# A METHOD TO MEASURE THE FOCUSING PROPERTIES (R_MATRIX) OF A MAGNET*

N. Tsoupas, P. Thieberger, K. Brown, D. Gassner, W. Glenn, L. Ahrens, T. Roser, Y.Y. Lee, and J. Wei
C-AD Dept., BNL Upton, NY 11973

*Abstract*

In this paper we discuss, and study the feasibility of a method that may be used to measure the focusing properties of a magnet. This method may prove valuable when applied to "non_conventional" magnets that deviate from the usual dipole magnets or other multipole magnets (quadrupoles /sextupoles etc.) which are commonly used in a synchrotron or a beam line. In this category of "non_conventional" magnets, fall special magnets, which come under the name "Snakes", which are being used in synchrotron accelerators[1,2,3,4] to introduce artificial spin resonances to help overcome the intrinsic and/or imperfection spin resonances which appear during the acceleration of polarized beams. This method of measuring the focusing properties of a magnet requires the use of "low energy" and "high rigidity" heavy-ions which may be obtained from the BNL Tandem accelerator.

In brief the method consists on, injecting "narrow_beamlets" of heavy ions into a magnet and measuring the coordinates, of these "narrow_beamlets", at the entrance and exit of the magnet.

From the measurement of the coordinates of the "narrow_beamlets" we can deduce information on the first order transfer matrix elements (R matrix) and higher order matrix elements that define the focusing properties of the magnet.

## INTRODUCTION

The common method, which is applied in the magnet division of the Brookhaven National Laboratory (BNL), to determine the focusing properties of a magnet, is to measure the magnetic field of the magnet by using various techniques[5]. The magnetic field measurements provide an accurate method to determine the focusing properties of a magnet like dipoles or higher order multipoles. The magnetic-field measurement method however is not very accurate when applied to measure the focusing properties of a magnet, like a Siberian Snake[3,4]. In this paper we study the possibility to measure the focusing properties of a Helical Snake[3,4] or of any other magnet that does not lend itself easily to any of the magnetic measurements technique. In the rest of the paper we discuss the principle of the proposed method, the apparatus for the required measurements, and the required accuracy of the measurements.

* Work performed under Contract # DE-AC02-98CH10886 with the auspices of the US Department of Energy.
† tsoupas@bnl.gov

## PRINCIPLE OF THE METHOD

The focusing properties of a magnet are considered known when the coordinates of a particle at the exit of a magnet can be determined, assuming that the coordinates of the particle at the entrance of the magnet are known. The above sentence can be expressed either, schematically in Figure 1, which shows a magnet with the coordinates of a particle at the entrance and exit coordinate systems, or mathematically in equation (1) below.

$$x_i(out)=\Sigma R_{ij}x_j(in) + \Sigma\Sigma W_{ikl}(in)x_k(in)x_l(in) + \Sigma\Sigma\Sigma T_{imno}x_m(in)x_n(in)x_o(in) + \ldots\ldots \text{ HOT} \quad (1)$$

The notation in equation (1) is:
$(x_1, x_2, x_3, x_4, x_5, x_6) \Leftrightarrow (x, x', y, y', dl, \delta p/p)$
Where x, y, x′, y′ are the lateral (x, y) and angular (x′, y′) deviations of the particle from the trajectory of the central particle. The quantity $\delta p$ is the momentum deviation of the particle's momentum p, from the momentum $p_0$ of the central-particle, and $\delta l$ is the path length difference of the particle's path from the path of the central particle.

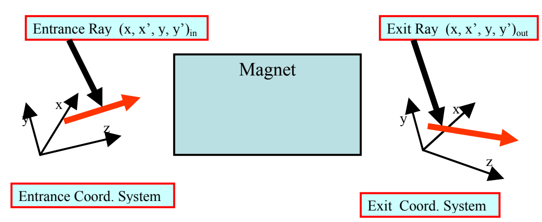


Figure 1. Schematic diagram of the magnet with the entrance end exit coordinate system. The beginning of the red arrows show the location of the particle and the direction of the arrow shows the direction of the particle at the entrance and exit coordinate systems.

The coefficients in equation (1) are defined as the partial derivatives of the output coordinates with respect to the input coordinates.

$R_{ij} = \partial x_i(out)/\partial x_j(in)$ 1st order
$W_{ikl} = \partial x_i(out)/\{\partial x_k(in)\partial x_l(in)\}$ 2nd order
$T_{imno} = \partial x_i(out)/\{\partial x_m(in)\partial x_n(in)\partial x_o(in)\}$ 3rd order
…………..
$T_{imno\ldots p} = \partial x_i/(\partial x_m\partial x_n\partial x_o\ldots\partial x_p)$ nth order

The knowledge of the coefficients $R_{ij}$, $W_{ijl}$, $T_{imno}$, and of the higher order coefficients, determine the focusing properties of any magnet.

In the "paraxial ray approximation", which assumes, that the momentum deviation $\delta p$ of the momentum p of the particle is much smaller than momentum $p_0$ ($\delta p/p_0 \leq 0.01$) of the central particle, and that the lateral coordinates x,y are much smaller than the radius of curvature[1] $\rho$, of the orbit of a particle moving in the magnetic field B of the magnet, most of the higher than the 1st order terms which appear in equation (1) ($W_{ikl}$, $T_{imno}$ ….etc) are usually negligible.

Therefore, the equation (1) above can be written

$$x_i(out)=\Sigma R_{ij}x_j(in) + \text{“ few HOT”}. \quad (2)$$

In equation (2) the expression “few HOT” includes the higher order term which are comparable in magnitude to the first order terms.

In summary, the first order coefficients $R_{ij}$, and the “most important” higher order coefficients appearing in equations (1,2) can be computed by measuring the coordinates $(x,x',y,y')_{exit}$ of an “adequate” number of particles at the exit of the magnet and the corresponding coordinates $(x,x',y,y')_{in}$ of the particles at the entrance coordinate system. By solving equation (2) we can determine the first order $R_{ij}$ terms as well as the “most important” higher order coefficients.

## DESCRIPTION OF THE APPARATUS

A schematic diagram of the proposed apparatus which can be used to measure the focusing properties of a magnet is shown in figure 2. The apparatus which schematically is shown in figure 2, consist of four separate devices.

The first device is the “1st ion-position defining device” which is placed at the entrance of the magnet and is located at a specified distance from the magnet iron, just before the start of the fringe field of the magnet. The “1st ion-position defining device” consist of a metal plate, labeled as “collimator“ in figure 2, which is thick enough to stop the incident ions. The collimator plate has “pin holes”[2] through which the ions will pass to enter the magnet. The location of the center of the pin holes defines the location $(x,y)_{in}$ of the particles at the entrance. Two sets of “jaws”, one horizontal and one vertical which can move parallel to the surface of the “collimator” will allow to select one pin hole at a time that the ions will enter the magnet. If the “jaw system” has an additional degree of freedom which allows the “jaw system” to rotate about the longitudinal axis (z-axis), this will permit the selection of a set of pin-holes which are lined up along any direction on the collimator plate. The “jaws” should be thick enough to stop the particles entering the magnet.

[1] The radius of curvature $\rho$ is defined in the equation $B\cdot\rho=k\cdot p/q$ (B is the field of the magnet, p is the momentum, and q is the charge state of the ion, k is a constant which depends on the units),

[2] The size of the holes depends on the magnet to be measured. For the Partial Snake magnet the diameter of the holes should be no greater than 0.5 mm.

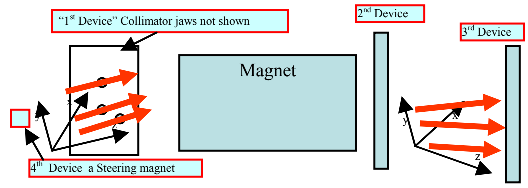


Figure 2. Schematic diagram of the magnet with the various devices which define the position (x,y) and direction (x′,y′) of the rays at the entrance and exit of the magnet.

The second device is the “2nd ion-position measuring device” which is located at a specified distance, just after the fringe field of the magnet, consists of a light emitting foil (visual Flag) and a CCD camera connected to a computer. The light emitting foil is a compound of Gadolinium Oxy-Sulfide doped with Terbium (Gd2O2S:Tb). The compound is bonded on an Al foil. The ions emerging from the pin holes of the “1st ion-position defining device” will generate light on the foils of the 2nd or 3rd measuring devices. The emitted light will be detected by the CCD camera and the computer will calculate the location of each “pin-hole image” on the visual flags (foils).

A schematic diagram of the detection system which utilizes a visual flag is shown in Fig. 3. The direction of the rays $(x',y')_{in}$ at the entrance of the magnet can be determined from the coordinates $(x,y)_{in}$ of the rays at the entrance, as defined by the “1st ion-position defining device”, and the coordinates $(x,y)_{out}$ of the rays at the exit as measured by the “2nd ion-position measuring device” when the field of the magnet is off.

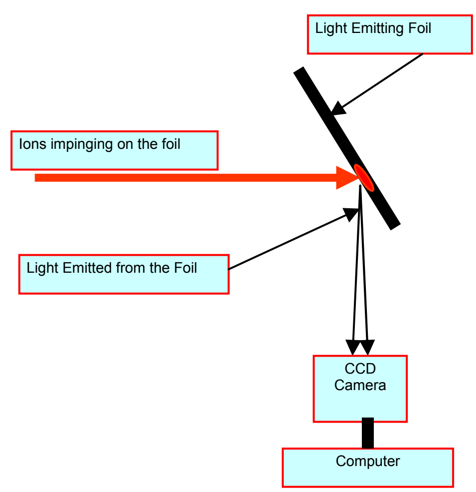


Figure 3. Schematic diagram of an ion-position measuring system to be used in the position measurement of the pencil-like ion beams which will enter and exit the magnet.

The third device “ 3rd ion-position measuring device” is located at the exit of the magnet at a specified distance, from the location of “2nd ion-position measuring device” otherwise it is identical to the “2nd ion-position measuring device". Position measurements of the ions taken from the 2nd and 3rd " ion-position measuring devices" will determine the direction of the ions at the exit of the magnet $(x',y')_{out}$.

The fourth device is the "ray-direction defining" magnet which is a dipole magnet that can change the direction $(x',y')_{in}$ of the rays at the entrance of the magnet and also serves to select the charge-state to be used when using heavy ions.

## COMPUTER SIMULATIONS

The method to measure the focusing properties of a magnet was tested using computer simulations on a 3D model of the warm helical snake [3] which is installed in AGS. The raytracing into the magnet was performed with the computer code SPRAY [6]. The required 3D magnetic fields of the magnet were computed using the computer code opera [7] on a 3D model of the magnet [4]. Error analysis of the proposed method as well as comparison of the method with the magnetic measurements method has been performed and details appear in [4]. The error analysis showed that the accuracy of the measurement of the beam centroids generated by the "beamlets" should be ±0.025 cm in order to obtain matrix elements of the same accuracy as we can obtain from the magnetic field measurements. We have assumed that the magnetic measurements are performed with a Hall probe which measures the field components at a given point with a relative error of $10^{-3}$. Zero position error has been assumed in the measurement of the Hall probe.

## SUMMARY

A method to determine experimentally the focusing properties of a magnet has been discussed. This method determines the first order matrix elements (R_matrix) of a magnet and the most "significant" higher order matrix elements. The error in the determination of the first order matrix elements is "reasonably small" to provide accurate measurements of the R_matrix of of a magnet.

## REFERENCES

[1] T. Roser, AIP Conf. Proc. 187 (1988) 1221.
[2] H.Huang, et. al. Phys. Rev. Lett. 73 (1994) 2982
[3] T. Roser et. al. "Acceleration of Polarized Beams using Multiple Strong Partial Siberian Snakes" These Proccedings.
[4] N. Tsoupas et. al. "Calculations of the Focusing Properties of a 7% Partial Snake Helical Magnet with Variable Helicity" BNL Tech. Note C-CA/AP/134 Feb. 2004.
[5] N. Tsoupas et. al. "A Method to Measure the Focusing Properties(R_matrix) of a Magnet" BNL Tech. Note C-CA/AP/135 Feb. 2004.
[6] N. Tsoupas BNL private communication "The computer code "SPRAY"
[7] Vector Field Inc.