CONCEPTION OF X-RAY SOURCE BASED ON COMPACT WAKEFIELD UNDULATOR

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Abstract

Conception of X-ray source based on wakefield undulator (WFU) with very short period is presented. In the base of photon generation by the WFU lies a new mechanism of undulator-type radiation emitted by an ultrarelativistic electron bunch that undulates due to non-synchronous spatial harmonics of its wakefields while the bunch moves along a periodic waveguide. The features of the hard radiation and yield of photons depending on waveguide sizes and charge distribution are considered. The creation of the WFU with sub-millimeter periods due to advanced accelerator technology opens possibilities to generate hard X-rays employing relatively low electron energies without external alternative fields.

INTRODUCTION

Wide development of advanced bio-medical application of X-ray imaging and diagnoses needs creation of compact sources of monochromatic hard X-rays. For today the mechanism of Thomson/Compton scattering of laser radiation on electron beams from compact rf linear [1] and ring [2,3] accelerators is promising for generating hard X-ray.

At the same time, recent researches [4-9] of new mechanisms of radiation, emitted by a beam of relativistic charged particles undulating in the alternating fields induced by this beam as it moves along a periodic rf structure, have specified new potential opportunities for generating ultra-short wavelength light. In Refs. [4,5], the static image charge fields produced by continuous sheet electron beam passing between two periodic grating metal surfaces were considered as wiggler type fields, and a new device called the image charge undulator was proposed.

If a beam is bunched, the static image charge electric force is compensated by the image current magnetic force with an accuracy $1/\gamma^2$ (γ - the Lorentz factor). In this case the fields of the coherent Cherenkov-type radiation (CR) become dominant wakefields (WF). (The term CR covers different names of radiation from a charged particle moving with constant velocity along a periodic structure [10].) In Refs. [6-9] it has been shown that transverse components of nonsynchronous spatial wakefield harmonics acting on particles can give rise to their undulating motion and consequently to generating the undulator-type radiation (UR). The theory of this mechanism of radiation for an infinitely long periodic rf structure was given in [6,7]. As follows from the theory, in the relatively long-wave spectral region, where diffraction of generated UR waves is essential, the radiation manifests itself in coherent interference of WF and UR. A pure UR takes place only in the relatively ultra-short wave range where UR wave diffraction can be neglected. The power of UR emitted coherently by a bunch of N particles is proportional to N^4 [7]. The power of spontaneous UR is proportional to N^3 [8].

The goal of the present paper is to show potential opportunities of production of high intensity hard X-rays emitted from an ultrarelativistic electron bunched beam moves in WFU representing a periodic structure of type corrugate waveguide.

WFU RADIATION MECHANISM

The essence of the radiation mechanism in WFU consists in the following. A short bunch of N charged particles with constant velocity v_z moving along a periodic waveguide/grating of a period D induces wakefields acting on the particles. The wake force can be expressed as a spatial harmonic Floquet's series

$$\vec{F}(\vec{r},t) = \sum_p \vec{F}^{(p)}(\vec{r}_\perp, t - z/\mathrm{v}_z)\, e^{i\frac{2\pi p}{D}z}. \qquad (1)$$

Here $\vec{F}^{(p)}(\vec{r}_\perp,\tau) = \int_0^\infty d\tau' \iint_{S_\perp} d^2\vec{r}'_\perp f_b(\vec{r}'_\perp, \tau-\tau') \vec{F}^{(p)}(\vec{r}_\perp, \vec{r}'_\perp, \tau')$ is the p^{th} space harmonic of the wake force, $\vec{F}^{(p)}(\vec{r}_\perp, \vec{r}'_\perp, \tau')$ is the p^{th} spatial harmonics of the force produced by a point charge eN (with a transverse coordinate $\vec{r}'_\perp$) acting on a particle with a charge e having a transverse coordinate $\vec{r}_\perp$ and moving at distance $\mathrm{v}_0\tau$ after the point charge; $f_b(\vec{r}_\perp,\tau)/\mathrm{v}_z$ is the normalized charge-density distribution; $S_\perp$ is the cross-section of the periodic rf structure.

Action of the wake force synchronous spatial harmonic (p=0) on the bunch particles results in energy losses associated with generating WF. The alternating transverse wake force ($p\neq0$) can give rise to undulating the particles with transverse velocity

$$\mathrm{v}_\perp(\vec{r}_\perp,\tau) = \frac{ic}{2\gamma}\sum_{p\neq 0} \vec{K}^{(p)}(\vec{r}_\perp,\tau) e^{i\frac{2\pi p}{D}z}, \qquad (2)$$

where $\gamma >> 1$, $\vec{K}^{(p)}(\vec{r}_\perp,\tau)$ is the undulator parameter

$$\vec{K}^{(p)}(\vec{r}_\perp,\tau) = \frac{Ne^2 \vec{u}_\perp^{(p)}(\vec{r}_\perp,\tau) D}{p\pi m c \mathrm{v}_z}. \qquad (3)$$

Here m is the particle mass of rest, c is the velocity of light, $\tau = t - z/\mathrm{v}_z$ is the time when a particle crosses the plane z=0; $\vec{u}_\perp^{(p)}(\vec{r}_\perp,\tau)$ is the traverse component of the p^{th} spatial harmonics of the wake function defined as

$$\vec{u}_\perp^{(p)}(\vec{r}_\perp,\tau) = -\frac{\vec{F}_\perp^{(p)}(\vec{r}_\perp,\tau)}{e^2 N}. \qquad (4)$$

As it is known undulation of charged particles with the velocity Eq.(2) can cause generating UR.

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SPECTRAL -ANGULAR CHARACTERISTICS

Single-particle radiation

For an infinitely long periodic rf structure spectral-angular UR characteristics were analysed in Ref.[9]. For WFU of a finite length consisting of N_u periods the average over period spectral-angular power density of hard UR emitted spontaneously by a single particle of the bunch has the following form in the limit $\left|K^{(p)}\right| << 1$

$$\begin{aligned}\frac{d^2P_e}{d\Omega d\omega} = \frac{e^2}{8\pi c}\frac{\omega^2 N_u}{\omega_u}\sum_{p=1}\Big\{&\left|K_x^{(p)}\right|^2\left[1-\sin^2\theta\cos^2\varphi R(\omega,\theta,p)\right]\\ &+\left|K_y^{(p)}\right|^2\left[1-\sin^2\theta\sin^2\varphi R(\omega,\theta,p)\right]\\ &-\mathrm{Re}\left(K_x^{(p)}K_y^{(p)}\right)\sin 2\varphi\sin^2\theta R(\omega,\theta,p)\Big\}\\ &\times\left[\frac{\sin\left(\left[\omega(1-\beta_z\cos\theta)-p\omega_u\right]\frac{\pi N_u}{\omega_u}\right)}{\left[\omega(1-\beta_z\cos\theta)-p\omega_u\right]\frac{\pi N_u}{\omega_u}}\right]^2\end{aligned}, \quad (5)$$

where $R(\omega,\theta,p)\equiv\left(1-\frac{\omega}{p\omega_u}\beta_z\cos\theta\right)^2-\beta_z^2\left(\frac{\omega}{p\omega_u}\right)^2$, ω is the UR frequency, $\omega_u = 2\pi \mathrm{v}_z/D$, $\beta_z = \mathrm{v}_z/c$, φ is the angle between the axis OX and XOY - plane projection of $\vec{k}$, $d\Omega = \sin\theta\, d\theta d\varphi$, θ is the polar angle between wave vector $\vec{k}$ of UR and the axis OZ of the WFU.

As it follows from Eq.(5) the radiation have a line spectrum with the resonant frequencies

$$\omega^{(p)}(\theta,\vec{r}_\perp,\tau) = 2p\omega_u\gamma^2\left[1+\sum_{q=1}\frac{\left|K^{(q)}(\vec{r}_\perp,\tau)\right|^2}{2}+\left(\gamma(\vec{r}_\perp,\tau)\theta\right)^2\right]^{-1}, \quad (6)$$

where p=1,2,… is numbers of the radiation harmonics.

Dividing Eq.(5) by photon energy $\hbar\omega$, and integrating over azimuth angle φ it can easy obtain the spectral-angular of photon flux density of the p^{th} harmonics which has the following form in the forward direction

$$\left.\frac{d^2\dot{n}_e^{(p)}}{d\theta d\omega/\omega}\right|_{\theta=0} = \frac{\omega_u N_u \gamma(\vec{r}_\perp,\tau)^2}{137}\left|pK^{(p)}(\vec{r}_\perp,\tau)\right|^2\left[\frac{\sin\left(\frac{\Delta\omega}{\omega^{(p)}}\pi p N_u\right)}{\frac{\Delta\omega}{\omega^{(p)}}\pi p N_u}\right]^2, \quad (7)$$

where $\Delta\omega = \omega - \omega^{(p)}$.

Beam radiation

Let us consider the UR from a beam which represents multi-bunch trains with bunch frequency f_{rf} and a pulse duty factor η. Since the bunch dimension σ is by many orders of magnitude greater than wavelengths of hard UR radiation $\sigma >> D/\gamma^2$, so UR emitted by the all particles of the bunch is spontaneous. In this case, the spectral-angular distribution of photon flux density of the p^{th} harmonics emitted by the bunched beam in the forward direction can be written as

$$\begin{aligned}\left.\frac{d^2\dot{n}_{beam}^{(p)}}{d\theta d\omega/\omega}\right|_{\theta=0} &= \frac{2\pi N_u}{\omega_u} f_{rf}\eta N\int_{-\infty}^{\infty}d\tau\iint_{S_\perp}d^2r_\perp f_b(\vec{r}_\perp,\tau)\left.\frac{d^2\dot{n}_e^{(p)}}{d\theta d\omega/\omega}\right|_{\theta=0}\\ &= \frac{2(r_0 D)^2}{137\pi}N^3N_u^2 f_{rf}\eta\int_{-\infty}^{\infty}d\tau\iint_{S_\perp}d^2r_\perp f_b(\vec{r}_\perp,\tau)\gamma(\vec{r}_\perp,\tau)^2\left|u^{(p)}(\vec{r}_\perp,\tau)\right|^2\left[\frac{\sin\left(\frac{\Delta\omega}{\omega^{(p)}}\pi p N_u\right)}{\frac{\Delta\omega}{\omega^{(p)}}\pi p N_u}\right]^2\end{aligned} \quad (8)$$

AXIALLY SYMMETRICAL WFU

To calculate analytically a photon yield it is convenient to consider WFU which represents a weakly corrugated circular metallic waveguide shown in Fig.1.

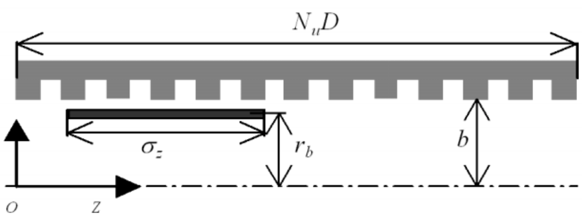

Figure 1: Schematic representation of WFU.

Let a monochromatic ultrarelativistic electron bunch of length σ_z with uniform charge distribution

$$f_b(r,\varphi,\tau) = \frac{c}{2\pi r_b\sigma_z}\left[H(\tau+\sigma_z/2c)-H(\tau-\sigma_z/2c)\right]\delta(r-r_b)\,\delta(\varphi) \quad (9)$$

moves through the WFU at distance r_b from the axis. Here $H(\tau)$ is the Heaviside function.

Wakefield spatial harmonics are found by the perturbation method [11]. The periodic radius of the waveguide can be written in Fourier series

$$b(z) = b_0\left[1+\varepsilon(z)\right] = b_0\left[1+\sum_{p=-\infty}^{\infty}\varepsilon_p\exp\left(i\frac{2\pi p}{D}z\right)\right] \quad (10)$$

where $\varepsilon(z)$ <<1 is the relative depth of corrugations, b_0 is the average radius of the waveguide. As per this approach the first order radial component of the p^{th} spatial harmonics of the wake function has the following form

$$\begin{aligned}u_r^{(p)}(r,\varphi,\tau) &= \frac{i8\pi p\varepsilon_p}{b_0 D}\sum_{m=0}^{\infty}\left(\frac{r_b}{b_0}\right)^m\frac{\cos(m\varphi)}{(1+\delta_{0,m})}\\ &\times\sum_{s=1}^{\infty}\left[A_{m,s}\frac{\sin\left(\omega_{m,s,p}\sigma_z/2c\right)}{\omega_{m,s,p}\sigma_z/2c}e^{i\frac{\omega_{m,s,p}}{2}\left(\tau+\frac{\sigma_z}{2c}\right)}-B_{m,s}\frac{\sin\left(\omega'_{m,s,p}\sigma_z/2c\right)}{\omega'_{m,s,p}\sigma_z/2c}e^{i\frac{\omega'_{m,s,p}}{2}\left(\tau+\frac{\sigma_z}{2c}\right)}\right]\end{aligned} \quad (11)$$

where $\delta_{0,m}$ is Kronecker's symbol,

$$A_{m,s}(r/b_0)\equiv\frac{J'_m(\mu_{m,s}r/b_0)}{J'_m(\mu_{m,s})}, \qquad B_{m,s}(r/b_0)\equiv\frac{b_0}{r}\frac{m^2}{m^2-\mu'^2_{m,s}}\frac{J_m(\mu'_{m,s}r/b_0)}{J_m(\mu'_{m,s})},$$

$\mu_{m,s}$ and $\mu'_{m,s}$ are the zeros of Bessel functions $J_m(\mu_{m,s}) = 0$ and $J'_m(\mu'_{m,s}) = 0$, respectively,

$$\omega_{m,s,p} = \frac{\pi pc}{D}\left[1+\left(\frac{D\mu_{m,s}}{2\pi pb_0}\right)^2\right] \qquad \omega'_{m,s,p} = \frac{\pi pc}{D}\left[1+\left(\frac{D\mu'_{m,s}}{2\pi pb_0}\right)^2\right]$$

are the frequencies of resonant WF modes.

Inserting Eq.(11) in Eq.(8) and taking into account the case of small beam energy disperse we obtain the spectral angular flux density emitted in the forward direction

$$\left.\frac{d^2\dot{n}^{(p)}_{beam}}{d\theta d\omega/\omega}\right|_{\theta=0}=\frac{32}{137\pi}N^3N_u^2 f_{rf}\eta\gamma^2\left(\frac{\pi r_0}{b_0}\right)^2\left|2p\varepsilon_p\right|^2\left[\frac{\sin\left(\frac{\Delta\omega}{\omega^{(p)}}\pi p N_u\right)}{\frac{\Delta\omega}{\omega^{(p)}}\pi p N_u}\right]^2$$

$$\times\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\left(r_b/b_0\right)^{m+n}}{\left(1+\delta_{0,m}\right)\left(1+\delta_{0,n}\right)}\sum_{s=1}^{\infty}\sum_{q=1}^{\infty}\left\{A_{m,s}A_{n,q}I\left(\omega_{m,s,p},\omega_{n,q,p}\right)+B_{m,s}B_{n,q}I\left(\omega'_{m,s,p},\omega'_{n,q,p}\right)\right.$$

$$\left.-A_{m,s}B_{n,q}I\left(\omega_{m,s,p},\omega'_{n,q,p}\right)-B_{m,s}A_{n,q}I\left(\omega'_{m,s,p},\omega_{n,q,p}\right)\right\} \tag{12}$$

where

$$I\left(\omega_1,\omega_2\right)\equiv\frac{c^2}{\omega_1\omega_2\sigma_z^2}\left[1-\frac{\sin\left(\omega_1\sigma_z/c\right)}{\omega_1\sigma_z/c}-\frac{\sin\left(\omega_2\sigma_z/c\right)}{\omega_2\sigma_z/c}+\frac{\sin\left(\left(\omega_1-\omega_2\right)\sigma_z/c\right)}{\left(\omega_1-\omega_2\right)\sigma_z/c}\right]$$

Let us estimate the possible values of 33 keV photon fluxes for example applied in coronary angiography [12]. In Tab.1 it is shown the WFU and electron bunch parameters needed for producing 33 keV photons.

Table 1: The WFU and bunch parameters.

Period	D	25 μm
Minimum radius	b_{min}	200 μm
Maximum radius	b_{max}	250 μm
Number of periods	N_u	300
Energy of electrons	W_e	295 MeV
Bunch length	σ_z	300 μm
Bunch distance from axis	r_b	0.9 b_{min}

Inserting the data from Tab.1 in Eq.(12) we obtain the dependences of photon fluxes density of the first harmonic on bunch charge. (The first Fourier harmonic of the relative depth of corrugations is ε_1=0.07). These dependences in terms of (photons/s/mrad/0.1% bandwidth) are represented in Fig.2. The curves (1,2,3) correspond to different products $f_{rf}\eta$ (3000, 300, and 30 kHz) respectively.

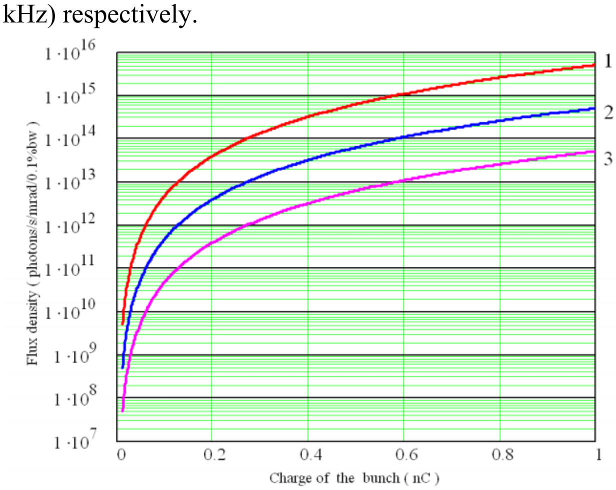

Figure 2: The 33 keV photon flux density produced by the WFU v.s. bunch charge for different $f_{rf}\eta$: the 1 curve - 3000 kHz, the 2 curve - 300 kHz, the 3 curve - 30 kHz.

CONCLUSIONS

A new conception of X-ray source based on compact wakefield undulator is presented. The spectral–angular characteristics of hard radiation generated by the WFU have been obtained. Using, as the WFU-model, a weakly corrugated circular waveguide excited by an thin uniform electron bunch it has been demonstrated the potential possibility of generating 33 keV photon fluxes applied in coronary angiography. The creation of the WFU with sub-millimeter periods due to the high aspect ratio microstructure technology such as deep X-ray lithography (see for example Ref.[13]), opens possibilities to generate hard X-rays employing relatively low electron energies without external alternative fields.

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