PARAMETERS OF X–RAY RADIATION EMITTED BY COMPTON SOURCES

Eugene Bulyak*, Vladislav Skomorokhov, NSC KIPT Kharkov, 61108, Ukraine

Abstract

Presented are results of analytical study on X–ray beam parameters generated in the Compton storage rings. A model with the given circulating electron bunch parameters and the laser splash as well is considered. For this model, the total yield of x–ray quanta and temporal duration is derived as a function of the crossing angle and geometric dimensions of both the bunch and splash. Also spectral characteristics of emitting X–ray beam are evaluated with account for the collimating conditions and both the angular and energy spreads in the bunch. As is shown, the width of X–ray energy spectrum is narrowest for the X–ray beam collimated along the bunch orbit. With increasing the scattering angle (with respect to the bunch orbit) the spectrum of emitting quanta is widening. Problems of X–ray beam generation with required energy and brightness with the Compton storage rings are discussed.

INTRODUCTION

In the report, scattering of low energy photons $E_{\rm las} \ll E_0 = mc^2$ by ultrarelativistic electrons ($\gamma = E_{\rm e}/E_0 \gg 1$ the Lorenz factor of electrons) is considering.

We will consider collision of a laser splash with an electron bunch in the laboratory frame. The electron and laser Cartesian frames are tilted by the angle φ, their origins are matched. The electron bunch moves at velocity v_e along the positive direction of y-axis, the laser splash – in the negative direction of y'–axis. The spatial densities of the both bunch and splash are distributed according to the gaussian law:

$$\begin{aligned} n_{\rm e} &= w_{\rm e} \exp\frac{1}{2}\left[-\frac{x^2}{\sigma_x^2} - \frac{(y-\beta T)^2}{\sigma_y^2} - \frac{z^2}{\sigma_z^2}\right] ,\\ n_{\rm ph} &= w_{\rm ph} \exp\frac{1}{2}\left[-\frac{x'^2}{\sigma_x'^2} - \frac{(y'+T)^2}{\sigma_y'^2} - \frac{z'^2}{\sigma_z'^2}\right] ,\\ w_{\rm e} &= \frac{N_{\rm e}}{(2\pi)^{\frac{3}{2}}\sigma_x\sigma_y\sigma_z} , w_{\rm ph} = \frac{N_{\rm ph}}{(2\pi)^{\frac{3}{2}}\sigma_x'\sigma_y'\sigma_z'} . \end{aligned}$$

Here $N_{\rm e,ph}$ is the total number of electrons and laser photons, resp.; $\sigma_{x,y,z}$ the dimensions of electron bunch; $\sigma'_{x,y,z}$ the dimensions of laser splash; $\beta = v_{\rm e}/c$ ratio of electron velocity to that of light in vacuum; $T = ct$ time expressed in units of length.

SPECTRAL CHARACTERISTICS

The energy of secondary X–ray quanta exhibits strong dependence upon the scattering angle ψ (between the electron and X–ray trajectories). By collimating the span of scattering angles, one can cut out the corresponding energy spectrum. In practice, the electrons in bunch have the finite energy spread; their trajectories have the angular spread around the bunch orbit. These effects contribute to the width of X–ray spectrum. (We will consider the laser photons being monoenergetic.)

The angular distribution of scattered quanta relating to the electron trajectory is strongly asymmetrical: the half of total number of quanta are emitted within the cone with opening angle $\psi_{1/2} = 1/\gamma \ll 1$. Hence, for the ultrarelativistic electrons ($\gamma \gg 1$) consideration would be limited by small scattering angles, $\psi \ll 1$.

We will characterize the angular distribution and energy spectrum of X–ray quanta with the spectral–angular density:

$$\nu(\epsilon,\psi) = \frac{1}{\sigma_{\rm c}}\frac{\partial^2\sigma_{\rm c}}{\partial\psi\partial\epsilon} , \qquad (1)$$

where $\sigma_{\rm c}$ is the Compton cross section; $\epsilon = E_{\rm X}/E_{\rm las}$ ratio of X–ray quantum energy to that of the laser photon.

For the small–angle approximation ($\psi \ll 1$), the spectral–angular density can be written as:

$$\nu(\epsilon,\psi) = 3\frac{x\left(1+x^4\right)}{\left(1+x^2\right)^4}\delta\left(\zeta - \frac{1}{1+x^2}\right) , \qquad (2)$$

where $x \equiv \gamma\psi$; $\zeta \equiv E_{\rm X}/E_{\rm X}^{(m)} = E_{\rm X}/2\gamma^2(1+\cos\varphi)$ ratio of X-ray energy to its maximal magnitude.

The energy spectrum deduced from (2) has a form:

$$G(\zeta) = \frac{3}{2}\left[1-2\zeta\left(1-\zeta\right)\right]\left(1-H\left(1-\zeta\right)\right) , \qquad (3)$$

where $H(\zeta)$ is the Heaviside step function.

Fig.1 presents the ideal spectral–angular density over the plane of (ϵ, x) variables. Also the collimation procedure is sketched in this picture.

Under assumptions of the Gaussian energy distribution in the bunch with a small spread ($\Delta\gamma/\gamma \le 0.03$ for the Compton rings) and making use of (2), we can derive the spectral–angular density with account for the electron energy spread:

$$G_\gamma(\zeta, x^2) = \frac{3\left[1-2\zeta\left(1-\zeta\right)\right]}{4\sqrt{2\pi}s_\gamma}\exp\left(-\eta^2\right) , \qquad (4)$$

where $\eta = \left(1/\zeta - 1 - x^2\right)/2\sqrt{2}s_\gamma$; $s_\gamma = \sigma_\gamma/\gamma$.

The collimated into $x_i \le x \le x_f$ energy spectrum has a form:

$$\begin{aligned} \mathcal{G}_\gamma(\zeta, x_{\rm i}, x_{\rm f}) &= \frac{3}{4}\left[1-2\zeta\left(1-\zeta\right)\right]\times \\ &\left[{\rm Erf}\left(\eta_{\rm i}\right) - {\rm Erf}\left(\eta_{\rm f}\right)\right] , \qquad (5) \end{aligned}$$

* bulyak@kipt.kharkov.ua

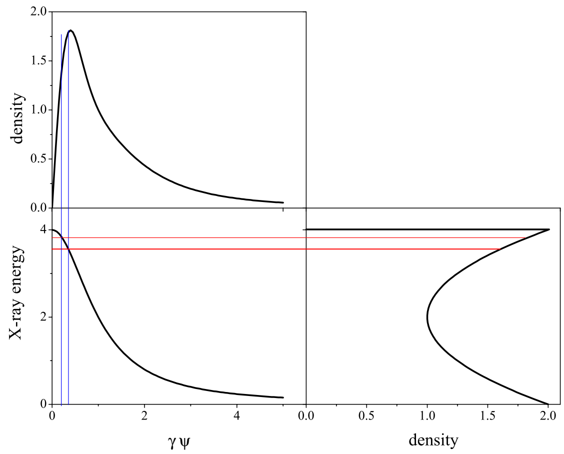

Figure 1: Spectral–angular density and collimation

where $\eta_{\rm i,f} = -\left(1/\zeta - 1 - x_{\rm i,f}^2\right)/2\sqrt{2}s_\gamma$; Erf(z) is the error function.

Suggest the angular spread of electron trajectories around the bunch orbit being Gaussian with the dispersion $\sigma_\psi^2 = \varepsilon/\beta_{\rm ip}$ (ε is the transverse emittance; $\beta_{\rm ip}$ the betatron function at the interaction point), the spectral–angular density with account for this spread has a form:

$$G_\psi(\zeta, x) = \frac{3\left[\zeta^2 + (1-\zeta)^2\right]}{2\sqrt{2\pi}s_\psi} \times \left(\exp\left(\eta^-\right) + \exp\left(\eta^+\right)\right) , \quad (6)$$

where $\eta^\pm = -\left(x \pm \sqrt{1/\zeta - 1}\right)^2/2s_\psi^2$; $0 < \zeta \le 1$; $x \ge 0$.

The collimated energy spectrum is:

$$\mathcal{G}_\psi(\zeta, x_{\rm i}, x_{\rm f}) = \frac{3}{4}\left[\zeta^2 + (1-\zeta)^2\right] \times \left[\mathrm{Erf}\left(\eta_{\rm f}^-\right) + \mathrm{Erf}\left(\eta_{\rm f}^+\right) - \mathrm{Erf}\left(\eta_{\rm i}^-\right) - \mathrm{Erf}\left(\eta_{\rm i}^+\right)\right] , \quad (7)$$

where $\eta_{\rm i,f}^\pm = \left(x_{\rm i,f} \pm \sqrt{1/\zeta - 1}\right)/\sqrt{2}s_\psi$.

Modification of the collimated spectrum caused by angular and energy spreads is presented in Fig.2.

As it can be seen from the figure, the width of spectrum depends on tree factors: the range and angle of collimation, the energy spread in electrons, and the angular spread of electron trajectories at the interaction point.

TOTAL YIELD

The total yield of X–ray quanta in the coordinate frame where the both bunches – electrons and laser photons – are moving with arbitrary velocities is determined by the Pauli formula (see [1, 2]):

$$N = \sigma\sqrt{(\vec{v_1} - \vec{v_2})^2 - \frac{[\vec{v_1}\vec{v_2}]^2}{c^2}} \int_V \int_t n_{\rm e} n_{\rm ph} {\rm d}V {\rm d}t , \quad (8)$$

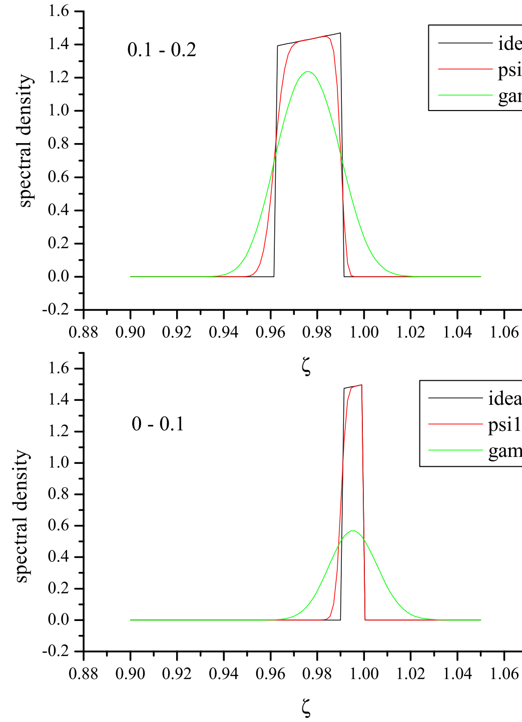

Figure 2: X–ray energy spectra: ideal, for the electron trajectories spread 0.1, and electron energy spread 0.5 % into collimating range 0. . . 0.1 (bottom) and 0.1. . . 0.2 (top)

where σ is the total cross section; V the radiating volume.

We have derived general expression which is rather complex, [3]. For a limiting case of ultrarelativistic electrons $\gamma \gg 1$ and $\psi < \pi - \gamma^{-1}$ this expression can be reduced to:

$$N_\varphi = \frac{N_{\rm e} N_{\rm ph} \sigma}{2\pi\sqrt{\sigma_z^2 + \sigma_z'^2}} \times \frac{1}{\sqrt{\sigma_x^2 + \sigma_x'^2 + (\sigma_y^2 + \sigma_y'^2)\tan^2\frac{\varphi}{2}}} . \quad (9)$$

As it can be seen from (9), for the head–on collision ($\varphi = 0$) the total yield id dependent only upon the transverse bunch and splash dimensions; for the arbitrary angle $\varphi \ne 0, \pi$ the yield is dependent on the longitudinal dimensions also. Response of the yield on crossing angle for the practical case is plotted in Fig.3. The yield monotonously decreases with increase in the crossing angle up to angles of $\varphi < \pi - 1/\gamma$; at $\varphi \approx \pi$ (back–on scattering angle) the yield jumps up to its maximal value.

TEMPORAL DURATION OF X–RAY SPLASH

For some applications, a figure of merit (besides the total yield and width of energy spectrum) is the duration of X–ray splash.

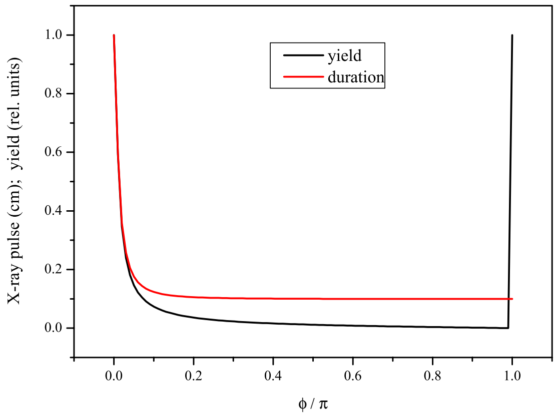

Figure 3: X–ray pulse duration (cm) and relative yield for $\sigma_y = 10\,\text{mm}$, $\sigma'_y = 1\,\text{mm}$, $\sigma_x = \sigma'_x = 50\,\mu\text{m}$; $\gamma = 88$

For observation of X–rays along the electron orbit $\psi = 0$, the temporal duration of splash τ in the ultrarelativistic case $\gamma \gg 1$ would be written as:

$$\tau^2 = \frac{\sigma_y^2 \left(\sigma_x^2 + \sigma_x'^2 + \sigma_y'^2 \tan^2 \frac{\varphi}{2}\right)}{\sigma_x^2 + \sigma_x'^2 + \left(\sigma_y^2 + \sigma_y'^2\right) \tan^2 \frac{\varphi}{2}} + \frac{1}{4\gamma^4} \frac{\sigma_x'^2 \sigma_y'^2 + \sigma_x^2 \left(\sigma_y'^2 \cos^2 \varphi + \sigma_x'^2 \sin^2 \varphi\right)}{\left(\sigma_x^2 + \sigma_x'^2\right)\left(1 + \cos\varphi\right)^2 + \left(\sigma_x^2 + \sigma_x'^2\right) \sin^2 \varphi} . \quad (10)$$

For the head–on collision ($\varphi = 0$) this expression is easily reduced to well known (see, e.g. [4]):

$$\tau^2 = \sigma_y^2 + \sigma_y'^2/16\gamma^4 .$$

For the right–angle collision ($\varphi = \pi/2$) we get (see [5]):

$$\tau^2 = \sigma_y^2 \left(\sigma_x^2 + \sigma_x'^2 + \sigma_y'^2\right) / \left(\sigma_y^2 + \sigma_x^2 + \sigma_x'^2 + \sigma_y'^2\right) .$$

For the crossing angles close to back–on collisions ($\varphi \approx \pi$) we get (comp. with [6]):

$$\tau^2 = \sigma_y'^2 \left[1 - \frac{4\gamma^4 \sigma_y'^2}{\sigma_x^2 + \sigma_x'^2} (\pi - \varphi)^2\right] \quad (11)$$

For a practical case ($\sigma_y \gg \sigma'_y \gg \sigma_x \sim \sigma'_x$), response of the X–ray splash duration on change in the crossing angle computed in accordance with the rigorous analytical formula is presented in Fig.3. As it can be seen from the curve, the temporal duration of splash is practically constant within the crossing angle range $\pi/3 \leq \varphi \leq \pi$.

SUMMARY

Within the approach of given parameters of the both electron bunch and laser splash, the main characteristics of X–ray splash generated by the Compton storage ring is derived in the form of close analytical expressions. These are:

- Collimated energy spectrum with account for the angular range of collimation; the energy and angular spread within electron trajectories.
- Total yield of X–ray quanta for whole span of crossing angles.
- Temporal duration of X–ray splash.

From analysis of the derived expressions, it follows that for the practical Compton X–ray source based on a storage ring:

- Main contribution to the width of spectrum of the collimated radiation arise from the collimation opening angle and angular spread in the electron trajectories at the interaction point (IP). Decrease in the betatron functions magnitude at IP causes enhancement in the total yield but spreads the spectrum.
- The spread of electrons energy results in twofold spread of the X–ray energy.
- The total yield of X–rays is maximal for the head–on collision. It decreases significantly with increasing of the collision angle up to $\varphi \approx (1 \ldots 3)/\gamma$ then remains almost constant up to near back–on angles. At the vicinity of the back–on collisions the yield is sharply increased and becomes equal to the head–on at $\varphi = \pi$.
- The duration of X–ray splash along the electron orbit decreases from the bunch length for the head–on collision down to that of the laser splash for the back–on collision in similar manner as the total yield (except for the sharp rise at the back–on collisions).

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