# CHOICE OF ARC DESIGN FOR THE ERL PROTOTYPE AT DARESBURY LABORATORY 

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## Abstract

The choice of arc design for the Energy Recovery Linac Prototype (ERLP) to be built at Daresbury Laboratory is investigated. Both the overall merits and disadvantages of a TBA arc and Bates bend are considered, and space restrictions particular to Daresbury Laboratory given. Some magnet parameters are given together with the layout chosen for ERLP.

## BEAM TRANSPORT REQUIREMENTS

The Energy Recovery Linac Prototype (ERLP), presently under construction at Daresbury Laboratory, is intended to allow flexible experiments in electron bunch transport [1]. It will study issues such as compression, synchronisation, energy recovery and coherent synchrotron radiation, which in conjunction with experiments planned elsewhere are needed to validate design choices for the proposed $4^{t h}$ Generation Light Source. The inclusion of an oscillator FEL [2] and the re-use of an existing building strongly influence the topology and design of the beam transport; details of the overall design and parameters are given elsewhere [1].


#### Abstract

ARC DESIGN Space requirements dictate the use of two compact 180degree arcs, with the oscillator FEL opposite a main singlepass linac [3]; two 4-dipole chicanes provide sufficient $R_{56}$ (we assume a sign convention where such chicanes have positive $R_{56}$ ) for full compression of the electron bunches, whilst providing a short cavity length to optimise the FEL gain. Since the chicanes provide positive $R_{56}$ either side of the FEL, the arcs must only supply zero or negative $R_{56}$ to give a full range of longitudinal manipulation. The chicanes supply a non-linear compression $T_{566}$ which scales with $R_{56}$ $$
\begin{equation*} \frac{T_{566}}{R_{56}}=-\frac{3}{2}-\frac{171}{40} \theta^{2}+\frac{99381}{5600} \theta^{4}+O\left(\theta^{6}\right) \tag{1} \end{equation*}
$$ where $\theta$ is the chicane bend angle (large in our case). This gives the well-known small-angle approximation $T_{566} / R_{56}=-3 / 2$. Since this cannot be varied in the chicanes themselves, any nonlinear variation must be performed in the arcs, and must be independent of the variations of $R_{56}$.


## TBA ARC

Guignard has shown [4] that a triple-bend achromat (TBA) contains the minimum number of dipoles necessary for isochronous transport - this is done by driving the dispersion to a negative value in the central dipole using quadrupoles; tuning of $R_{56}$ and $T_{566}$ is made by adjusting respectively quadrupoles and sextupoles. Given a maximum quadrupole strength, there are restrictions on what drift lengths are possible in the TBA cell to provide isochronous transport. The low beam energy of 35 MeV and compact building demand a single TBA cell for each arc with dipole field $\langle B\rangle=E \pi / 3 c e l_{m}$ (see Table 1 below). We assume that outer and central dipoles have the same angle $\phi$, bend radius $\rho$ and length $l_{m}$, so $R_{56}=$ $\int_{s_{1}}^{s_{2}} D(s) / \rho(s) \mathrm{d} s$. Guignard has derived conditions on the dispersion at the entrance to the centre magnet to provide an achromat with a particular $R_{56}$ value:

$$
\begin{gather*}
D_{j}=\rho\left[D_{j}^{\prime} \cot (\phi / 2)+1\right]  \tag{2}\\
D_{j}^{\prime}=-\frac{1}{\rho}\left[\frac{R_{56}}{2}-l_{m}\left(\frac{3}{2}-\frac{\sin \phi}{\phi}\right)\right] \tag{3}
\end{gather*}
$$

Note that $D_{j}^{\prime}$ is always negative if $R_{56}=0$.

## Conditions on the Cell Parameters

The simplest and most compact quadrupole configuration to set $D_{j}^{\prime}$ is a doublet [4], giving the cell layout shown in Figure 1. For isochronicity, the drift lengths $L_{1}, L_{2}, L_{3}$, and quadrupole strengths $k_{1}, k_{2}$ are set to satisfy equations 2 and 3 . It can be shown [4] that $L_{1}$ and $L_{2}$ satisfy:

$$
\begin{align*}
L_{1} & =a \frac{C_{2} q_{1}}{C_{1} q_{2}}\left(L_{3}-\frac{D_{j}}{D_{j}^{\prime}}+q_{2}\right)-l+q_{1}  \tag{4}\\
L_{2} & =q_{1}-q_{2}+\frac{b}{L_{3}-D_{j} / D_{j}^{\prime}+q_{2}} \tag{5}
\end{align*}
$$

where

$$
\begin{gather*}
l=\rho \tan (\phi / 2), \quad a=-D_{j}^{\prime} / \sin (\phi),  \tag{6}\\
b=\frac{q_{2}}{C_{2}}\left(\frac{q_{2}}{C_{2}}+\frac{q_{1}}{a C_{1}}\right), \quad q_{i}=\frac{C_{i}}{S_{i} \sqrt{k_{i}}}(i=1 \text { or } 2),  \tag{7}\\
C_{1}=\cos \left(l_{q} \sqrt{k_{1}}\right), \quad S_{1}=\sin \left(l_{q} \sqrt{k_{1}}\right),  \tag{8}\\
C_{2}=\cosh \left(l_{q} / \sqrt{k_{2}}\right), \quad S_{2}=\sinh \left(l_{q} \sqrt{k_{2}},\right) \tag{9}
\end{gather*}
$$

where $l_{q}$ is the quadrupole length, and $L_{3}$ is a free parameter.


Figure 1: General layout of an isochronous TBA cell.
Analytical formulae are available for physical solutions (i.e. $L_{1}, L_{2}>0$ ) [4, 5], but are complex and do not give optima on other criteria, e.g. on restricting maximum Twiss $\beta$-functions. We prefer to scan a reasonable range of $k_{1}, k_{2}$, $L_{3}$ values, and then filter non-physical and large drifts, for example, $0 \mathrm{~m}<L_{1}, L_{2}<2 \mathrm{~m}$. For each solution, an optically stable cell (not strictly necessary but convenient) is constructed by scanning $k_{3}$ and $k_{4}$. The optimal solution is then chosen by weighting $k_{i}, L_{i}$, and Twiss criteria - the advantage of this procedure is that with sufficiently fine scan steps the optimum solution will definitely be found. This procedure can be extended for arcs consisting of multiple cells.

## Properties of Solution

For ERLP we examined three dipole lengths (shown in Table 1), using a maximum quadrupole gradient of $6 m^{-2}$. The benefit of shortening the dipoles is offset by needing a longer drift length to correct the larger induced dispersion (for a given quadrupole strength); there is therefore only a weak dependence of overall cell length on dipole length. By increasing the quadrupole gradient one can of course arbitrarily reduce the cell length. The 0.245 T solution was chosen from these alternatives as it was short, and convenient for field and engineering reasons.

Table 1: Properties of the TBA solutions examined. It should be noted that the cell lengths arrived at are not the optimum possible, only the shortest ones found during the design procedure.

| Field (at 35 MeV) [T] | Length [m] | Cell Length [m] |
| :--- | :--- | :--- |
| 0.140 | 0.87 | 12.65 |
| 0.245 | 0.5 | 9.92 |
| 0.350 | 0.35 | 11.05 |

## Optics and Tuning of the TBA Arc

Twiss values for the 0.245 T solution are give in Figure 2. Figure 3 shows that the outer quadrupole $k_{1}$ is the more effective at tuning $R_{56}$, and it is straightforward to tune the cell to large negative values of $R_{56}$ of -0.6 m or less (corresponding to positive dispersion in the central dipole). However, driving the dispersion to a significantly negative
value is difficult, and only slightly positive $R_{56}$ of a few cm are possible. The large associated $T_{566}$ change must be compensated with more sextupole strength than is required in a Bates solution of similar size (see below), but the required values are still modest (around $100 \mathrm{~m}^{-3}$, or $12 \mathrm{Tm}^{-2}$ at 35 MeV ).


Figure 2: Optical properties of the 0.245 T solution.


Figure 3: Variation of $R_{56}$ (upper lines) and $T_{566}$ (lower lines) as a function of changing individual quad families $k_{1}$ (solid black, right) and $k_{2}$ (dashed red, left). Tuning either side of the values required for $R_{56}=0$ is shown; not all these values correspond to stable cell conditions.

## BATES ARC

A Bates arc consists of 5 dipole magnets, the central one bending 180 degrees to give $R_{56}=-L=-R \pi$ (where $R$ is the bending radius). 'Half chicanes'are placed either side which cancel out the $R_{56}$ by arranging the drift space between each dipole pair to satisfy

$$
\begin{equation*}
L_{1} \simeq \frac{\pi R}{6 \theta^{2}} \tag{10}
\end{equation*}
$$

assuming that the half-chicanes have the same bend radius as the main dipoles (see Figure 4). If the spacing $L_{2}$ between the half-chicane and the $\pi$ bend is small, a 45-degree chicane bend angle gives $L_{1} \simeq 0.8 R$. To some extent this condition restricts the width/length ratio of a Bates arc, independent of the bend radius. The addition of quadrupoles and sextupoles allows tuning of $R_{56}$ and $T_{566}$, and allow some geometrical adjustment.


Figure 4: Layout of Bates design using 1 m radius dipoles. The drift lengths $L_{1}$ and $L_{2}$ are shown. Quadrupoles and sextupoles are placed in each drift section. This layout is very similar to that used in the 43 MeV JLab IR-Demo.

## 1m Design Tunability

A 1 m bend radius design (corresponding to a dipole field of 0.22 T at 35 MeV ) is shown in Figure 4. Figure 5 shows the variation of over 1 m in $R_{56}$ for modest variations of quadrupole strength, and the concomitant variation of $T_{566}$. Independent adjustment of $T_{566}$ is performed using either family of sextupoles (see Figure 6), where a sufficient range is possible with around half the strength required in a similar-sized TBA.


Figure 5: Variation of $R_{56}$ (upper lines) and $T_{566}$ (lower lines) as a function of quadrupole strengths $k_{1}$ (solid black) and $k_{2}$ (dashed red). Quadrupole $k_{1}$ lies in $L_{1}$ etc.


Figure 6: Variation of $T_{566}$ as a function of quadrupole strengths $s_{1}$ (solid black) and $s_{2}$ (dashed red).

## COMPARISON OF THE ARC OPTIONS

With comparable dipole field strength the Bates arc is similar in size to the TBA option, although the TBA is wider and shorter - an advantage for the ERLP layout as length is limited [3]. Tunability of both $R_{56}$ and $T_{566}$ requires modest element strengths from either option, although the Bates is more linear and can readily be operated at positive $R_{56}$. Since ERLP only requires negative $R_{56}$ from the arcs, and the aperture requirement is dispersiondominated due to the large energy spread in the second arc, the TBA can readily provide the required aperture. The effect of non-linear dispersion appears to be similar in both designs.

As well as providing $R_{56}$ and $T_{566}$ adjustment the ERLP return arc must also allow path length adjustment ( $R_{55}$ ) of more than $1 \lambda(\lambda=23 \mathrm{~cm}$ at the 1.3 GHz RF frequency). In both designs this can be achieved mechanically by moving the arcs, which is straightforward for an arc of this size. In the Bates design path length may also be adjusted within the $\pi$-bend using symmetrically-powered correctors to produce a kick $\theta$ which gives

$$
\begin{equation*}
\delta l=-\frac{2 L_{\pi} x^{\prime}}{\pi} \tag{11}
\end{equation*}
$$

where $L_{\pi}$ is the length of the $\pi$-bend; this requires a larger aperture in the central dipole. In principle trajectory adjustment can be performed in the TBA dipoles with 4 correctors per dipole, but the complexity is too great to be of practical use.

Both types of arcs can meet ERLP requirements. Although the Bates design requires lower quadrupole/sextupole strengths, the single dipole design and layout of the TBA design is an advantage for ERLP and has therefore been chosen [3].

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