CONTROLLING EMITTANCE GROWTH IN AN FEL BEAM CONDITIONER

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Abstract

It has been proposed [1] to 'condition' an electron beam prior to the undulator of a Free-Electron Laser (FEL) by increasing each particle's energy in proportion to the square of its transverse betatron amplitude, or action. This conditioning enhances FEL gain by reducing the axial velocity spread within the electron bunch. In this paper we extend analysis proposed by A. Wolski for requirements to the conditioner which does not suffer from large emittance growth. We also present a possible *quadrupole-undulator* conditioner, with simulations showing that the emittance growth can be suppressed in a reasonable length system, but alignment tolerances are extremely tight.

INTRODUCTION

The most demanding requirement for future X-ray FELs [2, 3] is the generation of a sufficiently small transverse electron emittance. To mitigate this problem, ideas have been proposed to 'condition' an electron beam [1, 4].

Previously [5] we presented a system that allows conditioning of the beam on a relatively short length, however, it suffers from projected emittance growth to the extent that makes it impractical for application. It was conjectured in Ref. [5] that the emittance growth associated with conditioning is an inevitable consequence of Hamiltonian equations of motion. Later A. Wolski [6] pointed out examples of conditioners which do not suffer from emittance growth. Unfortunately, all such examples are characterized by a very small conditioning per unit length of the system.

In this paper we extend analysis proposed by A. Wolski and apply it to a simple FODO lattice. We show that weak conditioning in such a system is related to small slippage of large amplitude particles in the beam, of the order of (un-normalized) beam emittance. We also present a possible implementation of a beam conditioner consisting of a quadrupole undulator and discuss difficulties associated with implementation of such device.

CONDITIONING AND EMITTANCE

In an FEL undulator, particles with high energy travel a shorter path (increased axial velocity), while a large betatron amplitude lengthens its path. Conditioning establishes a correlation which cancels these two effects, resulting in a reduction of the axial velocity spread, enhancing the FEL gain. The relative energy conditioning requirement, for natural undulator focusing, can be written as [1]

$$\delta_u = \frac{1}{2}\frac{1}{\beta_u}\frac{\lambda_u}{\lambda_r}J_u\,, \tag{1}$$

where γ_u is the electron energy in the undulator (in units of rest mass), $\beta_u (= \beta_x = \beta_y)$ is the constant beta-function in the undulator, λ_u is the undulator period, λ_r is the FEL radiation wavelength, and $J_u = (J_{u_x} + J_{u_y})$ is the 4D action, or square of the betatron amplitude of the particle.

Let us consider a conditioner consisting of three parts. First, an RF cavity generates an energy chirp in the beam; then the beam passes through an optical system delaying particles with large transverse amplitude, introducing a slippage Δz proportional to the square of the amplitude; and finally the chirp is removed with another RF cavity (see Fig. 1). The level of conditioning in such a system is proportional to the energy chirp and the slippage Δz. The slippage of a particle is given by $\Delta z = \frac{1}{2}\int([x'(z)]^2 + [y'(z)]^2)dz$, where $x(z)$ and $y(z)$ are the horizontal and vertical coordinates, respectively. The angle $x'(z)$ is expressed in terms of the initial values x_0 and x'_0, and the transport matrix elements $R_{21}(z)$ and $R_{22}(z)$: $x'(z) = R_{21}(z)x_0 + R_{22}(z)x'_0$ yielding,

$$\Delta z = \frac{1}{2}\int (x')^2 dz = \frac{1}{2}x_0^2\int R_{21}^2 dz + x_0 x'_0\int R_{21}R_{22}dz + \frac{1}{2}(x'_0)^2\int R_{22}^2 dz\,, \tag{2}$$

with a similar expression for the vertical coordinate y (assuming uncoupled motion). For conditioning, we require Δz to be proportional to the sum of Courant-Snyder invariants, $\Delta z = A(J_x + J_y)$, where $J_x = (\gamma_0 x_0^2 + 2\alpha_0 x_0 x'_0 + \beta_0 (x'_0)^2)/2$, β_0, α_0 and γ_0 are the Twiss parameters in the horizontal plane at the entrance to the conditioner, with a similar expression holding for J_y.

The projected emittance growth generated in the conditioner is due to chromatic effects: unless specially designed, the optical system typically introduces energy dependance in β and α which causes a mismatch for particles of different energies. The resulting emittance growth can be evaluated using the $B_{\rm mag}$ beta-mismatch parameter [7], which gives the ratio of the final and initial emittance due to a mismatch in β and α functions:

$$B_{\rm mag} = \frac{1}{2}\left[\frac{\beta(E)}{\beta(E_0)} + \frac{\beta(E_0)}{\beta(E)} + \left(\alpha(E)\sqrt{\frac{\beta(E_0)}{\beta(E)}} - \alpha(E_0)\sqrt{\frac{\beta(E)}{\beta(E_0)}}\right)^2\right]\,, \tag{3}$$

where $\beta(E)$ and $\alpha(E)$ are the values at the exit of the conditioner. Assuming a small energy spread, and using a Taylor expansion of $\beta(E)$ and $\alpha(E)$, one finds for the emittance increase $\Delta\epsilon = \epsilon(B_{\rm mag}-1)$ the following expression:

$$\Delta\epsilon = \frac{\epsilon\sigma_E^2}{2}\left[\left(\frac{\partial\alpha}{\partial E}\right)^2 - 2\frac{\alpha}{\beta}\frac{\partial\beta}{\partial E}\frac{\partial\alpha}{\partial E} + \frac{\gamma}{\beta}\left(\frac{\partial\beta}{\partial E}\right)^2\right],$$

where σ_E^2 is the rms energy spread in the beam due to the chirp. The equation above shows that to avoid emittance growth in the lowest order, one has to design the system in a such a way, that $\partial\alpha_{x,y}/\partial E = 0$ and $\partial\beta_{x,y}/\partial E = 0$. Below we will show, for a simple case of a FODO lattice, that the requirement of small emittance growth in the system results in a relatively small conditioning per unit cell.

QUADRUPOLE UNDULATOR

As an example conditioner we take a compact FODO lattice of quadrupole magnets, as in Fig. 1, forming effectively a *quadrupole undulator*.

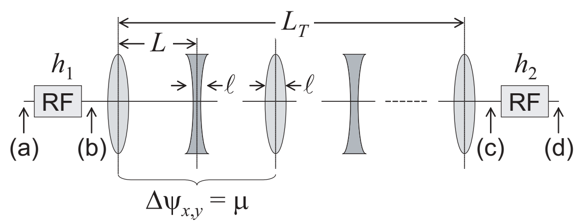

Figure 1: Quadrupole undulator FEL beam conditioner.

The exit values of the β function in the FODO lattice are equal to $\beta_{\rm max}$ and $\beta_{\rm min}$ in two orthogonal planes, correspondingly. For a FODO lattice,

$$\beta_{\rm max} = 2L\frac{1+\sin(\mu/2)}{\sin\mu}, \quad \beta_{\rm min} = 2L\frac{1-\sin(\mu/2)}{\sin\mu},$$

where L is the half-cell length and μ is the phase advance per cell. Note that in this formula μ, and hence $\beta_{\rm max}$ and $\beta_{\rm min}$, are functions of energy E. One can then derive:

$$\frac{1}{\beta_{\rm max}}\frac{\partial\beta_{\rm max}}{\partial E} = \tan\frac{\mu}{2}\frac{\cos\frac{\mu}{2} - 2\cot(\mu)(1+\sin\frac{\mu}{2})}{E(1+\sin\frac{\mu}{2})}, \quad (4)$$

from which it follows that $\partial\beta_{\rm max}/\partial E = 0$ at $\mu = 76.35°$. However, one can also show that $\partial\beta_{\rm min}/\partial E \neq 0$ for any value of μ. Note, that $\partial\beta_{\rm max}/\partial E$ and $\partial\beta_{\rm min}/\partial E$ do not grow with the number of cells in the system.

Using the well known transport matrix for a FODO lattice, one can calculate the slippage of a particle Δz:

$$\Delta z = 2N_c J_i \tan(\mu/2), \quad (5)$$

where $J_i = J_x + J_y$ and N_c is the number of cells in the system. Typically, if μ is not close to π, the slippage generated in one cell per one degree of freedom is of the order of the beam emittance, since $\langle J_x\rangle = \langle J_y\rangle = \epsilon$.

Equation (5) shows that Δz, and hence conditioning, increases when $\mu \to \pi$. This however causes an increase of the derivative $\partial\beta_{\rm max}/\partial E$ and results in large emittance growth. This situation is qualitatively similar to the one found in Ref. [5], where conditioning in a relatively short system generated an enormous emittance growth.

The FODO section requirements are calculated below. The inverse focal length of a quadrupole magnet is $1/f = G\ell e/p_i$, where G is the focusing magnetic field gradient, ℓ is the length of the quadrupole magnet, e is the electron charge, and p_i is the longitudinal beam momentum. Using thin-lens FODO cells we can apply the standard formula:

$$1/f = G\ell e/p_i = \frac{2}{L}\sin(\mu/2). \quad (6)$$

Taking the quadrupole length as $\ell = L/4$, the total length of N_c cells as $L_T = 2N_cL$, and the momentum as $p_i = mc\gamma_i$, the total length of quadrupole undulator is

$$L_T = 4N_c\sqrt{\frac{mc\gamma_i\sin(\mu/2)}{eG}}. \quad (7)$$

The bunch is energy chirped prior to entrance into the quadrupole undulator (point (a) in Fig.1) according to $h_1 \approx \sigma_{\delta_i}/\sigma_{z_i}$, where σ_{δ_i} is the rms relative energy spread induced by the first RF section and σ_{z_i} is the rms bunch length. This chirp changes each particle's energy to $\delta = h_1 z_i$ at point (b), where z_i is the longitudinal coordinate within the bunch. The slippage in the quadrupole undulator causes a path length delay, Δz in Eq. (5), which produces a new longitudinal position at point (c) of $z = z_i + 2N_cJ_i\tan(\mu/2)$, while the energy is not changed. Finally, at point (d), after the final reversed RF chirp section, the particle's energy becomes

$$\delta = h_1z_i + h_2[z_i + 2N_cJ_i\tan(\mu/2)] = 2N_chJ_i\tan(\mu/2), \quad (8)$$

where $h_2 = -h_1 \equiv h$. Equation (8) shows that the energy is now conditioned, varying with transverse action, J_i.

This conditioned energy spread shrinks when accelerating from γ_i up to the FEL energy, γ_u, but is also amplified when compressing the bunch from σ_{z_i} down to σ_{z_u}, producing the conditioned energy deviation in the FEL

$$\delta_u = 2N_ch\frac{\sigma_{z_i}}{\sigma_{z_u}}\frac{\gamma_i}{\gamma_u}J_i\tan(\mu/2). \quad (9)$$

Equating this to Eq. (1) to produce the proper level of conditioning for the FEL, and using $h = \sigma_{\delta_i}/\sigma_{z_i}$, we have

$$2N_c\sigma_{\delta_i}\frac{J_i}{\sigma_{z_u}}\frac{\gamma_i}{\gamma_u}\tan(\mu/2) = \frac{1}{2}\frac{1}{\beta_u}\frac{\lambda_u}{\lambda_r}J_u. \quad (10)$$

This can be solved for the number of FODO cells needed for conditioning, using the invariant action $\gamma_iJ_i = \gamma_uJ_u$.

$$N_c = \frac{1}{4\sigma_{\delta_i}\tan(\mu/2)}\frac{\sigma_{z_u}}{\beta_u}\frac{\lambda_u}{\lambda_r} \quad (11)$$

This is used in Eq. (7) to calculate the total length of quadrupole undulator needed.

$$L_T = \frac{1}{\sigma_{\delta_i}}\frac{\sigma_{z_u}}{\beta_u}\frac{\lambda_u}{\lambda_r}\frac{\cos(\mu/2)}{\sqrt{\sin(\mu/2)}}\sqrt{\frac{mc\gamma_i}{eG}}. \quad (12)$$

Table 1: FEL parameters for LCLS.

parameter	symbol	value	unit
und. energy/mc^2	$\gamma_u mc^2$	14	GeV
undulator period	λ_u	3	cm
rad. wavelength	λ_r	1.5	Å
und. $\beta_{x,y}$	β_u	30	m
und. bunch length	σ_{z_u}	20	μm

Table 2: Conditioner parameters for LCLS.

parameter	symbol	case-1	case-2	unit
energy	$\gamma_i mc^2$	100	100	MeV
phase adv/cell	μ	135	135	deg
quad gradient	G	600	235	T/m
beta max.	$\beta_{\max}$	220	351	mm
beta min.	$\beta_{\min}$	10.6	17.0	mm
energy spread	σ_{δ_i}	2.5	1.0	%
bunch length	σ_{z_i}	1.0	1.0	mm
N cells	N_c	552	1381	
total length	L_T	50	200	m

This length is reduced as $\mu \rightarrow \pi$, as the rms energy chirp, σ_{δ_i}, is increased, and weakly with higher quadrupole gradients, G, and lower conditioner energy, γ_i. The large energy spread in the strong focusing channel adds a significant chromatic emittance growth as $\mu \rightarrow \pi$, ultimately limiting the choice of μ to somewhat less than π.

Table 1 lists LCLS [2] FEL parameters and Table 2 lists parameters for its quadrupole undulator conditioner for two cases: (case-1) an aggressive, short system with strong quadrupole gradients, and (case-2) a less aggressive system with weaker gradients and less chirped energy spread.

TRACKING

Both quadrupole undulator conditioners (case-1 and case-2) of Table 2 are evaluated with particle tracking using *Elegant* [8] to 2nd-order. In Fig. 2 the relative energy deviation after the case-1 conditioner, at 100 MeV, is plotted against x position, and its distribution is also shown. The energy-action correlation is shown in Fig. 3 and agrees reasonably well with the expected slope indicated in Eq. (8). The spread around the proper conditioning slope is due to emittance growth through the conditioner. The less aggressive system (case-2) has less emittance growth ($\sim$ 1%), and produces a more highly correlated, linear conditioning.

When scaled to the FEL energy, as in Eq. (9), the rms conditioned energy spread at 14 GeV becomes 1.7×10^{-4}. The residual energy spread which is not correlated with the action is at an rms level of 6×10^{-5} (at 14 GeV in case-1) and should not significantly affect the FEL performance. The transverse emittance growth through the case-1 quadrupole undulator is $\sim$ 3%, but blows up quickly with larger energy spread or phase advance. Unfortunately, without room for steering correction in this compact quadrupole undulator, the transverse alignment tolerances are quite severe at $<$0.1 μm.

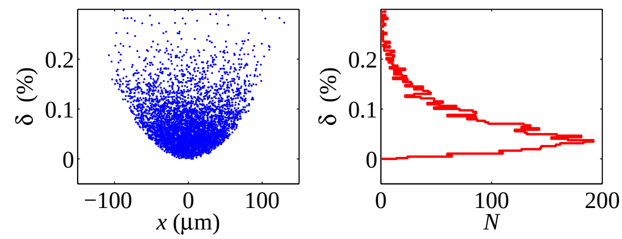

Figure 2: Tracking of quadrupole undulator FEL conditioner (case-1), plotting relative energy deviation at 100 MeV vs. x position (left), and energy distribution (right).

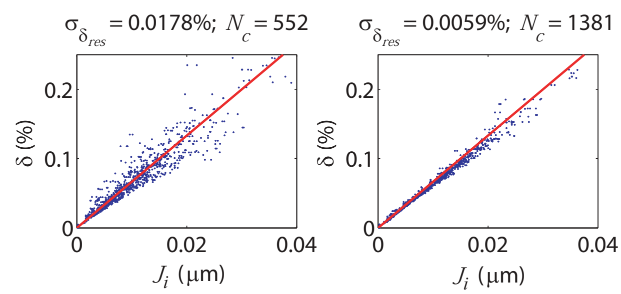

Figure 3: Tracking of case-1 (left) and case-2 (right), plotting relative energy deviation at 100 MeV vs. action, J_i. The red line is the expected slope in Eq. (8).

CONCLUSIONS

We have shown that FEL conditioning in one FODO cell is of the order of the emittance and have described a simple conditioner composed of FODO cells. Such a quadrupole undulator may only become practical if the extreme alignment tolerances can be attained, or a beam-based steering correction is included in this compact device.

We emphasize that our results are obtained for a beam matched to the conditioner. One can show the effect of conditioning is increased if the beam is mismatched, however this causes large projected emittance growth.

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