STOCHASTIC COOLING POWER REQUIREMENTS *

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Abstract

A practical obstacle for stochastic cooling in high-energy colliders like RHIC is the large amount of power needed for the cooling system. Based on the coasting-beam Fokker-Planck (F-P) equation, we analytically derived the optimum cooling rate and amplifier power for a beam of uniform energy distribution and a system of linear gain function. The results indicate that the usual back-of-envelope formulae over-estimate the cooling power by a factor of the mixing factor M. A longitudinal and transverse stochastic cooling system of 4 – 8 GHz frequency bandwidth in RHIC can effectively counteract intra-beam scattering (IBS), preventing longitudinal beam debunching, balancing transverse emittance growth, and improving luminosity.

INTRODUCTION

Stochastic cooling [1, 2] has long been recognized as a viable approach to counteract the emittance growth and beam loss caused by intra-beam scattering in RHIC [3, 4]. Theoretically, with a transverse cooling system of frequency bandwidth from 4 to 8 GHz, the (normalized 95%) emittance of a gold beam of 10^9 particles per bunch can be preserved at 30 $\pi\mu$m. With a longitudinal cooling system of the same frequency bandwidth, the debunching caused by the particles escaping from the RF bucket can be eliminated [5]. Over a 10-hour store, stochastic cooling can significantly increase the luminosity and reduce the experimental background.

A possible technical difficulty is the existence of very strong coherent components at GHz frequency range that would saturate the electronics of the cooling system and swamp the true stochastic information. Due to this problem, attempts at implementing bunched-beam stochastic cooling at the Tevatron and the SPS were unsuccessful. On the other hand, cooling of the heavy ion beam in RHIC has the advantage that the signal-to-noise ratio is high due to the high charge state, and that longitudinally the beam occupies a large fraction of the RF bucket approaching coasting-beam cooling conditions. Furthermore, the strong IBS diffusion in the gold beam is expected to break-down soliton-like coherent structure in the bunched beam [6]. According to the recent measurements of Schottky signals, stochastic cooling of the gold beam in RHIC would not be impeded by the anomalous coherent components in the GHz-range Schottky signals [7, 8].

Practically, the obstacle for stochastic cooling in RHIC is the large amount of amplifier power needed for the cooling system [3]. Early study using the bunched-beam Fokker-Planck approach indicated that the power needed is proportional to the energy spread of the beam to the forth power [4]. With a total kicker coupling-resistance of 6.4 kΩ, the power needed for longitudinal cooling at beam storage is several kilo Watts at a frequency from 4 to 8 GHz. However, a comparison between this Fokker-Planck calculation [4] and the estimate given in Ref. [3] indicates a difference in the scaling behavior of the cooling power when the mixing factor [2] is larger than unity. According to the estimate, the power needed for stochastic cooling in RHIC would be much larger.

This paper presents analytically derived scaling laws for the longitudinal and transverse cooling power, and discusses applications in RHIC.

LONGITUDINAL F-P EQUATION

Assume that the evolution of the beam distribution is slow during a synchrotron-oscillation period. The evolution of the longitudinal density function $\Psi_L(W)$ can be described by the Fokker-Planck equation [9, 10, 2]

$$\frac{\partial \Psi_L}{\partial t} = -\frac{\partial}{\partial W}(F_L\Psi_L) + \frac{1}{2}\frac{\partial}{\partial W}\left(D_L\frac{\partial \Psi_L}{\partial W}\right). \quad (1)$$

with the boundary condition

$$\begin{cases} -F_L\Psi_L + \frac{1}{2}D_L\frac{\partial \Psi_L}{\partial W} = 0, & W = 0 \\ \Psi_L = 0, & W = W_{max} \end{cases} \quad (2)$$

where $W \equiv \Delta E/\omega_s$ is the scaled energy deviation, and ω_s is the revolution frequency. Neglect the thermal noise which is small compared with the Schottky noise for heavy-ion beams. The drifting and diffusion coefficients are

$$\begin{aligned} F_L(W) = & \lim_{\Delta t\to 0}\frac{1}{\Delta t}\langle\int_0^{\Delta t} dt\, U_W(t)\rangle \\ D_L(W) = & \lim_{\Delta t\to 0}\frac{1}{\Delta t}\langle\int_0^{\Delta t} dt \int_0^{\Delta t} dt'\, U_W(t)U_W(t')\rangle \end{aligned} \quad (3)$$

where

$$U_{W,i} = \frac{Ze}{\omega_s}\sum_{n=-\infty}^{\infty} V^K(t)\,\delta\left(t - \frac{2\pi n}{\omega_i} - \frac{\phi_i^0}{\omega_s} - \frac{\theta^K}{\omega_s}\right), \quad (4)$$

$$V^K(t) = \frac{Ze}{2\pi}\int_{-\infty}^{\infty} d\omega e^{i\omega t} G_L(\omega)\sum_{j=1}^{N}\sum_{m=-\infty}^{\infty} e^{-i\omega\left(\frac{2\pi m}{\omega_j}+\frac{\phi_j^0}{\omega_s}+\frac{\theta^P}{\omega_s}\right)} \quad (5)$$

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where ϕ_i^0 is the initial phase of the particle, θ^K and θ^P are the azimuthal angles along the ring, the superscripts K and P indicate the kicker and pick-up, the subscript i indicates the test particle, and $G_L(\omega)$ is the gain function. With randomized initial phases and the factor $e^{-im\left(\theta^P-\theta^K\right)}$ absorbed by the gain function, Eq. 3 becomes

$$F_L = \frac{z^2e^2\omega_i}{4\pi^2}\sum_{m=-\infty}^{\infty} G_L(m\omega_i)e^{-im\left(\theta^P-\theta^K\right)\frac{\Delta\omega_i}{\omega_s}}$$
$$D_L = \frac{z^4e^4\omega_i^2}{8\pi^3}\sum_{j=1}^{N}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{|G_L(m\omega_j)|^2}{|m|}\rho(\omega_j)|_{\omega_j=\frac{n}{m}\omega_i} \tag{6}$$

where the factor $e^{-im\left(\theta^P-\theta^K\right)\frac{\Delta\omega_i}{\omega_s}}$ represents the "bad mixing" between the pick-up and the kicker, and the summation is over the effective frequency range of the cooling system. The average power required for cooling is

$$\bar{P}_L = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_0^{\Delta t} dt\,\frac{\langle (V^K(t))^2\rangle}{n^K R^K} \tag{7}$$

where n^K is the number of kicker, and R^K is the coupling resistance of each kicker.

TRANSVERSE F-P EQUATION

Evolution of the transverse density function $\Psi_T(I)$ is described by the Fokker-Planck equation [9, 10, 2]

$$\frac{\partial\Psi_T}{\partial t} = -\frac{\partial}{\partial I}(F_T\Psi_T) + \frac{1}{2}\frac{\partial}{\partial I}\left(D_T\frac{\partial\Psi_T}{\partial I}\right) \tag{8}$$

with the boundary condition

$$\begin{cases} -F_T\Psi_T + \frac{1}{2}D_T\frac{\partial\Psi_T}{\partial I} = 0, & I = 0 \\ \Psi_T = 0, & I = I_{max} \end{cases} \tag{9}$$

where I is the transverse action, and

$$F_T(I) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\langle\int_0^{\Delta t} dt\, U_I(t)\rangle$$
$$D_T(I) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\langle\int_0^{\Delta t} dt\int_0^{\Delta t} dt'\, U_I(t)U_I(t')\rangle \tag{10}$$

with

$$U_{I,i} = -\sqrt{2I_i\beta_x^K}\sin\phi_{\beta,i}\sum_{n=-\infty}^{\infty}\frac{U^K(t)}{\omega_s}\,\delta\left(t-\frac{2\pi n}{\omega_i}-\frac{\phi_i^0}{\omega_s}-\frac{\theta^K}{\omega_s}\right) \tag{11}$$

$$U^K(t) = \frac{Z^2e^2}{2\pi}\int_{-\infty}^{\infty} d\omega e^{i\omega t}G_T(\omega)\sum_{j=1}^{N}\sqrt{2I_j\beta_x^P}\sum_{m=-\infty}^{\infty}\cos\phi_{\beta,j}e^{-i\omega\left(\frac{2\pi m}{\omega_j}+\frac{\phi_j^0}{\omega_s}+\frac{\theta^P}{\omega_s}\right)} \tag{12}$$

Here, ϕ_β is the betatron phase, and β_x^P and β_x^K are the betatron functions. Assume that the gain G_T is the same at the upper and lower betatron sidebands, and merge the factor $e^{-i(m\pm Q_x)\left(\theta^P-\theta^K\right)}$ into the gain function, we have

$$F_T = \frac{z^2e^2\sqrt{\beta_x^K\beta_x^P}\omega_i}{4\pi^2}\sum_{m=-\infty}^{\infty}G_T\left[(m\pm Q_x)\omega_i\right] e^{-im\left(\theta^P-\theta^K\right)\frac{\Delta\omega_i}{\omega_s}}\sin\left[Q_x(\theta^P-\theta^K)\right]I$$
$$D_T = \frac{z^4e^4\omega_i^2\beta_x^K\beta_x^P I}{8\pi^3}\sum_{j=1}^{N}I_j\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{|G_T[(m\pm Q_x)\omega_j]|^2}{|m|}\rho(\omega_j)|_{\omega_j=\frac{n}{m}\omega_i} \tag{13}$$

where Q_x is the transverse tune, $e^{-i(m\pm Q_x)\left(\theta^P-\theta^K\right)\frac{\Delta\omega_i}{\omega_s}}$ represents the "bad mixing" between the pick-up and the kicker, and $\sin\left[Q_x(\theta^P-\theta^K)\right]$ indicates that a betatron phase advance of $\pi/2$ between the pick-up and the kicker optimizes the performance. The average power required for cooling is

$$\bar{P} = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_0^{\Delta t} dt\frac{\langle (V^K)^2\rangle}{n^K R^K},\quad V^K = \frac{2\pi\beta^2 E_s\Delta_x^K}{ZeL^K}U^K(t) \tag{14}$$

where n^K is the number of kicker, R^K and L^K are the coupling resistance and length of each kicker, respectively, $2\Delta_x^K$ is the kicker gap size, and $E_s = Am_0c^2\gamma$ is the synchronous energy.

COOLING RATE AND POWER

Longitudinally, the cooling rate for the average beam energy spread $\langle W\rangle = 2\int_0^{W_{max}} W\,\Psi_L(W)\,dW$ is given by

$$\frac{\partial\langle W\rangle}{\partial t} = 2\int_0^{W_{max}} dW\left(F_L + \frac{1}{2}\frac{\partial D_L}{\partial W}\right)\Psi_L(W) \tag{15}$$

Assume a linear gain function "notched" at multiples of the revolution frequency,

$$G_L(m\omega_i) = gmW,\quad \Delta\omega_i = -\frac{\eta\omega_s^2}{E_s\gamma^2}W$$

where $\Delta\omega_i = \omega_i - \omega_s$, and η is the momentum slip factor. Denote the effective frequency range of cooling from $n_1\omega_s$ to $n_2\omega_s$, $\Delta n = n_2 - n_1$, and $\bar{n} = \frac{n_1+n_2}{2}$. Consider the case that the Schottky bands are non-overlapping, i.e., $M > 1$, and the energy distribution is uniform,

$$\rho[\omega(W)] = \begin{cases}\frac{N}{2\Delta\omega_s} & |W| < W_0 \\ 0 & \text{otherwise}\end{cases} \tag{16}$$

where N is the number of particle, and $\Delta\omega_s$ is the frequency spread. Neglecting the effect of "bad mixing", the maximum cooling rate that corresponds to the minimum cooling time τ_{min} is obtained from Eq. 15 as

$$\tau_{L,min}^{-1} \approx -\frac{\Delta n\omega_s}{2\pi NM} \tag{17}$$

The mixing factor M is given by

$$M = \frac{1}{\bar{n}|\eta|(\hat{\Delta p}/p)_0} = \frac{1}{\sqrt{3}\bar{n}|\eta|\sigma_p}, \quad \text{for } M > 1 \qquad (18)$$

where σ_p is the rms spread in momentum $\Delta p/p$. The average power needed for the optimum longitudinal cooling is

$$\bar{P}_L \approx \frac{2}{f_s n^K R^K} \frac{1}{\tau_{L,min} M} \left(\frac{\beta^2 E_s \sigma_p}{Ze} \right)^2 \qquad (19)$$

where $f_s = \omega_s/2\pi$ is the revolution frequency.

Transversely, the cooling rate for the average beam action $\langle I \rangle = \int_0^{I_{max}} I \, \Psi_T(I) \, dI$ is given by

$$\frac{\partial \langle I \rangle}{\partial t} = \int_0^{I_{max}} dI \, \left(F_T + \frac{1}{2} \frac{\partial D_T}{\partial I} \right) \Psi_T(I) \qquad (20)$$

Assume a constant gain function at the betatron sidebands of the multiple revolution frequency, $G_T[(m \pm Q_x)\omega_s] = g$. The maximum cooling rate for the transverse action (emittance) is obtained as

$$\tau_{T,min}^{-1} = \frac{1}{\langle I \rangle} \frac{\partial \langle I \rangle}{\partial t}|_{max} \approx -\frac{\Delta n \omega_s}{\pi N M} \qquad (21)$$

The average power needed for the optimum transverse cooling is

$$\bar{P}_T \approx \frac{2\langle \epsilon_x \rangle (\Delta_x^K)^2}{f_s n_k R_k \beta_x^K} \frac{1}{\tau_{T,min} M} \left(\frac{\beta^2 E_s}{ZeL^K} \right)^2 \qquad (22)$$

where $\langle \epsilon_x \rangle = 2\langle I \rangle$ is the unnormalized average emittance.

RHIC EXAMPLE

Consider longitudinal and transverse stochastic cooling of a gold beam at RHIC storage. As shown in Table 1, the beam grows under intra-beam scattering during a typical 10-hour store [11]. Due to the growth in momentum spread, the mixing factor M varies from 9.4 to 5.6. The optimum cooling time varies from 8.7 to 3.2 hours for the momentum spread, and from 4.4 to 1.6 hours for the transverse emittance. With 128 units of kickers, each at 50 Ω coupling resistance, the average power for longitudinal cooling varies from 0.15 kW to 2.0 kW. Again with 128 units of 50 Ω kickers, each of effective length 1 cm and gap height ($2\Delta_x^K$) 4 cm at locations of $\beta_x^K = 20$ m, the average power for transverse cooling varies from 10 W to 114 W.

Table 1: Parameter example for stochastic cooling of a gold beam at RHIC storage.

Machine circumference	3833	m
Mass number, A	197	
Change state, Z	79	
Energy per nucleon, E_s/A	100	GeV/u
Revolution frequency, $f_s = \omega_s/2\pi$	78	kHz
Bunch intensity	1	10^9
Beam storage time	10	hour
Momentum slip factor, $\|\eta\|$	1.9	10^{-3}
RF voltage	6	MV
RF harmonic, h	2520	
Bunch length rms (begin - end)	0.11 - 0.19	m
Bunch length rms (begin - end)	27° - 45°	
Bunching factor (begin - end)	0.19 - 0.31	
Eff. bunch intensity (begin - end)	1.33 - 0.81	10^{13}
Momentum spread rms (begin - end)	0.44 - 0.71	10^{-3}
Transverse norm. 95% emittance	15 - 40	μm
Cooling bandwidth	4 - 8	GHz
Mixing factor, M (begin - end)	9.4 - 5.6	
Momentum cooling time	8.7 - 3.2	hour
Emittance cooling time	4.4 - 1.6	hour

DISCUSSIONS AND SUMMARY

Based on the coasting-beam Fokker-Planck equation, we analytically derived the optimum cooling rate and required power for the longitudinal and transverse stochastic cooling. The results indicate that the usual back-of-envelope formulae [3] over-estimated the cooling power by a factor of the mixing factor M in both cases. On the other hand, the scaling laws derived from the coasting-beam Fokker-Planck approach agree with those derived from the bunched-beam Fokker-Planck approach [4] if the peak beam intensity is used as the effective coasting-beam intensity. Although we have ignored signal suppression for the entire discussion, the conclusion holds.

A longitudinal stochastic cooling in RHIC with 4 – 8 GHz bandwidth can effectively counteract IBS-induced longitudinal beam growth, reducing the experimental background resulting from the residual beam that escaped the RF bucket and debunched around the ring. Combining with a transverse stochastic cooling of the same frequency bandwidth to contain the growth of the transverse emittance, we expect a significant increase in the average luminosity during a 10-hour store.

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