ACCELERATION OF ELECTRONS BY SPATIALLY MODULATED LASER WAVE

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Abstract

The acceleration of electrons in a system of bands of a linearly polarized laser wave propagating at small angles to the direction of motion of the electrons is considered. The parameters of the electron bunch and the laser wave are chosen so, that during driving electrons in a band of a wave, the electric field of a wave does not change its direction. We derive the requirements for the maximum rate of acceleration, and consider the influence of light diffraction on the process of acceleration. It is shown that the discussed scheme of acceleration allows the possibility for high acceleration rates due to existence of modern powerful lasers.

INTRODUCTION

The possibility of the acceleration of electrons by the laser field in the vacuum has been actively discussed in recent years [1-14]. In most cases of schemes of electron acceleration, the electron - laser interaction occurs on short length. The electron-laser interaction is restricted spatially because of use of a narrow focal point of a laser bunch [1-4] or use of narrow bands of a laser light [5-14]. However, these methods may cause some problems. At use of the focused laser beam high requirements is presented to the cross-section sizes of electron beam. At use of the light strips, proposed in some papers, there may be some problems of phasing of the laser beam in different strips. In the paper [1], a scheme is proposed where these requirements are considerably softened. In this paper we consider the possibility of the acceleration of the electron beam by scheme [1] in more detail. We find the conditions of deriving of maximum rate of acceleration, correct for any intensity of the laser, and consider the influence of light diffraction on process of acceleration.

THE SCHEME OF ACCELERATION OF ELECTRONS

The principle scheme of acceleration of electrons is shown on Fig. 1. Here the electron intersects the system of bands of linearly - polarized waves under the angle α. We assume that the accelerating wave is propagated along axis z, electric field E of a wave is directed on axes y, and magnetic field H - on axis x. Then:

$$E = E_y = E_m \sin\varphi,\quad H = H_x = -E_m \sin\varphi, \tag{1}$$

$$\varphi = \omega t - kz, \tag{2}$$

where, ω is frequency of a wave, $k = \omega/c$ is wave number. It follows from the equations of motion of electron in the field (1) of wave that:

$$\frac{d\gamma}{dt} = \frac{1}{mc}\frac{dp}{dt} = \frac{d(\gamma\beta_z)}{dt} = \frac{eE_m}{mc^2}\beta_x \sin\varphi, \tag{3}$$

$$\frac{d\gamma}{dz} = \frac{eE_m\beta_y}{mc^2\beta_z}\sin\varphi, \tag{4}$$

$$\frac{dp_y}{dt} = eE_m(1-\beta_z)\sin\varphi,\qquad \frac{dp_x}{dt} = 0, \tag{5}$$

where $\gamma = \varepsilon/mc^2$ is relativistic factor of the electron, $\vec{p} = m\vec{v}_e\gamma$ and $\vec{v}_e$ are the momentum and velocity of the electron. $\beta_y = v_{e,y}/c, \beta_z = v_{e,z}/c$. $v_{e,y}$ and $v_{e,z}$ are y and z components of the electron's velocity accordingly. From (3) follows that

$$\gamma(1-\beta_z) = \gamma_0(1-\beta_{z,0}) = const = b, \tag{6}$$

$$p_x = mv_{e,x}\gamma = mv_{e,x,0}\gamma_0 = const = f, \tag{7}$$

where $\gamma_0, \beta_{z,0}$ and $v_{e,x,0}$ are initial values of corresponding parameters. To simplify, in what follows we shall consider the case when $v_{e,x,0} = 0$. Then, according to (7) $v_{e,x} = 0$ and the electron's motion will occur in the plane (yoz).

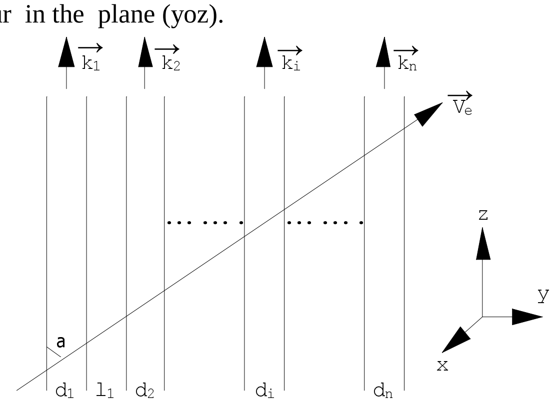

Fig.1: The principle scheme of acceleration of electrons in structure consisting from n light bands.

Firstly we can find the dependence of the γ - factor from the phase φ. Indeed it follows from (2) that

$$\frac{d\varphi}{dz} = \frac{k}{\beta_z}(1-\beta_z), \tag{8}$$

and using (4) and (8) we have

$$\frac{d\gamma}{d\varphi} = \frac{\xi\beta_y \sin\varphi}{1-\beta_z}, \tag{9}$$

where $\xi = eE_m/m\omega c$ is parameter of the laser intensity. Furthermore, using (6), (9) and the kinematical relation $\beta^2 = \beta_y^2 + \beta_z^2 = 1 - 1/\gamma^2$, we find:

$$b\int_{\gamma_0}^{\gamma} \frac{d\gamma}{\sqrt{2\gamma b - 1 - b^2}} = \xi \int_{\varphi_0}^{\varphi} \sin\varphi d\varphi \,, \qquad (10)$$

where φ_0 is the initial phase of the wave. After integration (10), we find the gain of the γ, correct for any parameter of the laser intensity ξ :

$$\gamma - \gamma_0 = \frac{\xi^2}{2b}(\cos\varphi_0 - \cos\varphi)^2 + \frac{\xi}{b}\sqrt{2\gamma_0 b - 1 - b^2}\,(\cos\varphi_0 - \cos\varphi). \qquad (11)$$

According to (6) the constant b can be written as:

$$b = \gamma - \sqrt{\gamma^2 - 1}\cos\alpha \,. \qquad (12)$$

Using (12), one can find from (11) the angle α_m when γ takes the maximal value:

$$\cos\alpha_m = \frac{4\gamma_0\sqrt{\gamma_0^2 - 1}}{4\gamma_0^2 + \xi^2(\cos\varphi - \cos\varphi_0)^2} \pm \frac{\xi(\cos\varphi - \cos\varphi_0)\sqrt{4 + \xi^2(\cos\varphi - \cos\varphi_0)}}{4\gamma_0^2 + \xi^2(\cos\varphi - \cos\varphi_0)^2} \qquad (13)$$

The dependence of γ from the angle α for some values of parameters γ_0 , φ , φ_0 , ξ is represented on Fig.2. From (13) it follows that in case where $\gamma >> 1$ and $\xi << 1$ we have: $\alpha_m \approx 1/\gamma_0$ and $b \approx 1/\gamma_0$.

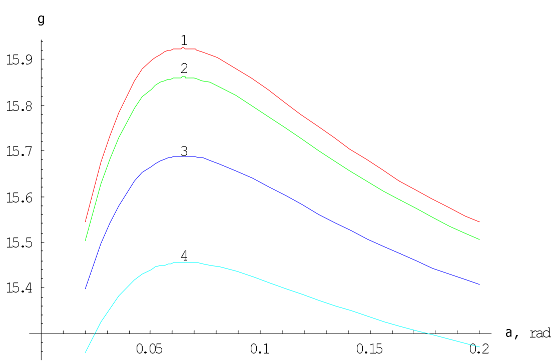

Fig. 2: The dependence of the γ - factor of electrons from angle α for: $\gamma_0 = 15$; $\xi = 3\cdot 10^{-2}$; $\varphi_0^{(1)} = 0$; $\varphi_0^{(2)} = \pi/6$; $\varphi_0^{(3)} = \pi/3$; $\varphi_0^{(4)} = -\pi/2$;. $\varphi = \pi$;

.Now, we can find the dependence of z from the phase φ . First, using (6) and (9) we have:

$$\int_{\varphi_0}^{\varphi} (\gamma - b)d\varphi = \frac{2\pi b}{\lambda}\int_0^z dz \,, \qquad (14)$$

where λ is the wavelength. After substituting the expression for γ from (11) into (14) and integration, we find:

$$z = \frac{\lambda}{2\pi b}\{[\gamma_0 + \frac{\xi}{b}\sqrt{2\gamma_0 b - 1 - b^2}\cos\varphi_0 + \frac{\xi^2}{4b}(1 + 2\cos{\varphi_0}^2) - b](\varphi - \varphi_0) - \frac{\xi}{b}\left(\xi\cos\varphi_0 + \sqrt{2\gamma_0 b - 1 - b^2}\right)(\sin\varphi - \sin\varphi_0) + \frac{\xi^2}{8b}(\sin 2\varphi - \sin 2\varphi_0)\}. \qquad (15)$$

The dependence of the γ-factor of the electrons from their z-coordinate can be found using relations (11) and (15). The dependence of the γ-factor from the z - coordinate for some parameters γ_0 , φ , φ_0 , ξ is represented on Fig.3.

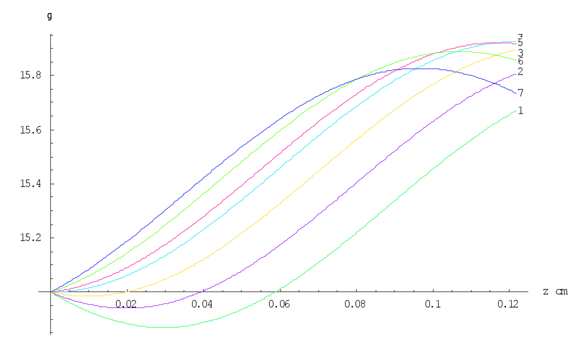

Fig.3: The dependence of the γ-factor of electrons from z at movement of electrons in the first light band, for: $\gamma_0 = 15$; $\xi = 3\cdot 10^{-2}$; $\alpha_0 = 0.067$, $\varphi_0^{(1)} = -\pi/4$; $\varphi_0^{(2)} = -\pi/6$; $\varphi_0^{(3)} = -\pi/12$; $\varphi_0^{(4)} = 0$; $\varphi_0^{(5)} = \pi/24$; $\varphi_0^{(6)} = 3\pi/24$; $\varphi_0^{(7)} = 5\pi/24$.

The bandwidth d of the waves can be found using (15) from requirements that during the movement of the electron in a zone of a light band, the electric field of the wave does not change its direction. The maximal value of z one finds from (15) for electrons when φ_0 =0 and $\varphi = \pi$. Then the thickness of the first light band is:

$$d_1 \le \frac{\lambda\gamma_0 tg\alpha_0}{2b}(1 + \frac{\xi}{b\gamma_0}\sqrt{2\gamma_0 b - 1 - b^2} + \frac{3\xi^2}{4b\gamma_0}). \qquad (16)$$

The electron beam will be accelerated when passing the system of light bands if on distance $l_v = z_v tg\alpha$ between the i –th and i+1-th bands the phase of a wave changes by π . Because the electron beam passes the distance l_v

in time $t_{v,i} = z_{v,i} / v_{e,i} \cos\alpha_i$, we find:

$$z_{v,i} = \frac{\lambda}{2}\frac{\gamma_i}{b_i}\beta_{z,i}\,. \qquad (17)$$

Integrating the left side of the relation (10) from γ_{i-1} to γ_i and the right side from φ_{i-1} to φ_i, we find the γ-factor of the electrons crossing of the i-th light band:

$$\gamma_i^{(j)} = \gamma_{i-1}^{(j)} + \frac{\xi^2}{2b}(\cos\varphi_{i-1}^{(j)} - \cos\varphi_i^{(j)})^2 + \frac{\xi}{b}\sqrt{2\gamma_{i-1}^{(j)}b - 1 - b^2}(\cos\varphi_{i-1}^{(j)} - \cos\varphi_i^{(j)}). \qquad (18)$$

Using (18) and integrating the left side of the expression (14) from z_{i-1} to z_i and the right side from φ_{i-1} to φ_i, we find the change of z- coordinates of electrons crossing the i-th light band:

$$z_i - z_{i-1} = \frac{\lambda}{2\pi b}\{[\gamma_{i-1} + \frac{\xi}{b}\sqrt{2\gamma_{i-1}b - 1 - b^2}\cos\varphi_{i-1} + \frac{\xi^2}{4b}(1 + 2\cos\varphi_i^{\,2}) - b](\varphi_i - \varphi_{i-1}) - \frac{\xi}{b}\left(\xi\cos\varphi_{i-1} + \sqrt{2\gamma_{i-1}b - 1 - b^2}\right)(\sin\varphi_i - \sin\varphi_{i-1}) + \frac{\xi^2}{8b}(\sin 2\varphi_i - \sin 2\varphi_{i-1})\}\,\cdot \qquad (19)$$

Using the maximal value for $(z_i - z_{i-1})_{\max}$ (at φ_{i-1}=0 and $\varphi_i = \pi$) we can find the width of i-th light band:

$$d_i = (z_i - z_{i-1})_{\max} tg\alpha_{i-1} \qquad (20)$$

From (19) and (20) it follows that the value of $z_i - z_{i-1}$ and d_i grows at the increase of quantity γ_{i-1}.

INFLUENCE OF DIFFRACTION OF LIGHT ON PROCESS OF ACCELERATION OF ELECTRONS

We assume that at the diffraction of light, the distribution of intensity on the directions is approximately described by the Fraunhofer diffraction, which is determined by the following expression:

$$I(\theta) = I_0\left[\frac{\sin\psi(\theta)}{\psi(\theta)}\right]^2, \qquad (21)$$

where I_0 is the intensity of light in a direction of the incident wave, $\psi = \pi d\sin\theta/\lambda$ and d is the width of the light beam. From (21) it follows that the main part of the intensity of radiation is concentrated in the interval of the angles $0 \le \theta \le \theta_m \approx \lambda/d$. Using (21) we can estimate the diffraction angle of the i-th light band. In particular, if $\xi << 1$ and $\alpha_{i-1,m} \approx 1/\gamma_{i-1} << 1$ we find $0 \le \theta_i \le \theta_m \approx 2/\gamma_{i-1}$. Thus, because of the light diffraction we have a spectrum of injection angles (the angle between vectors $\vec{k}(\theta)$ and ($\vec{v}_e$) in the given light band. For example, in the interval of $0 \le \theta_i \le \theta_m/2$ according to (22), increase of θ decreases $I(\theta) \sim \xi^2$ and, therefore, according to (19) decreases γ. In addition, as follows from Fig.3, the increase or reduction of angle α from the value of α_m, results in the decrease of growth γ. For the calculation of effective γ_{eff} that takes into account the diffraction of light, it is necessary to take the sum $\gamma(\xi_n)$, with contribution of actions from various $\xi_n(\theta)$. Practically, it leads to the replacement of ξ in formulas (11) and (18) by $\xi_{eff} < \xi$ and to some reduction of γ.

Thus, in spite of small value of angle α and losses of intensity of light owing to diffraction the rate of acceleration of electrons can be high, than at conventional methods due to existence of modern powerful lasers.

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