ON PARASITIC CROSSINGS AND THEIR LIMITATIONS TO E$^+$E$^-$ STORAGE RING COLLIDERS

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Abstract

We treat the problem of parasitic crossing in e$^+$e$^-$ storage ring colliders analytically. Analytical formulae for the beam lifetime limited by the combined effects of beam-beam interactions at interaction point and at parasitic crossings are derived, and applied to the by-2 colliding mode of PEP-II low energy ring.

INTRODUCTION

Among others, one of the efficient way to increase the luminosity of a storage ring collider is to increase the number of colliding bunches. Taking two ring collider for example, when the distance between the adjacent bunches are two small, near interaction point (IP), the colliding bunches instead of being separated by two vacuum chambers, they have to travel in the same beam pipe and they have chance to have so-called parasitic crossings (PC) [1]-[9] before and after making collision at IP as shown schematically in Fig. 1. The long rang nonlinear beam-beam forces at PCs will have extra contributions to the limitation from the beam-beam interaction at IP. As far as PEP-II B-Factory is concerned, runing in by-2 mode and higher current, the parasitic crossing effect will be important even dominant [10]. As for Super-B factory [11][12] this effect could be more important. In this paper we make a theoretical analysis on the parasitic crossing and its combined effects together with beam-beam interaction at IP, and apply it to PEP-II low energy ring running in by-2 mode.

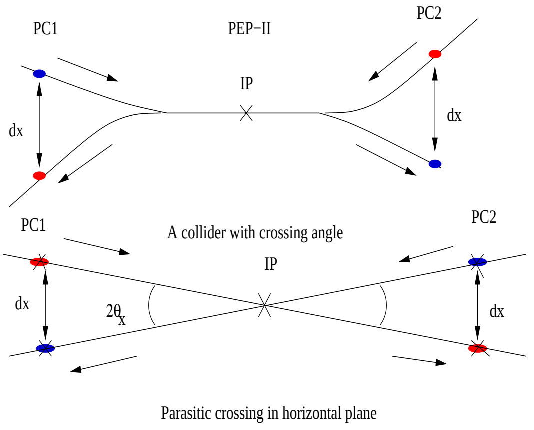

Figure 1: A schematic illustration of parasitic crossings.

BEAM-BEAM PARASITIC INTERACTIONS

The coherent kick felt by a test particle in head-on collision with a round Gaussian oppositely charged bunch can be calculated as [13]:

$$\delta r' = -\frac{2N_e r_e}{\gamma_* r}\left(1-\exp\left(-\frac{r^2}{4\sigma^2}\right)\right) \quad (1)$$

where $r=\sqrt{x^2+y^2}$, $r'=dr/ds$, N_e is the particle population in the counter-rotating bunch, r_e is the electron classical radius (2.818×10^{-15} m), σ is the standard deviation of the transverse charge density distribution of the counter-rotating bunch at IP, γ_* is the normalized particle's energy, and $*$ denotes the test particle. Quite different from the head-on collision of two bunches, during parasitic crossing the core particles of each bunch behave like a macro particle in the long range field of the other passing bunch, therefore the beam-beam kick formula expressed in eq. 1 should be modified to adapt to a large bunch separation distance, d, which is much larger than σ. A detailed discussion on long-range beam-beam kick could be found in ref. [4]. A good approximation for this long range kick is to replace σ in eq. 1 by Σ_{PC}, $\Sigma_{PC}=\sqrt{d_x^2+d_y^2}$, with d_x and d_y being the bunch horizontal and the vertical separation distance of two crossing bunches at the parasitic crossing point, respectively. Expanding the right hand side of eq. 1 into Taylor series and looking at the vertical kick, one gets

$$\delta y'_{PC} = \frac{N_e r_e}{\gamma_*}\left(\frac{1}{2\Sigma_{PC}^2}y - \frac{1}{16\Sigma_{PC}^4}y^3 + \frac{1}{192\Sigma_{PC}^6}y^5 - \cdots\right) \quad (2)$$

The Hamiltonian of a test particle in a linear storage ring perturbed by one parasitic crossing beam-beam force in vertical plane is expressed as follows:

$$H_{PC,y} = \frac{p_y^2}{2} + \frac{K_y(s)}{2}y^2 + \frac{N_e r_e}{\gamma_*}\left(\frac{1}{4\Sigma_{PC}^2}y^2 - \frac{1}{64\Sigma_{PC}^4}y^4 + \frac{1}{1152\Sigma_{PC}^6}y^6 - \frac{1}{24567\Sigma_{PC}^8}y^8 + \cdots\right)\sum_{k=-\infty}^{\infty}\delta(s-kL) \quad (3)$$

where $p_y = dy/ds$.

Let's stop here and recall what we know about in the head-on collision case [13]. As said above, a test particle which represents a whole bunch in head-on collision with another counter circulating bunch will suffer from a horizontal kick which is different from that shown in eq. 1. A whole set of corresponding formulae for head-on collision in the vertical plane are given below:

$$\delta r'_{IP} = -\frac{2N_e r_e}{\gamma_* r}\left(1-\exp\left(-\frac{r^2}{4\sigma^2}\right)\right) \quad (4)$$

Expanding eq. 4 into Taylor series, one gets

$$\delta y'_{IP} = \frac{N_e r_e}{\gamma_*}\left(\frac{1}{2\sigma^2}y - \frac{1}{16\sigma^4}y^3 + \frac{1}{192\sigma^6}y^5 - \cdots\right) \qquad \text{(RB)} \tag{5}$$

The Hamiltonian of a particle which represents the whole bunch in a linear storage ring perturbed by one hean-on collision at one IP in vertical plane is expressed as follows:

$$H_{IP,y} = \frac{p_y^2}{2} + \frac{K_y(s)}{2}y^2 + \frac{N_e r_e}{\gamma_*}\left(\frac{1}{4\sigma^2}y^2 - \frac{1}{64\sigma^4}y^4 + \frac{1}{1152\sigma^6}y^6 - \cdots\right)\sum_{k=-\infty}^{\infty}\delta(s-kL) \qquad \text{(RB)} \tag{6}$$

Started from eq. 6, in ref. [13], that beam-beam effects limited beam lifetimes could be expressed as [13]:

$$\tau_{bb,y,RB} = \frac{\tau_y}{2}\left(\mathcal{R}_{y,IP,RB}\right)^{-1}\exp\left(\mathcal{R}_{y,IP,RB}\right) = \frac{\tau_y}{2}\left(\frac{4}{\pi\xi_y}\right)^{-1}\exp\left(\frac{4}{\pi\xi_y}\right) \tag{7}$$

where τ_y are the damping times in horizontal and vertical planes and ξ_y are the head on collision beam-beam parameters. Comparing simply eq. 3 with eq. 6, by analogy, one gets the beam lifetime limited by one parasitic crossing

$$\tau_{PC,y,RB} = \frac{\tau_y}{2}\left(\mathcal{R}_{y,PC,RB}\right)^{-1}\exp\left(\mathcal{R}_{y,PC,RB}\right) = \frac{\tau_y}{2}\left(\frac{4}{\pi\xi_{PC,y}}\right)^{-1}\exp\left(\frac{4}{\pi\xi_{PC,y}}\right) \tag{8}$$

with

$$\xi_{PC,y} = \frac{r_e N_e \beta_{PC,x}}{2\pi\gamma_*\Sigma_{PC}^2} = \frac{r_e N_e \beta_{PC,y}}{2\pi\gamma_* d_x^2} \tag{9}$$

where $\beta_{PC,y}$ is the vertical beta function value at the parasitic crossing point, and d_y has been set to zero as a special case. What we should do now is to combine the effects from the beam-beam interactions at IP and PC to obtained the corresponding resultant beam lifetime. Recalling the discussions made in ref. [14] on the beam-beam interactions with the perturbation from the nonlinear electron cloud effect, by analogy, one gets

$$\tau_{bb,total} = \frac{\tau_y}{2}\left(\mathcal{R}_{total}\right)^{-1}\exp\left(\mathcal{R}_{total}\right) \tag{10}$$

where

$$\mathcal{R}_{total} = \frac{1}{\frac{1}{\mathcal{R}_{y,IP,FB}} + \frac{1}{\mathcal{R}_{y,PC,RB}}} \tag{11}$$

$$\mathcal{R}_{y,IP,FB} = \frac{3}{\sqrt{2}\pi\xi_y} \tag{12}$$

$$\mathcal{R}_{y,PC,RB} = \frac{4}{\pi\xi_{PC,y}} \tag{13}$$

If there are N_{PC} parasitic crossings per turn, eq. 12 should be replaced by

$$\mathcal{R}_{total} = \frac{1}{\frac{1}{\mathcal{R}_{y,IP,FB}} + \sum_{i=1}^{N_{PC}}\frac{1}{\mathcal{R}_{y,PC,RB,i}}} \tag{14}$$

where

$$\mathcal{R}_{y,PC,RB,i} = \frac{4}{\pi\xi_{PC,y,i}} \tag{15}$$

$$\xi_{PC,y,i} = \frac{r_e N_e \beta_{PC,y,i}}{4\pi\gamma_*\Sigma_{PC,y,i}^2} = \frac{r_e N_e \beta_{PC,y,i}}{2\pi\gamma_* d_{x,i}^2} \tag{16}$$

where d_y is set to zero. According to ref. [15], eqs. 11 and 12 should be replaced respectively by the following two expressions

$$\mathcal{R}_{y,IP,FB} = \frac{3\xi_{y,max,em,flat}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y} \tag{17}$$

and

$$\mathcal{R}_{y,PC,RB} = \frac{\xi_{y,max,em,flat}}{\pi\xi_{y,max,0}\xi_{PC,y}} \tag{18}$$

with

$$\xi_{y,max,em,flat} = \frac{H_0}{2\pi F}\sqrt{\frac{T_0}{\tau_y\gamma}} \tag{19}$$

and

$$F = \frac{\sigma_s}{\sqrt{2}\beta_{y,*}}\left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right)^{1/2} \tag{20}$$

where $H_0 \approx 2845$, $\xi_{y,max,0}$ is rigid beam case limiting value, and eq. 19 corresponds to single interaction point case. Taking $\xi_{y,max,0} = 0.0447$ means that we quantify the term "beam-beam limit" for the beam-beam limited beam lifetime being one hour at $\tau_y = 30$ ms. When $\sigma_s = \beta_{y,*}$ one has $F = 1$.

Till now we have established the necessary analytical formulae to estimate the effects due to parasitic crossing points.

APPLICATION TO PEP-II LOW ENERGY RING

When PEP-II collides in by-2 mode, the bunch spacing is 1.26 m which results in two nearest parasitic crossings at 0.63 m each side away from IP (the effects from other parasitic crossings are neglected). The PEP-II low energy ring parameters are: $\gamma = 6120$, $d_x = 3.217$ mm, $\epsilon_x = 23$ nm, $\epsilon_y = 1.5$ nm, $\beta_{IP,x}$=0.25 m, $\beta_{IP,y}$=0.012 m, $\beta_{PC,x}$=1.837 m, and $\beta_{PC,y}$=33.087 m. In Fig. 2 we give the relation between the particle population inside a bunch of high energy ring, N_e, and the vertical parasitic beam-beam parameter, $\xi_{PC,y}$. Taking $\xi_y = 0.063$ for PEP-II low energy ring as the theoretically achievable maximum vertical beam-beam parameter without parasitic crossings [14], from eq. 14 we give theoretically maximum achievable vertical beam-beam parameter (the same beam-beam effect limited beam lifetime as that of without parasitic crossings), $\xi_{y,Max}$, as a function of per parasitic crossing beam-beam parameter, $\xi_{PC,y}$, as shown in Fig. 3. As an example, taking $N_e = 4.65 \times 10^{10}$, one finds from Fig. 2 that $\xi_{PC,y} = 0.0108$, and then from Fig. 3 that the achievable maximum vertical beam-beam parameter at IP is $\xi_{y,Max} = 0.05486$, which is 87% of

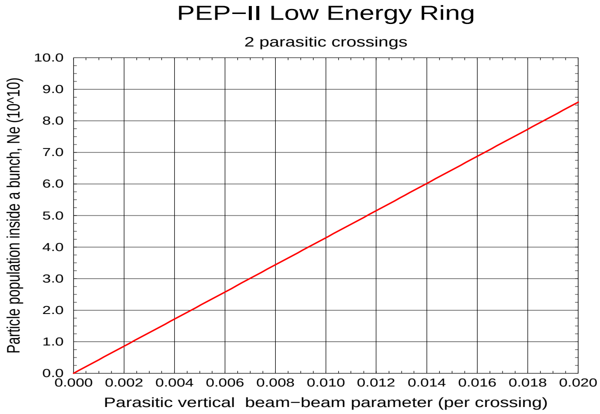

Figure 2: PEP-II Low Energy Ring (2 parasitic crossings): maximum vertical beam-beam parameter, $\xi_{y,Max}$, with the presence of parasitic crossings vs the parasitic crossing vertical beam-beam paramebbparasit4 ter per crossing, $\xi_{PC,y}$.

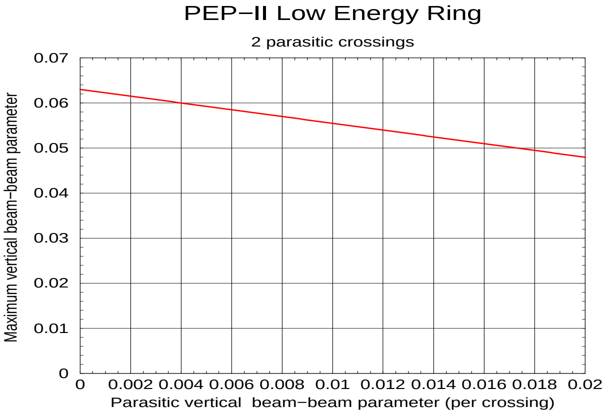

Figure 3: PEP-II Low Energy Ring (2 parasitic crossings): maximum vertical beam-beam parameter, $\xi_{y,Max}$, with the presence of parasitic crossings vs the parasitic crossing vertical beam-beam parameter per crossing, $\xi_{PC,y}$.

that without the two nearsest parasitic crossings, or a drop of 13%. Experimentally, when PEP-II passes from by-3 mode (no parasitic crossings) to by-2 mode, the bunch current in the high energy ring drops from $I_{bunch} = 1.139$ mA ($I_{bunch} = I_{beam}[1.07A]/N_b[939]$) [16] to $I_{bunch} = 0.976$ mA ($I_{bunch} = I_{beam}[1.2A]/N_b[1230]$, or $N_e = 4.65 \times 10^{10}$) [17], or a drop of 14%, which is close to the case theoretically calculated above.

CONCLUSIONS

In this paper we established analytical relation between vertical parasitic beam-beam parameter and the achievable maximum vertical beam-beam parameter at IP, and it is applied to the PEP-II low energy ring working in by-2 mode.

ACKNOWLEDGEMENTS

The author thanks J. Seeman for helpful discussions and his critical comments. Many thanks go to W. Kozanecki, Y. Cai and M. Sullivan for their providing useful PEP-II operation parameters.

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