and ultimate beam intensities are concerned [1]. As analytical frequency domain calculations do not allow for uneven filling schemes nor transient effects (e.g. after injection) to be explored, a computer simulation was developed to be able to study those problems. The primary challenge for such a simulation is to efficiently implement the multi-bunch multi-turn effects for a large number of bunches like in LHC ($N_b \approx 3000$).

SIMULATION SCHEME

The code employed is based on previous works [2, 3] dedicated to single bunch collective effects in electron machines. Concerning the LHC, the emphasis was now put on the transverse long-range regime $\tau \gg \sigma_\tau$, that is the region where wake fields affect succeeding bunches at a given turn or at subsequent turns. Here τ is the time interval between the passage of the exciting and the trailing charge, σ_τ is the bunch length. Impedances with correspondingly long-lasting wake fields are the resistive-wall impedance and narrow-band impedances like Higher Order Modes (HOMs) of cavities. The latter are very well described by a resonator model with three parameters: R, the shunt impedance, Q, the quality factor and ω_r, the resonator frequency.

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Working with wake tables in the long-range regime is not particularly practicable, hence analytical formulae are preferred and used. Concerning the LHC and its collimation system, a better resistive wall impedance model has been developed and the corresponding wake function has been calculated in closed analytical form [4].

The longitudinal distribution is neglected and accordingly the wake potential is approximated by its wake function. Then the description of HOMs becomes particularly simple: The resonator wake function $W_\perp(\tau) \propto \exp(j\,\omega_1\tau)$, where $\omega_1 = \omega_r/Q\,(j/2 + Q')$ and $Q' = \sqrt{Q^2 - 1/4}$, can be represented by a complex quantity (Phasor) and time evolution and summation of wake contributions reduce to simple multiplication with an complex exponential and vector addition in the complex plane [5, Chap.3]. This allows a large number of different resonators to be efficiently treated in the simulation.

The square-root dependence of the resistive wall wake function $W_\perp(\tau) \propto 1/\sqrt{\tau}$ does not allow for such simple mathematical operations. Therefore a fast wake summation algorithm based on FFT convolution has been developed [6] and is shortly described in the next section.

Approximating the wake potential $W^\perp_{pot}(\tau)$ by its wake function $W_\perp(\tau)$, i.e. neglecting bunch oscillation modes, is especially justified for the resistive wall wake function, which is almost constant (less than 5% variation) in the time interval of a normal bucket length $t \in [-t_\sigma, t_\sigma]$,

$$W^\perp_{pot}(\tau) = \int_{-\infty}^{\tau} \xi(t)\,\lambda(t)\,W_\perp(\tau - t)\,dt = \int_{-t_\sigma}^{t_\sigma} \xi(t)\,\lambda(t)\,W_\perp(\tau - t)\,dt = \approx W_\perp(\tau) \underbrace{\int_{-t_\sigma}^{t_\sigma} \xi(t)\,\lambda(t)\,dt}_{\approx\, q\,\cdot\,\bar{\xi}}, \quad (1)$$

where $\lambda(t)$ is the line charge density and $\xi(t)$ are the normalized varying displacements of the bunch. q denotes the charge of the bunch and $\bar{\xi}$ its average displacement. Note that this means that no head-tail mode excitation can be conveyed by the resistive wall wake function in the long-range regime.

THE PROGRAM MULTI-TRISIM

The predecessor code TRISIM3D [2] has been extended to handle multiple bunches, incorporating all the previously described ideas. A hybrid approach has been chosen to allow for a fast multi-bunch tracking: one detailed bunch,

consisting of up to 10^5 macro-particles plus one macro-particle each for the total set of bunches. Only the rigid dipole motion of the bunches in the long-range regime can be simulated. The reason to preserve the detailed bunch is the possibility to study single bunch effects as well and in particular to see possible head-tail modes, that might arise from high-Q resonator wakes in the long-range regime.

The longitudinal motion is described by the energy deviation ϵ and time delay t with respect to a synchronous particle. For the transverse motion the horizontal and vertical positions x, y and slopes x', y' are represented by the 4-dimensional vector $\vec{z} = \{x, x', y, y'\}$. The different (impedance) elements of the accelerator are taken into account by point-like interactions (IPs), i.e. a kick is applied to the particle while the position remains the same. The arc sections between the IPs are described by linear transfer matrices. Note that the distributed effect of the resistive wall is condensed into a point-like IP.

Non-linear elements such as octupoles have been added to the code to provide a source of Landau damping, which is important for protons, which are not subject to radiation damping. The geometry is properly taken into account for each element using Yokoya factors and also using the quadrupolar (detuning) wake component which depends on the witness particle offset.

FAST WAKE SUMMATION VIA FFT CONVOLUTION

The problem of summing up the wake contributions over previous turns and bunches can be summarized as follows. The kick $\Delta x'$ on bunch j at turn n, in the case of the resistive wall impedance, is apart from constants given by

$$\Delta {x'}_n^j = \textit{const.} \cdot \left[\sum_{i=0}^{j-1} \frac{\langle x \rangle_n^i}{\sqrt{(j-i)\cdot\tau_{buc.}}} + \right.$$
$$\left. + \sum_{\substack{k= \\ n-n_{\text{mem}}}}^{n-1} \sum_{i=0}^{N_b-1} \frac{\langle x \rangle_k^i}{\sqrt{(n-k)\cdot\tau_{rev.} + (j-i)\cdot\tau_{buc.}}} \right] . \quad (2)$$

Note that the summation is truncated for $k < n - n_{\text{mem}}$, because the fields penetrate the vacuum chamber proper after some diffusion time [7, Eq.(13)]. Here n_{mem} is usually in the order of a few up to 100 turns. One summation is taken over turns k, the other over bunches i, where the total number of bunches is N_b. The revolution time is denoted by $\tau_{rev.}$, $\tau_{buc.}$ is the bucket spacing and $\langle x \rangle_k^i$ denotes the offset of bunch i at turn k.

Expression (2) can be cast into the framework of convolution theory. This is especially visible if the problem is restricted to one bunch $N_b = 1$ only. Then (2) reads

$$\Delta x'_n = c \cdot \sum_{k=0}^{n-1} \frac{\langle x \rangle_k}{\sqrt{(n-k)\cdot\tau_{rev.}}} = c \cdot \sum_{k=0}^{n-1} g(k) \cdot f(n-k) \ , \quad (3)$$

Table 1: Fixed Target beam in CERN SPS

Fixed Target Beam	SPS @ inj.
Momentum p @ injection [GeV/c]	14
Revolution time $\tau_{\text{rev.}}$ [μs]	23.07
Tunes Q_H / Q_V	26.64 / 26.59
Gamma transition γ_T	23.2
Maximum number of batches	2
Number of bunches per batch	2100
Bunch Intensity	$4.8 \cdot 10^9$
Batch spacing [ns]	1050
Bunch spacing [ns]	5
Full bunch length [ns]	4
Trans. emittance $\epsilon_{H,V}$ [μm]	<10 / <7.5
Long. emittance ϵ_L [eVs]	0.2

Summation starts with $k = 0$ to make it more illustrative. The offsets can be seen as a function $g(k) \equiv \langle x \rangle_k$ and the rest can be interpreted as a function $f(n-k) \equiv \textit{const.} \cdot \frac{1}{\sqrt{(n-k)\cdot\tau_{rev.}}}$. Noting that the discrete convolution $h_n = (g * f)_n$ for functions g and f is defined as $\sum_{k=0}^{N-1} g(k)\, f(n-k) = \sum_{k=0}^{N-1} g_k\, f_{n-k}$, the (circular) FFT convolution algorithm — consisting in finding the individual FFTs of g and f, their multiplication $H_j = G_j\, F_j$ and finally the inverse FFT of H — can be used to calculate the sum of kicks

$$\Delta x'_n = h_n = \sum_{k=0}^{N-1} g_k\, f_{n-k} \ \overset{\overset{FFT}{\Longrightarrow}}{\underset{IFFT}{\Longleftarrow}} \ G_j F_j = H_j . \quad (4)$$

Modifications have to be applied to adjust the standard circular convolution algorithm to the multi-bunch multi-turn case (2), which are described in detail in [6]. Compared to direct summation (2), which scales as $\mathcal{O}(N_b^2)$, computation time can be largely reduced using the FFT convolution algorithm, because the FFT scales only as $\mathcal{O}(N_b \log_2 N_b)$, where N_b is the number of bunches. This makes simulations of multi-bunch multi-turn effects feasible. Note that this fast summation algorithm is not restricted to resistive wall only.

APPLICATION TO THE SPS

To verify the simulation code, growth rate and tune measurements in both transverse planes in the SPS were carried out in 2003. The parameters of these runs with fixed target beam are summarized in Tab. 1. The fixed target beam was chosen because it is considered to show dominantly the effect of the resistive wall. This is in particular true when the vacuum chamber wall has been conditioned before by scrubbing runs.

We measured growth rates for 1 batch (total intensity $1 \cdot 10^{13}$) in the machine of $(2\pi\, \text{Im}\, \Delta Q_H)^{-1} = 183.5 \pm 23.5$ turns and $(2\pi\, \text{Im}\, \Delta Q_V)^{-1} = 77 \pm 4$ turns. Unfortunately we could not measure with varying intensities, there-

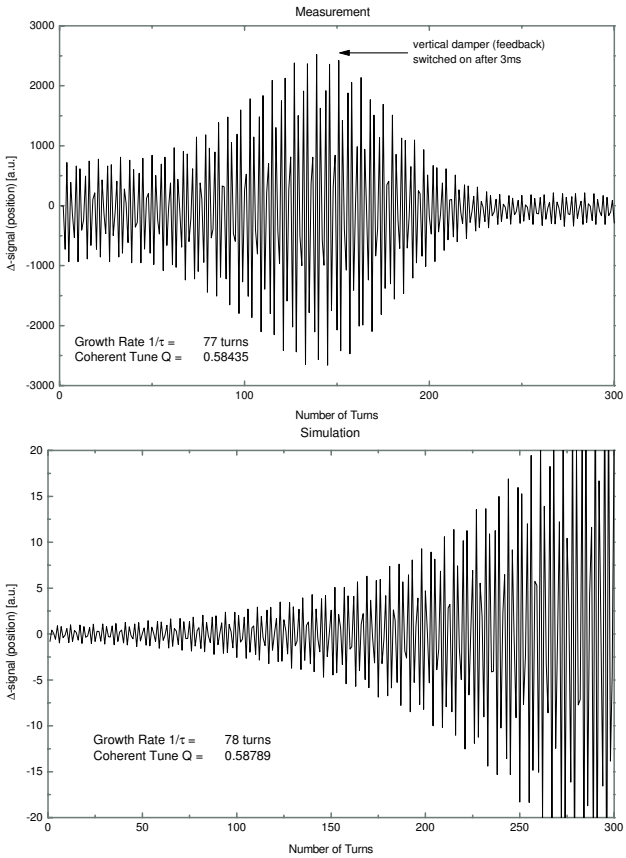

Figure 1: SPS measurement and simulation in the vertical plane for parameters given in Tab. 1. Excellent agreement of growth rates. The transition region visible after injection in the measurement data is not easily modelled by simulation.

fore no tune slope could be established. The measured coherent tunes with respect to the design tunes would give tune shifts of $\mathrm{Re}\,\Delta Q_H = 0.0038 \pm 0.0009$ and $\mathrm{Re}\,\Delta Q_V = 0.0057 \pm 0.0009$.

For the simulation only the resistive wall effect (with 'inductive bypass') was considered and the parameters from Tab. 1 have been used. Growth rates found by simulation are $(2\pi\,\mathrm{Im}\,\Delta Q_H)^{-1} = 150$ turns and $(2\pi\,\mathrm{Im}\,\Delta Q_V)^{-1} = 78$ turns. The tune shifts are $\mathrm{Re}\,\Delta Q_H = 0.0013$ and $\mathrm{Re}\,\Delta Q_V = 0.0021$. The simulated values for the growth rates are in excellent agreement with the measured quantities within a few percent. Unsurprisingly we can not make any statement concerning tune shifts, but the order of magnitude fits well to the obtained growth rates. Measurement and simulation results are portrayed in Fig. 1.

SIMULATIONS IN LHC

A major concern in LHC are the collimators made of graphite, which come very close to the beam, details can be found in [1]. At injection energy a transverse feedback system will damp rigid dipole motion. At top energy the feedback is turned off to avoid emittance growth. Hence all head-tail modes have to be Landau damped, which will be done by using octupoles.

Table 2: Simulated tune shifts for LHC at 7TeV

Intensity [p/bunch]	$-\,\mathrm{Re}\,\Delta Q$	$-\,\mathrm{Im}\,\Delta Q$
Nominal $1.15{\cdot}10^{11}$	$2.13 \cdot 10^{-4}$	$0.253 \cdot 10^{-4}$
Ultimate $1.67{\cdot}10^{11}$	$3.11 \cdot 10^{-4}$	$0.379 \cdot 10^{-4}$

Tab. 2 lists the growth rates and tune shifts obtained with the simulation. To obtain results more rapidly the energy was scaled down by a factor 10, which gives faster growth and reduces needed computation time. The simulated tune shifts are then scaled back to the original energy.

The simulated results are lower than analytically calculated tune shifts [1]. The difference has to be studied further, but it is consistent with expectation: in the simulation the actual LHC bunch pattern layout has been used. For analytical estimates a uniform machine filling is assumed which gives the worst case results and thus higher tune shifts.

CONCLUSION AND OUTLOOK

The multi-bunch multi-turn simulation code MULTITRISIM has been described. Presented results are mostly related to the resistive wall effect which was and still is a major concern in the LHC project. Comparison of simulation and measurements in the SPS with respect to resistive wall growth rates show good agreement. The results obtained for LHC have to be studied further.

A major goal achieved was the efficient implementation of the resistive wall effect in the code by FFT convolution. Computation times now are in the order of minutes for a full simulation of up to 10^5 turns.

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REFERENCES

[1] *The LHC Design Report*, Vol. I, The LHC Main Ring. 2004.
[2] A. Wagner. *3-dimensional simulation of collective effects in particle accelerators*. PhD thesis, TU Graz, Austria, 1997.
[3] G. Sabbi. Simulation of single bunch collective effects in LEP CERN-SL-95-25-AP.
[4] A. Koschik, et al. Transverse resistive wall impedance and wake function with inductive bypass. These Proceedings.
[5] B. W. Zotter and S. A. Kheifets. *Impedances and wakes in high-energy particle accelerators*. World Scientific, 1998.
[6] A. Koschik. *Simulation of multi-bunch instabilities of proton beams in LHC*. PhD thesis, TU Graz, Austria, 2004. In preparation.
[7] A. Chao, S. Heifets, and B. Zotter. Tune shifts of bunch trains due to resistive vacuum chambers without circular symmetry. *Phys. Rev. ST Accel. Beams*, 5:111001, 2002.