EMITTANCE GROWTH AND BEAM LIFETIME LIMITATIONS DUE TO BEAM-BEAM EFFCTS IN E$^+$E$^-$ STORAGE RING COLLIDERS

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Abstract

In this paper we give analytical expressions for the maximum beam-beam parameter and related beam-beam limited beam lifetime in e$^+$e$^-$ storage ring colliders. After analyzing the performances of existing or existed machines, we make some discussions on the parameter choice for the Super-B factory design.

INTRODUCTION

For about four decades, beam-beam effect has been a subject of scientific research for its limiting nature on the performance of storage ring colliders, and countless publications have been dedicated to it. As a very comprehensive and classical review on beam-beam effect, readers are directed to ref. [1] for detailed information. In this paper we treat the beam-beam limitations from two directions, firstly, from emittance blow-up point of view (see ref. [2], which is modified in this paper), secondly, from the point of view of beam-beam limited beam lifetime (see ref. [3]), and finally, we combine them to a unified theory. Since there are some modifications to ref. [2], we spend more inks in section 2 to clarify emittance blow-up mechanism, and section 3 is devoted to the unified beam-beam effect theory. In section 4 experimental results obtained in different machines are compared with analytical ones, and finally, in section 5, parameter choice for Super-B factory has been briefly discussed.

BEAM-BEAM PARAMETER LIMIT COMING FROM BEAM EMITTANCE BLOW-UP

In e$^+$e$^-$ storage ring colliders, due to strong quantum excitation and synchrotron damping effects, the particles are confined inside a bunch. The state of the particles can be regarded as a gas, where the positions of the particles follow statistic laws. When two bunches undergo collision at an interaction point (IP, denoted by "*") the particles in each bunch will suffer from additional heatings. Taking the vertical plane for example, one has beam-beam induced kicks in y and $y' = dy/ds$ expressed as:

$$\delta y = -\frac{\sigma_s}{f_y} y \qquad (1)$$

$$\delta y' = -\frac{1}{f_y} y \qquad (2)$$

$$\frac{1}{f_y} = \frac{2N_e r_e}{\gamma \sigma_{y,*,+}(\sigma_{x,*,+} + \sigma_{y,*,+})} \qquad (3)$$

where σ_s is the bunch length, N_e is the particle number inside the bunch, r_e is the electron classical radius, $\sigma_{x,*,+}$ and $\sigma_{y,*,+}$ are bunch transverse dimensions just before the two colliding bunches overlapping each other, and $\sigma_{x,*}$ and $\sigma_{y,*}$ are defined as the transverse dimensions when the two bunches are fully overlapped at IP. The invariant of vertical betatron motion can be expressed as [4]:

$$a_y^2 = \frac{1}{\beta_y^*}\left(y_*^2 + \left(\beta_{y,*} y_*' - \frac{1}{2}\beta_{y,*}' y_*\right)^2\right) \qquad (4)$$

From eqs. 1 and 2 one finds that

$$\delta a_y^2 = \frac{1}{\beta_{y,*}}\left(\frac{\sigma_s}{f_y}\right)^2 y_*^2 \left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right) \qquad (5)$$

where y_* is the vertical displacement of the test particle with respect to the center of the colliding bunch. Due to the gaseous nature of the particles, one has to take an average of all possible values of y_* according to its statistical distribution function, and from eq. 5 one obtains:

$$< \delta a^2 >= \frac{1}{\beta_{y,*}}\left(\frac{\sigma_s \sigma_{y,*}}{f_y}\right)^2 \left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right) \qquad (6)$$

The resultant particles' vertical dimension combining the synchrotron radiation and beam-beam effects can be expressed as follows:

$$\sigma_{y,*}^2 = \frac{1}{4}\tau_y \beta_{y,*} \times \left(Q_y + \frac{1}{T_0 \beta_{y,*}}\left(\frac{\sigma_s \sigma_{y,*}}{f_y}\right)^2 \left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right)\right) \qquad (7)$$

where T_0 is the revolution time, τ_y is the radiation damping time, and Q_y is defined according to ref. [4] as $\sigma_{y,*,0}^2 = \frac{1}{4}\tau_y \beta_{y,*} Q_y$ with $\sigma_{y,*,0}$ being bunch natural vertical dimension at IP. Solving eq. 7, one finds

$$\sigma_{y,*}^2 = \frac{\sigma_{y,*,0}^2}{\left(1 - \frac{\tau_y}{4T_0}\left(\frac{e^2 N_e K_{bb,y}}{E_0}\right)^2\right)} \qquad (8)$$

where E_0 is particles' energy, and

$$K_{bb,y} = \frac{\sigma_s}{2\pi\epsilon_0 \sigma_{y,*,+}(\sigma_{x,*,+} + \sigma_{y,*,+})} \times \left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right)^{1/2} \qquad (9)$$

Since $\sigma_y(s) = \sqrt{\epsilon_y \beta_y(s)}$, from eq. 8 one gets:

$$\epsilon_y = \frac{\epsilon_{y,0}}{\left(1 - \frac{\tau_y}{4T_0}\left(\frac{e^2 N_e K_{bb,y}}{E_0}\right)^2\right)} \quad (10)$$

where $\epsilon_{y,0}$ is the natrual transverse emittance. For a flat bunch ($\sigma_{y,*,+} << \sigma_{x,*,+}$), from eq. 10 one knows that

$$\sigma_{x,*,+}\sigma_{y,*,+} > \left(\frac{3RN_{IP}(e^2 f N_e \beta_{y,*})^2}{8\pi^2 \epsilon_0 m_0 c^2 \gamma^5}\right)^{1/2} \quad (11)$$

Defining

$$H = \frac{\sigma_{x,*,+}\sigma_{y,*,+}}{\sigma_{x,*}\sigma_{y,*}} \quad (12)$$

where H is a measure of the plasma pinch effect, assuming that H can be expressed as follows

$$H = \frac{H_0}{\sqrt{\gamma}} \quad (13)$$

and recalling the beam-beam parameter definition:

$$\xi_y = \frac{N_e r_e \beta_{y,*}}{2\pi\gamma\sigma_{y,*}(\sigma_{x,*} + \sigma_{y,*})} \quad (14)$$

where β_y^* is the beta function value at the interaction point, σ_x^* and σ_y^* are the bunch transverse dimensions after the plasma pinch effect, respectively, and finally, by combining eqs. 11, 13 and 14 one gets in general case

$$\xi_y \leq \xi_{y,max,em,flat} = \frac{H_0}{2\pi F}\sqrt{\frac{T_0}{\tau_y \gamma N_{IP}}} \quad (15)$$

or for isomagnetic case

$$\xi_y \leq \xi_{y,max,em,flat} = \frac{H_0 \gamma}{F}\sqrt{\frac{r_e}{6\pi R N_{IP}}} \quad (16)$$

where $H_0 \approx 2845$, R is the local dipole bending radius, and F is expressed as follows

$$F = \frac{\sigma_s}{\sqrt{2}\beta_{y,*}}\left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right)^{1/2} \quad (17)$$

The subscript em in eqs. 15 and 16 denotes the emittance blow-up limited beam-beam parameter. When $\sigma_s = \beta_{y,*}$ one has $F = 1$.

BEAM-BEAM PARAMETER LIMIT COMING FROM BEAM-BEAM INDUCED BEAM LIFETIME

In ref. [3] we have derived beam-beam effect limited beam lifetimes for a rigid flat beam

$$\tau_{bb,y,flat} = \frac{\tau_y}{2}\left(\frac{3}{\sqrt{2}\pi\xi_y N_{IP}}\right)^{-1} \exp\left(\frac{3}{\sqrt{2}\pi\xi_y N_{IP}}\right) \quad (18)$$

$$\tau_{bb,x,flat} = \frac{\tau_x}{2}\left(\frac{3}{\pi\xi_x N_{IP}}\right)^{-1} \exp\left(\frac{3}{\pi\xi_x N_{IP}}\right) \quad (19)$$

and a rigid round beam

$$\tau_{bb,y,round} = \frac{\tau_y}{2}\left(\frac{4}{\pi\xi_x N_{IP}}\right)^{-1} \exp\left(\frac{4}{\pi\xi_x N_{IP}}\right) \quad (20)$$

From eqs. 18 and 19 one finds that for the same $\tau_{y,bb,flat}/\tau_y$, $\tau_{x,bb,flat}/\tau_x$, and $\tau_{y,bb,round}/\tau_y$, one has $\xi_{x,flat} = \sqrt{2}\xi_{y,flat}$, and $\xi_{y,round} = \frac{4\sqrt{2}}{3}\xi_{y,flat} = 1.89\xi_{y,flat}$.

Now taking into account of the emittance blow-up effect due to beam-beam interactions, in a heuristic way, one gets

$$\tau_{bb,y,flat} = \frac{\tau_y}{2}\left(\frac{3\xi_{y,max,em,flat}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}}\right)^{-1} \exp\left(\frac{3\xi_{y,max,em,flat}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}}\right) \quad (21)$$

and

$$\tau_{bb,y,round} = \frac{\tau_y}{2}\left(\frac{3\xi_{y,max,em,round}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}}\right)^{-1} \exp\left(\frac{3\xi_{y,max,em,round}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}}\right) \quad (22)$$

with

$$\xi_{y,max,em,round} = 1.89\xi_{y,max,em,flat} \quad (23)$$

where $\xi_{y,max,0}$ is rigid beam case limiting value. Taking $\xi_{y,max,0} = 0.0447$ means that we quantify the term "beam-beam limit" for the beam-beam limited beam lifetime being one hour at $\tau_y = 30$ ms.

COMPARISON OF SOME MACHINE PERFORMANCES WITH RESPECT TO THEORETICAL ESTIMATIONS

We start with the machine parameters [5] shown in Table 1 where the beam energy ranges from half GeV (DAFNE) up to almost hundred GeV, LEP-200, among which there are two machines make the collisions with non zero crossing angle, i.e., DAFNE and KEK-B. Using Table 1 and eq. 15 and assuming $F = 1$, the theoretical head-on collision beam-beam parameter limits are given in Table 2. The experimentally achieved maximum beam-beam parameters are shown in Table 2 also with or without crossing angle. The agreement between the two sets of values is quite well. Two machines, KEK-B factory and DAFNE, which have finite crossing angles, deserve further analyses. From Table 2 one finds that with Piwinski crossing angle $\Phi = 0.69$ the experimentally achieved KEK-B low energy ring's (positron) maximum vertical beam-beam parameter is 20% lower than that of head-on collision. On the contrary, the maximum achieved vertical beam-beam parameter of high energy ring seems not have been affected by

Machine	N_{IP}	γ	τ_y (ms)	T_0 (μs)
DAFNE	1	10^3	36	0.325
BEPC	1	3.7×10^3	28	0.8
PEP-II(L)	1	6.12×10^3	62	7.33
KEKB(L)	1	6.86×10^3	43	10.05
KEKB(H)	1	1.57×10^4	46	10.05
PEP-II(H)	1	1.76×10^4	37	7.33
LEP-100	4	8.82×10^4	38	88.9
LEP-200	4	1.58×10^5	5	88.9

Table 1: The machine parameters

the large crossing angle. As for DAFNE, according the theoretical analysis method described in ref. [6], it seems that the experimentally achieved rather low vertical beam-beam parameter (0.02) should not be due to Piwinski angle of $\Phi = 0.22$, but might be due to bunch lengthening effect [7] in addition to nonlinear electron cloud effect [8] (the electron cloud density $\rho_{ec} \approx 5 \times 10^{12}$).

Machine	$\xi_{y,max,theory}$	$\xi_{y,max,exp}$
DAFNE	0.043	0.02
BEPC	0.04	0.04
PEP-II(L)	0.063	0.06
KEKB(L)	0.084	0.069
KEKB(H)	0.053	0.052
PEP-II(H)	0.048	0.048
LEP-I	0.037	0.033
LEP-II	0.076	0.079

Table 2: The theoretical maximum and experimentally achieved beam-beam parameters

SOME DISCUSSIONS CONCERNING SUPER-B FACTORY DESIGNS

Super-B factory, or so-called next generation B factory, should have a luminosity larger than 10^{36}/cm^2/s [9][10], which could be expressed as follows [9]:

$$L_{max} = 2.17 \times 10^{34}(1+r)\xi_{y,max}\frac{E_0(GeV)N_bI_b(A)}{\beta_{y,*}(cm)} \tag{24}$$

where L_{max} has units of $\frac{1}{cm^2 s}$, $r = \sigma_{y,*}/\sigma_{x,*}$, N_b is the number of bunches inside a beam, and I_b is the average current of a bunch. Since ξ_y depends I_b, beam transverse dimensions at IP, and $\beta_{y,*}$, the critical thing in pushing ξ_y to its maximum value $\xi_{y,max}$ expressed in eq. 15 for flat beam and eq. 23 for round beam is to choose carefully $I_{b,max}$ at which $\xi_{y,max}$ is reached that the bunch length $\sigma_s(I_{b,max})$ should be almost same as $\beta_{y,*}$, and the rest thing is to push N_b to realize the required luminosity. In the following, based on the beam-beam effect theory developed above, we will discuss the initial parameters for a 10^{36} B-Factory proposed by Seeman in ref. [9]. Given $E_0 = 3.1$ GeV, $I_bN_b = 19.2$ A, $N_b = 3492$, $\beta_{y,*} = 0.32$ cm, $\sigma_{s,0} = 3.5$ mm, $\xi_y = 0.14$, $r = 1$, and $\tau_y = 63$ ms, from eqs. 15, 22, 23 (where $\xi_{y,max,em,flat} = 0.06$ is used), and 24, one finds $L_{max} = 1.13 * 10^{36}$ and the beam-beam limited beam lifetime $\tau_{bb,y,round} = 9$ minutes, which agree quite well with Seeman's estimation [9].

CONCLUSIONS

In this paper we have presented analytical expressions for the maximum beam-beam parameters and the corresponding beam-beam limited beam lifetimes for flat and round colliding beam cases. By applying these analytical formula to the 10^{36} B-Factory parameters given in ref. [9], one finds a similar beam-beam effect limited beam lifetime.

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