# LOW ENERGY ION BEAM DYNAMICS IN AXI-SYMMETRIC RF UNDULATOR LINAC

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### Abstract

The ion beam acceleration and focusing in an axisymmetric periodic RF undulator structure is considered. In this structure the RF field has no synchronous wave and the accelerating force is to be driven by a combination of two non-synchronous waves. A new approach to RF focusing accelerator channel parameters choice is suggested. The efficiency of beam bunching and acceleration is confirmed by the numerical simulation.

### **INTRODUCTION**

The stable motion of an ion beam in linac can be provided by using external focusing devices or a special configuration of RF fields (RF focusing). For the low energy ion accelerators, the latter approach seems to be more promising. Today, several approaches of RF focusing are known: Alternating Phase Focusing (APF), Radio Frequency Quadrupole (RFQ), Axisymmetric RF focusing (ARF), and Undulator RF focusing (URF). The fundamental principles of APF in the single-wave approximation have been formulated in [1-2]. Using of existing APF method did not allow to improve all the RF focusing capabilities. In the scope of ordinary APF description, it is impossible to achieve a large value of longitudinal acceptance and beam current. The RFQ structure generally are used as initial part of DTL linac [3]. The beam current and transmission coefficient can be made large, but the acceleration gradient is small In RFQ. There is also the matching problem between the RFQ and axisymmetric DTL. A new type of axially symmetric RF field focusing (ARF) in a drift-tube accelerating structure was proposed and studied in [4]. The transverse and longitudinal beam stability are achieved by the variation of drift tube aperture and appropriate phasing. The ARF linac has a high current transmission coefficient, and the rate of energy gain is larger than in the RFQ structure.

In a conventional RF linac beams are accelerated by a synchronous wave. Another method to accelerate ions in the fields without synchronous wave was suggested in Ref. [5-6]. In this case the accelerating force is to be driven by a combination of two non-synchronous waves (two undulators). In undulator linac one of the undulator must be the RF type (it is driven non-synchronous RF wave field), the second one being, optionally, of magnetic, electrostatic or RF types. A few versions of URF linac were considered.

- 1. UNDULAC-E that employs a combination of RF field and field of electrostatic undulator;
- 2. UNDULAC-RF that employs a combination of two space harmonics of RF field.

These two undulator linacs for high intensity ribbon ion beam are investigated earlier [6-7]. In this paper the axisymmetric UNDULAC-RF structure is studied.

### **RF FIELD IN PERIODICAL RESONATOR**

Axisymmetric Undulator RF linac (AURF) can be designed as an interdigital structure (see Fig. 1). The longitudinal and transverse electric RF field components in periodical resonant structure can be described by:

$$E_{z} = E_{n} I_{0}(h_{n}r) \cos(\int h_{n}dz) \cos(\omega t),$$

$$E_{r} = E_{n} I_{1}(h_{n}r) \sin(\int h_{n}dz) \cos(\omega t),$$
(1)

where  $h_n = \mu / D + 2\pi n / D$  is the wave number of field harmonic,  $\mu$  is a phase advance of the RF field per period of structure, *n* is the harmonic number;  $I_0$  and  $I_1$  are the modified Bessel functions.



Figure 1: The structure of UNDULAC-RF ( $\mu = \pi$  mode).

Let us consider the simplest AURF structure with two fundamental harmonics n=0, n=1. In our case the phase velocities  $\beta_{ph,n,l} = \omega_{n,l} / ch_{n,l}$  of two waves differ significantly from the average velocity of the particles  $\beta_b$ . The acceleration and the focusing are provided by a combined wave, the phase velocity of which,  $\beta_w$ , is close to the beam velocity.

$$\beta_b \approx \beta_w \equiv 2\omega/c(h_0 + h_1). \tag{2}$$

The investigation of beam dynamics is not easily problem in URF because there is no synchronous RF field harmonic. The motion equation can be average over the rapid oscillations as in Ref. [5-6]. Then one can obtain equation in the Hamilton's form (1):

$$\frac{\mathrm{d}^2 \bar{\boldsymbol{r}}}{\mathrm{d}t^2} = -\frac{\mathrm{d}}{\mathrm{d}\bar{\boldsymbol{r}}} U_{eff} \,. \tag{3}$$

The effective potential function,  $U_{\text{eff}}$ , is included in this equation. Such function depends on the slow varying transverse co-ordinate,  $\rho = 2\pi r / \lambda$ , and particle phase in combined wave  $\varphi = \int h_s d\xi - \tau$  only. Here  $\xi = 2\pi z / \lambda$ 

and  $\tau = \omega t$  are the dimensionless longitudinal coordinate and time,  $\lambda$  and  $\omega$  are the length and the frequency of the wave. The condition of the acceleration and the transverse focusing can be investigated by means of  $U_{\text{eff}}$ . In UNDULAC-RF for  $\mu = \pi$  mode of RF field the effective potential function is equal

$$U_{eff} = \frac{1}{4} \left\{ \frac{10}{9} e_0^2 g\left(\frac{\rho}{2}\right) + \frac{26}{25} e_1^2 g\left(\frac{3\rho}{2}\right) + 2e_0 e_1 \left( I_0 \left(\frac{\rho}{2\beta_s}\right) I_0 \left(\frac{3\rho}{2\beta_s}\right) - I_1 \left(\frac{\rho}{2\beta_s}\right) I_1 \left(\frac{3\rho}{2\beta_s}\right) \right) \sin(2\varphi) \right\}$$
(4)

and for the zero mode

$$U_{eff} = \frac{1}{8} \left\{ e_0^2 + \frac{5}{9} e_1^2 g(2\rho) + 2e_0 e_1 I_0 \left( \frac{2\rho}{\beta_s} \right) \sin(2\phi) \right\}.$$
 (5)

Here  $e_{r,z} = e\lambda E_{r,z} / 2\pi mc^2$  are the dimensionless RF field harmonic amplitudes,  $g(\rho) = I_0^2 (\rho / \beta_c) + I_1^2 (\rho / \beta_c)$ . It should be noted that the frequency of ion beam bunching is doubled  $\omega_b = 2\omega$ .

# ANALYSIS OF LONGITUDINAL AND TRANSVERSE MOTION

Let us consider the AURF linac for deuterium ions. This accelerator consists of two subsections: the gentle buncher subsection and acceleration subsection. The effective beam acceleration can be realized if the reference particle velocity  $\beta_s$  is equal to  $\beta_w$ . In the buncher the reference particle phase in combined wave is reduced linearly, and the RF field harmonics amplitudes are increased sine proportional. The equation for the velocity of the reference particle can be written using the effective potential function (4) or (5):

$$\frac{\mathrm{d}\beta_s}{\mathrm{d}\tau} = e_{\mathrm{eff}} \sin 2\varphi_s \,. \tag{6}$$

Here  $e_{eff} = v \cdot e_0 \cdot e_1 / \beta_s$  is the effective amplitude of combined wave, v=1 for  $\mu = \pi$  mode of the RF field and v=1/2 for  $\mu = 0$  mode,  $\varphi_s$  is the phase of the reference particle in the combined wave.

Let us suppose that the particle velocity  $\beta$  is close to the phase velocity of the combined wave,  $\beta_w$ , and differs significantly from the phase velocities of zero,  $\beta_{s0}$ , and first,  $\beta_{s1}$ , RF field harmonics ( $\beta_{s0} = 2\beta_w$ ,  $\beta_{s1} = 2\beta_w/3$ for  $\mu = \pi$  and  $\beta_{s0} = \infty$ ,  $\beta_{s1} = \beta_w/2$  for  $\mu = 0$  modes). For  $\mu = \pi$  mode RF field the separatrix of the combined wave is shown in Fig. 2 (curve 3). This figure is plotted for the next parameters:  $E_0=120$  kV/cm, the bunching subsection length  $L_b=1.2$  m, accelerator subsection length L=1.2 m,  $\lambda=1.5$  m, the initial energy of the deuterium ions  $W_{in}=150$  keV ( $\beta_{in} = 0.013$ ).

The separatrixes of the zero (curve 1) and the first (curve 2) RF field harmonics are shown in this figure also

in assumption that the beam velocity is close to  $\beta_{s0}$  or to  $\beta_{s1}$  accordingly. The parameter  $\chi = 1.0$  in Fig. 2a and  $\chi = 0.5$  in Fig. 2b. In our case the separatrixes of the first harmonic of the RF field and the combined wave are overlapped. The ion longitudinal velocity simulated in the field of the two harmonics as the function of the longitudinal co-ordinate *z* is ploted in the same figure (curve 4). The longitudinal velocity oscillations can be larger than the combined wave separatrix size. The separatrixes are overlapped if the ratio of RF field harmonic amplitudes is large ( $\chi \ge 1$ ) (Fig. 2a). In this case the ions can be recatched by the first harmonic of the RF field or loused and the averaging method is not correct. These results show that the choice of the parameter  $\chi$  for the harmonics influences on the beam motion.

In the UNDULAC-RF for  $\mu = 0$  mode the separatrixes of combined wave and first RF field harmonic are distant because the phase velocity of combined wave is two times larger. Here, the influence of the velocity oscillations are smaller comparatively  $\mu = \pi$  mode.



Figure 2: The separatrixes of combined wave and RF field harmonics.

The transverse beam motion can be analyzed by means of the effective potential function too. The existence of a total minimum for  $U_{eff}$  is the necessary condition for the stability of the transverse and the longitudinal beam motion. The shape of the effective potential function is

shown in Fig. 3. The  $U_{eff}$  is the function of the particle phase  $\psi = \varphi - \varphi_s$  and the transverse coordinate *r*. The sections of  $U_{eff}$  are obtained by cutting by the plane r=0for different  $\psi$  (curve 1) and by the plane  $\varphi_s = \pi/2$  for different *r* (curve 2). It can be shown that the transverse focusing condition is satisfied for the different  $\chi$  value in UNDULAC-RF when  $\mu = \pi$  mode of the RF field is used. The Fig. 3a is plotted with  $W_{in}=150$  keV,  $E_0=130$ kV/cm,  $\chi = 0.9$  for  $\mu = \pi$  mode.

The amplitudes of RF field harmonics must be equal for the effective transverse beam focusing in UNDULAC-RF when  $\mu = 0$  mode field is used. The effective potential function for this type of UNDULAC is plotted at Fig. 3b with  $\chi = 0.5$ . One can see that  $U_{eff}$  has a local maximum in this case and the hollow beam can be observed. The transverse focusing is no effective because it is provided by means of the first RF field harmonic.



Figure 3: The sections of the effective potential function in UNDULAC-RF.

# NUMERICAL SIMULATION OF BEAM DYNAMICS

The computer simulation of a high-current deuterium ion beam dynamics in AURF linac described above was carried out by means of the cloud-in-cell method. The especially computer code BEAMDULAC was created for this purpose. This investigation is necessary to verify the averaging method and to find the optimal linac parameters. In order to test the averaging method applicability, the simulation was carried out for both the averaged field and the RF polyharmonic field. All results obtained in smooth approximation and in polyharmonic field for the zero current coincide to 5% -10%. An influence of a space charge field on the 3D ion beam dynamics can be studied by means of this code in order to define more exactly the limit current for input/output deuterium beam energy 0.15/1.2 MeV for the maximum value of the zero harmonic amplitude  $E_0$ =130 kV/cm.

At first, it was investigated the beam dynamics in UNDULAC-RF for  $\mu = \pi$  mode of the RF field. The optimal transverse focusing condition was evaluated. The current transmission coefficient is equal  $K_T$ =65-70 % if the parameter  $\chi = 1.0$ . The particles loses are caused by fast oscillations of particle phases and velocities and by the influence of space charge field. The limit current is equal 130 mA in this type of UNDULAC.

One can show that the transverse focusing is not effective in UNDULAC-RF for  $\mu = 0$  mode. The current transmission coefficient  $K_T \approx 30$  % for this case. It is clear, that this type of undulator linac can not be used for ion beam acceleration.

#### CONCLUSION

The method of AURF linac realization based on the modified interdigital cavity was described The analytically studies and the numerical simulation show that axi-symmetric UNDULAC-RF for  $\mu = \pi$  mode of the RF field is the new perspective system for ion beam acceleration. This periodical structure is simpler for realization, than ARF where parameter  $\chi = 3-5$  [4]. The zero RF field harmonic amplitude is also smaller than in ARF linac. In the consider range of the energy, the AURF structure can be used instead of ARF and RFQ because the energy gain in UNDULAC-RF is larger than in these conventional linacs.

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