OVERCOMING PERFORMANCE LIMITATIONS DUE TO SYNCHRO-BETATRON RESONANCES IN THE HERA ELECTRON RING

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Abstract

Performance limitations of the HERA Electron Ring due to synchro-betatron resonances are analysed and successful cures have been developed and implemented which enable high luminosity operation at the desired working point for high electron spin polarization.

INTRODUCTION

After the HERA luminosity upgrade [2], the HERA [1] Electron Ring suffered from synchro-betatron resonances (SBR: $Q_x+m\cdot Q_s=q$, m = 2, 3, q integer). This was mainly because of a lower synchrotron tune of $Q_s=0.051$ due to stronger transverse focusing in the arcs (from 60° per FODO cell to 72°) and a shift in the damping distribution in favour of transverse damping which increases the longitudinal emittance. At the preferred working point for high luminosity and for high electron spin polarization [3], ($Q_x=0.13$, $Q_y=0.21$), between the 2nd and the 3rd SBR satellite of the horizontal integer resonance operation proved to be very difficult. SBR [4] are driven by chromaticity, by dispersion in the RF cavities, in sextupoles, or at the interaction point (IP), by collisions under a crossing angle, or by any other effects where transverse forces depend on longitudinal position or momentum deviation. In HERA, with a dispersion-free IP and head-on collisions, the main concern is chromaticity and dispersion in the sextupoles and RF cavities.

Near a SBR, the amplitude of transverse betatron oscillations will be strongly increased due to coupling with the longitudinal oscillations. The beam may then be lost at the transverse aperture limitation even before the motion becomes unstable. The driving terms for SBRs can be calculated using the formalism for resonances in the transverse plane following the procedure by Ripken et al [5]. In this formalism the transverse coordinates are measured with respect to the dispersion orbit. The motion is then expressed in terms of longitudinal and transverse oscillation modes. These are only weakly coupled so that perturbation theory can be applied. The coupling terms in the Hamiltonian which drive the SBRs $Q_x+2Q_s+q=0$ and $2Q_x+Q_s+q=0$ are

$$K_{21} = -\frac{1}{2}\varepsilon\cdot p^2 + \frac{1}{2}mD\cdot\varepsilon\cdot x^2 + \frac{1}{2}W\cdot\sigma\cdot(Dp - D'x)^2$$

$$K_{12} = -D'\varepsilon^2 p + \frac{1}{2}mD^2\varepsilon^2 x + \frac{1}{2}W\sigma^2(Dp - D'x)$$

$$W = \left(\frac{2\pi h}{L}\right)^2\cdot\frac{eU}{E_0}\cdot\cos(\phi_0)\cdot$$

Here *x*, *p* are the transverse and ε, σ are the longitudinal phase space variables. The independent variable is the path length *s* along the design orbit. *D* is the dispersion function, the m are the sextupole strengths, the U are the cavity voltages, h is the harmonic number, E_0 is the reference energy, *L* is the accelerator circumference and ϕ_0 is the synchronous phase. Primes denote derivatives with respect to *s*. There are three contributions, representing chromatics, dispersion in sextupole magnets and dispersion in RF cavities. In evaluating driving terms, one assumes solutions for the uncoupled linear motion in terms of amplitude and phase $\beta(s)$ and $\psi(s)$, $z=\sqrt{2J_z\beta_z(s)}\cdot\cos(\psi_z(s)+\varphi_z)$ for both longitudinal and transverse motion. The longitudinal β-function β_s is nearly constant around the ring and is in good approximation $\beta_s = L\sqrt{\frac{\alpha_{mc}E_0}{2\pi heU\cos(\Phi_0)}}$. The longitudinal phase function ψ_s is then a linear function in s, $\psi_s(s)=\frac{2\pi\cdot s}{L}\sqrt{\frac{eU\cos(\Phi_0)h\alpha_{mc}}{2\pi E_0}}$; α_{mc} is the momentum compaction factor. The Hamiltonian described in reference [5] is expanded up to third order in the coordinates. The vertical phase space coordinates, which are less important, are not considered. Expansion to higher order terms is not expected to produce important contributions. The fourth order SBR resonance $Q_x+3Q_s=0$, however, which may be produced by interference between the driving terms for the resonances $Q_x+2Q_s=0$ and $2Q_x+Q_s=n$, turns out to be non-negligible. To evaluate the driving terms of the SBRs, the standard procedures as described for example in reference [6] can be used whereby the vertical phase space variables are replaced by the longitudinal ones. In the case that the tunes are close to a resonance, the motion is dominated by a single harmonic of the non-linear potential (called driving term), for which the equations of motion can be solved and the stability limits of the motion can be evaluated. The driving term is an integral of field coefficients and optical functions around the accelerator. For the term $\frac{1}{2}W\cdot D'\cdot\sigma^2\cdot x$ for example, the corresponding driving terms for the resonance $Q_x+2Q_s=q$ reads

$$k_{1212q} = \left|\oint\frac{ds}{8\sqrt{2\pi}}W(s)D'\beta_x^{1/2}\beta_s e^{(i(\psi_x+2\psi_s-(Q_x+2Q_s)2\pi s/L))}\right|$$

The driving term determines the width of a resonance. This is defined as the distance of the tune from resonance for which particles with a given amplitudes become unstable. We calculate the width for $n_\sigma=10$ times the natural beam size. For the resonance $Q_x+n\cdot Q_s+q \approx 0$, we have

$$Q_x + nQ_s + q = \frac{\left(n^2+1\right)^n}{\sqrt{\left(n^2+1\right)^2 - 4\cdot n}} \cdot \frac{n_\sigma}{2} \cdot k_{1212q} \cdot \sqrt{\left(\frac{\varepsilon_s + n\varepsilon_x}{2}\right)^{n-1}}$$

$\varepsilon_x \ll \varepsilon_s$ are the unperturbed horizontal and longitudinal equilibrium emittances, n_σ = amplitude /rms-beam size.

RESONANCE ANALYSIS

The resonance strengths are analysed for several HERA-e beam optics. The optics used before the HERA luminosity upgrade (HELUMV6), has a betatron phase advance of 60° per FODO cell and has six sextupole families for chromaticity correction. The low emittance optics used after the luminosity upgrade (HELUM72GJ) has 72° per FODO cell. An RF frequency shift of 300Hz is applied. Only two sextupole families are used for chromaticity compensation. An improved version (HELUM72SM) still has only two sextupole families but has intrinsic cancellation of the non-linear chromaticity of the two colliding beam interaction regions (IR) due to appropriate choice of betatron phase advance. The relevant beam parameters are listed in Table 1. The results are listed in tables 2, 3 and in figure 1. The tables give values for driving terms and resonance widths. The linear resonance Q_x+Q_s, driven by dispersion in the RF cavities is quite small. The strength of the resonance Q_x+2Q_s is dominated by the term $\frac{1}{2}\cdot D'\cdot\varepsilon^2\cdot p$. There is, however, no evidence for a significant disadvantage of the 72°optics which would explain the difficulties with the resonance Q_x+2Q_s=q near the desired working point which were experienced in operations and confirmed by tracking calculations [7]. However SBR satellites of the half integer resonance, far from the desired tunes are much stronger for the 72° optics HELUM72GJ. This can be attributed to the non-compensated non-linear chromaticity, in particular for the optics HELUM72GJ. Figure 1 shows how the driving term accumulates for the three optics. One recognizes the compensation of IR chromaticity contributions by the sextupoles in the arc for HELUMV6. One also notes that there is no non-linear chromatic compensation for HELUM72GJ and one can see the effect of the intrinsic compensation between the two IRs for the optics HELUM72SM.

Table1. Beam Parameters

Optics	HELUMV6	HELUM72
Phase Adv./FODO [2π]	60	72
Momentum Compaction	6.81 x 10^{-4}	4.75 x 10^{-4}
RF Frequency shift [Hz]	0	+350
Synchrotron Tune	-0.061	-.0515
RMS-Energy Spread	9.54 x 10^{-4}	12.8 x 10^{-4}
Horizontal Emittance [nm]	42	20
Long. Emittance [μm]	10.24	15.4
Long. Beta Function [m]	11.25	9.4
Beam Energy [GeV]	27.5	
Harmonic number	10560	
RF Voltage [MV]	125	
Synchronous Phase [°]	44.3	
Circumference [m]	6335.826	

Table 2: Driving terms of SBR Resonances [$m^{-1/2}$]

Optics/ resonance	Q_x+Q_s	Q_x+2Q_s	$2Q_x+Q_s$
HELUM6V6	5.07E-04	0.088	0.538
HELUM72GJ	4.68E-04	0.065	2.357
HELUM72SM	4.22E-04	0.035	1.053

Table 3: Widths of SBR (n_σ=7)

Optics/ resonance	Q_x+Q_s	Q_x+2Q_s	$2Q_x+Q_s$
HELUM6V6	0.000507	0.00467	0.0202
HELUM72GJ	0.000465	0.00759	0.1136
HELUM72SM	0.000422	0.00551	0.0502

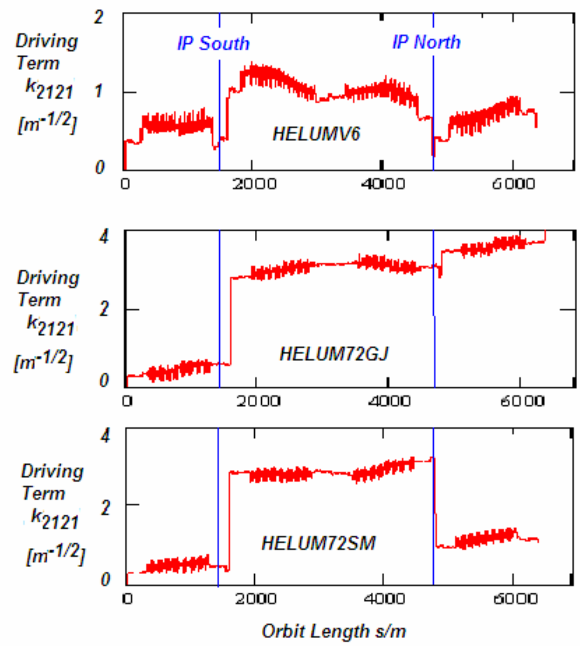

Figure 1: Build up of the driving term of the satellite resonance $2Q_x+Q_s$=q around the circumference

HIGHER ORDER RESONANCES

The above considerations do not offer an explanation for the operational difficulties with SBR in HERA since calculation of resonance strength based on expansion to higher order does not provide a noticeable contribution to the resonance strength Q_x+3Q_s=q. However the resonance may be driven by an interference of the resonances Q_x+2Q_s=q and $2Q_x+Q_s$=q. If the tune is not close to these resonances, the driving terms can be absorbed by a canonical transformation, which generates in turn the driving term for higher order resonances. The evaluation of the driving terms of such higher order resonances involves a double integral around the ring. The procedure is described in reference [8]. Comparing the build-up of the driving term of the resonance Q_x+3Q_s=q for the optics HELUM72GJ and HELUM72SM (figure 2) one can see, that the 3rd order satellite is much weaker if the seeding resonance $2Q_x+Q_s$=q is intrinsically compensated. The absolute value of the resonance width is for HELUM72GJ approximately Q_x+3Q_s+q=0.0015 (n_σ=7). Thus the space around the desired tune is limited by two SBR resonances and the horizontal betatron amplitudes are enhanced by a factor of at least 2 for $n_\sigma=7$. This produces significant enhancement of the positron beam size and explains the degradation of the beam lifetime seen in operations.

CLOSED ORBIT EFFECTS

Closed orbit distortions may strongly enhance SBR. A single kick will generate a closed orbit oscillation and the corresponding dipole component in quadrupole magnets oscillates in phase, with the betatron frequency. This generates a large contribution to the dispersion which accumulates around the circumference. The contribution is proportional to the accelerator circumference L. It oscillates with the betatron frequency and it will drive the SBR resonance $Q_x+2Q_s=q$. The driving term for this resonance samples the dispersion in phase with the horizontal betatron oscillation. The corresponding contributions to the driving term will accumulate around the lattice providing a strong contribution proportional to L. Thus an oscillatory closed orbit distortion generates a contribution to the resonance strength which is proportional to L^2. For a large accelerator like HERA with L = 6355.826 m one then expects a large contribution to the strength of satellite resonances. The contribution to the resonance driving term can be estimated for a regular FODO lattice with N FODO cells and a circumference L with a horizontal tune of Q_x and beta functions β_x and β_s as

$$h_{1212}^{chr} = \frac{L^2 \cdot \hat{x} \cdot k \cdot l \cdot (\hat{\beta}_x - \breve{\beta}_x)}{32\pi \cdot L_{FODO} \sin^2(\pi Q_x)\beta_x^{3/2}\beta_s} = 0.125 \cdot \frac{\hat{x}}{mm} m^{-1/2}$$

($\hat{x}$ is the peak closed orbit amplitude).

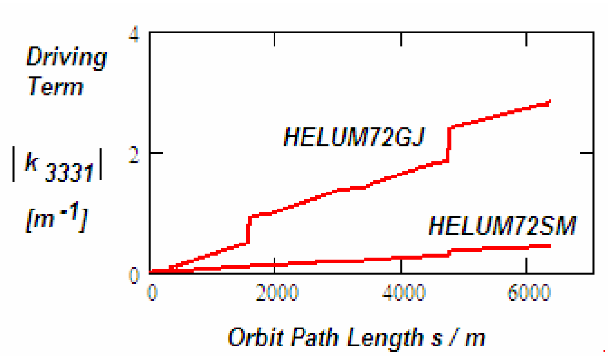

Figure 2: Build-up of the leading driving term of the SBR $Q_x+3 \cdot Q_s=q$ using the formalism of reference [8]. The optics HELUM72SM with intrinsic chromatic compensation has a considerably reduced resonance driving term.

The result of an exact evaluation for HERA is that an orbit oscillation of only 1mm amplitude leads to an increase of the resonance strength by a factor of three (see figure 3). This analysis explains the extreme orbit dependence of the lepton beam lifetime which is observed if the HERA electron ring is operated with a working point near $Q_x+2Q_s=q$. Another strong contribution is produced by the asymmetric component of the closed orbit at the interaction points (IP) because of the large β. If one applies a closed orbit bump of 5mm amplitude with an angle at the IP, the resonance strength is increased by a factor of two (see figure 4). The conclusion is that HERA can only be operated near the desired working point with extremely precise orbit control.

CONCLUSIONS

The analysis described in the previous sections explains the operational difficulties as resulting from SBR. It led to cures which now allow HERA to operate safely near the desired working point between the 2nd and 3rd satellite SBR of the horizontal integer resonance. These measures are:

a) The use of an optic HELUM72SM with 90° betatron phase advance between the two IP's provides sufficient headroom near the desired working point. This optics provides a reasonable, intrinsically compensated higher order chromaticity and suppresses the 3rd order satellite resonance. This optics has the additional advantage, that the beam-beam beta-beat is cancelled in first order.

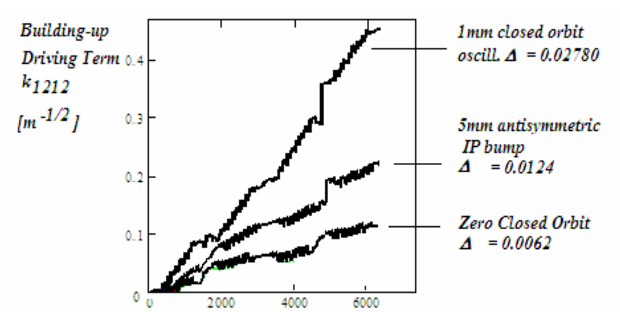

Figure 3: Contributions to the driving term of the resonance Q_x+2Q_s by closed orbit distortions

b) Tight closed orbit control during the whole operation cycle is provided using orbit stabilization feedback [9]. It controls the orbit to a level of 0.1mm and efficiently avoids –among other undesired closed orbit effects- the generation of a strong contributions to the SBR.

The initial severe distortion of HERA operation at the desired working point after the luminosity upgrade was completely removed by these measures. They provide the basis for the present high luminosity and high polarization operation with the upgraded lattice.

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