

# MAGNET SORTING ALGORITHM APPLIED TO THE LNLS EPU

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## Abstract

A sorting algorithm for magnet blocks of the LNLS EPU was implemented and used for a 10-period prototype. The algorithm takes into account the magnetization vectors of the blocks measured in a Helmholtz coils system and their interactions. Field quality was optimized to within specified tolerances for all undulator phases. This paper describes the optimization parameters used in the sorting code and the results obtained with it.

## INTRODUCTION

The Brazilian Synchrotron Light Laboratory (LNLS) is about to start a new phase in its history with the addition of the first insertion devices (IDs) to its 1.37 GeV, 93m-long electron storage ring. There are four non-dispersive long straight sections with 3m-long clear regions for IDs.

The first ID to be installed is a 2T wiggler that has been build and delivered by STI Optronics. Its installation is scheduled for the next machine shutdown in October of 2004. The second ID is going to be an Elliptically Polarizing Undulator (EPU) based on pure permanent magnet technology and of the APPLE-II type[1]. This EPU will be constructed at LNLS and it will have a period length of 50mm, minimum gap of 22mm and will cover the energy range of  $\sim 250$ -900 eV with the first, third and fifth harmonics. Its construction will start sometime in mid 2005. But before the full 56-period EPU is built, LNLS will construct a 10-period prototype. Work on it started in the beginning of 2004. In this paper, the algorithm used for the sorting of the magnet blocks of the prototype is explained and detailed. This sorting is very important in order to minimize the effects that magnetization errors of the magnets have on beam parameters and on radiation quality.

## MAGNET BLOCKS

The material NdFeB has been chosen for the magnet blocks because of its high remanent field and coercivity, and because of its thermal stability. The blocks for the prototype have been supplied by Vaccumschmelze and their magnetic properties have been characterized in a Helmholtz coils system. The data is summarized in Table 1.

There are four distinct types of magnet blocks: types C1 and C2 are of blocks used at the central part of the undulator structure and their magnetization vectors are aligned along the vertical and longitudinal directions, respectively. Block types T1 and T2 are used at the termination structure of the cassettes.

All blocks have square transverse sections with sides of 40mm. For fixation purposes, they also have 6x6mm<sup>2</sup> cutouts on two opposite edges. This choice provides a higher symmetry for the sorting problem and leads to a larger number of available configurations of blocks. Therefore solutions of the sorting problem, for a given set of tolerances on field quality, can be found faster.

	Nr	L[mm]	$\mathbf{m}_x$ (mT)	$\mathbf{m}_y$ (mT)	$\mathbf{m}_z$ (mT)
C1	118	12.3	$-12 \pm 4$	$-2 \pm 8$	$1248 \pm 4$
C2	108	12.3	$-1 \pm 4$	$1246 \pm 6$	$-1 \pm 3$
T1	61	3.13	$-2 \pm 2$	$-1 \pm 8$	$1187 \pm 9$
T2	30	3.13	$8 \pm 4$	$1250 \pm 5$	$1 \pm 8$

Table 1: Magnetization of blocks. Averages and r.m.s. values of the magnetization vectors are listed, as well as the total number of blocks of each type and their thickness.

## OPTIMIZATION PARAMETERS

Ideally an ID should have as little effect as possible on the beam dynamics. An infinitely wide EPU with perfect mechanical and magnet structures, appropriate terminations and non-interacting magnet blocks is the closest one can get to an ideal undulator. Even in this case there are unavoidable tune shifts. When construction errors of the mechanical and magnetic structures of the EPU are considered, the quality of the field can deteriorate substantially.

It turns out that for a given set of magnet blocks with magnetic imperfections, the specific ordering in which these blocks are used in the EPU structure influences both beam dynamics and radiation quality. The purpose of the sorting algorithm is to find a configuration of the magnets that minimizes the effects of the blocks' imperfections. For this, the most relevant parameters to optimize are the *first* and *second field integrals* and the *phase error*.

### Field Integrals

To first order in the inverse of the beam energy  $E$ , the final slope and steering experienced by the beam after it traverses an ID are proportional to the first and second field integrals, respectively[2]:  $(x', z', \Delta x, \Delta z) = (ec/E)(I_1^z, -I_1^x, I_2^z, -I_2^x)$ , with  $I_1^{x,z}(x_0, z_0) \equiv \int_{-\infty}^{\infty} dy B_{x,z}(x_0, y, z_0)$  and  $I_2^{x,z}(x_0, z_0) \equiv \int_{L_{inf}}^{L_{sup}} dy \int_{L_{inf}}^y dy' B_{x,z}(x_0, y', z_0)$ . For ideal devices these quantities vanish for any initial beam transverse position  $(x_0, z_0)$ . But for real undulators, due to magnetic imperfections, they have undesired residual values. Within the model of uniformly magnetized blocks assumed in this work, the magnetic field of a magnet block can be expressed as the product of a matrix and the block's magne-

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tization vector[3]. This matrix contains information on the position and geometry of the block. Field integrals can as well be expressed the same way. Furthermore, this matrix and its first and second integrals can be calculated analytically, even for finite integration limits. This nice property simplifies considerably field calculations and additionally renders a more efficient sorting algorithm.

### Phase Error

The two optimization parameters described previously are concerned with the impact of the blocks imperfections on the beam parameters. But if the field experienced by beam particles has low quality, the radiation produced by the undulator will diverge from specification. In particular, it is very important that as particles traverse the field region of the undulator, they radiate in the correct phase so that radiation coming from the same particle at different orbit positions interferes constructively at the harmonic energies. Field deviations from pole to pole of the EPU imply different trajectory arc-lengths for the particles and consequently to phase errors with which the photons are emitted. There may be significant suppression of the harmonic peaks due to this effect. It has been shown[4] that there is a strong correlation between the simple parameter

$$\sigma_\phi^2 = \frac{4\pi^2}{M} \sum_{p=1}^M \left( \frac{l_p - \bar{l}}{\lambda_{ph}} \right)^2 \quad (1)$$

and the peak intensities of the radiation harmonics. In this expression  $l_p$  is the trajectory arc-length in the pole  $p$ ,  $\bar{l}$  is its average over all  $M$  poles and  $\lambda_{ph}$  the wavelength of the radiation harmonic. If the phase error  $\sigma_\phi$  vanishes, the harmonic peaks are intense as for the ideal undulator. The trajectory length  $l_p$  at each pole can be evaluated in terms of the first field integrals:

$$l_p = \int_{pole} dy \sqrt{1 + (ec/E)^2 [I_1^x(y)^2 + I_1^z(y)^2]} \quad (2)$$

Since most particles of the beam are distributed very closely to the reference axis it suffices to minimize the phase error only at the central axis.

## MAGNETIC INTERACTIONS

One major challenge with the construction of EPUs based on pure permanent magnets is to guarantee that optimized parameters remain so as the polarization of the field, or undulator phase, is varied. Different states of polarization are accomplished by shifting Halbach cassettes longitudinally with respect to each other and if the sorting of the magnets is optimized for a specific phase, most likely it will not be optimum for other phases. This comes from the fact that the magnet blocks' material has non-unit relative permeability and therefore these blocks interact and respond to each other's fields, changing their effective magnetization vectors.

In the literature this interaction effect is not considered at all and the sorting of the magnets characterized with the Helmholtz coil system is done with the assumption of unit permeability. But even with the relative permeability of NdFeB differing from one only by a few percentages, the optimized parameters may change beyond tolerances when the interaction is turned on. For example, simulation with RADIA[5] of an APPLE-II EPU with 56 periods shows a residual first integral of  $B_x$  that rises from zero to 30 G.cm when the interaction of the blocks is considered. As a matter of fact, if a field component is symmetric with respect to the center of the device, as it is for  $B_x$  in the LNLS case, its integrals will then be affected by interaction. Since with APPLE-II EPUs field components have opposite symmetries, the interaction of the blocks should be considered in the sorting algorithm.

For typical external fields  $\mathbf{H}$  experienced by the blocks in the undulator structure, NdFeB can be modeled as a linear, anisotropic material with an easy axis direction:  $\mathbf{m} = \mathbf{m}_0 + \chi \cdot \mathbf{H}$ , where  $\chi$  is a diagonal matrix with two independent parameters describing the susceptibility constants along and perpendicular to the easy axis. The values for these parameters, provided by the vendor of the magnet blocks and used in the sorting code, are 0.06 and 0.17, respectively. The field  $\mathbf{H}$  is generated by all the magnet blocks and calculated at the center of the block for which the equation is being considered. This equation can be applied to each block and inverted to give effective magnetization  $\mathbf{M} = (\mathbf{m}_1, \mathbf{m}_2, \dots)$  vectors in terms of the nominal vectors  $\mathbf{M}_0 = (\mathbf{m}_1^0, \mathbf{m}_2^0, \dots)$  measured in the Helmholtz coils system:  $\mathbf{M} = \mathbf{I}^{-1} \cdot \mathbf{M}_0$ , with

$$I_{\alpha\beta}^{ij} = \delta_{ij}\delta_{\alpha\beta} - (1 - \delta_{ij})\chi_\alpha^i Q_{\alpha\beta}^{ij}, \quad (3)$$

where the matrix element  $Q_{\alpha\beta}^{ij}$  gives the  $\alpha$ -component contribution to the field at the center of the block  $i$  due to the  $\beta$ -component of the magnetization vector of block  $j$ . The inverse of the interaction matrix  $\mathbf{I}$  is constructed and stored for each undulator gap and phase considered in the sorting process.

## SORTING CODE

The implementation of the sorting algorithm was done in C++. The code was written in such a way that it is as general as possible and hence accepts input parameters that describe a large number of possible APPLE-II geometries.

The algorithm, in general terms, is straightforward: given a configuration of magnet blocks, the optimization parameters are calculated as explained in previous sections. A new configuration is generated with an interchange of a randomly chosen pair of blocks and optimization parameters are calculated again. New configurations are always kept if they are better than previous ones. But if they result in worse parameters, they are kept with a small probability (0.5% was used). This simple strategy avoids stagnation of the algorithm at a local minimum of the configuration space.

Typically there are many optimization parameters and the best configuration of magnets for one parameter will not be optimum for others. A compromise has to be achieved. Usually a single objective function that weights all these parameters is devised and minimized. The choice of weights in this function is somewhat arbitrary. In our implementation of the sorting algorithm we adopted more restrictive *criteria*. The code first minimizes the first field integrals to acceptable values, then it switches to optimizing the second integrals by generating configurations of magnets and accepting only those that still keep the first integrals below specifications. At last, the code minimizes the phase error while keeping the field integrals within acceptable values.

## RESULTS

The tolerance used for the first integral was 15 G.cm. Integration of both  $B_x$  and  $B_z$  on 21 different longitudinal lines on the midplane was considered. These lines are evenly spaced in the interval  $\pm 10$ mm around the reference axis. After few thousand iterations of the sorting algorithm, the code was able to reduce the integrals for the EPU prototype from a typical value of 200 G.cm to values below the tolerance, on all lines. For each new configuration tried, the optimization parameters were checked for seven different EPU phases, ranging from horizontal to vertical polarization, going through circular polarization and intermediate phases as well.

As for the second integral, a tolerance value of 10 T.cm<sup>2</sup> was used for integrations over the length interval  $\pm 4.13$ m around the center of the undulator structure. This tolerance corresponds to an acceptable beam steering of approximately 0.2mm at the ring energy. This value is rather easy to obtain with the sorting algorithm. In fact, because of the strong correlation between first and second integrals, by the time the code had reached the tolerance of the first integral, the configuration of blocks generated also satisfied the tolerance of the second field integrals. This was achieved for the same set of 21 integration lines and seven EPU phases used in the first integral case.

Random configurations of blocks lead to typical phase errors of 4.4 degrees for the set of magnets available (Table 1). This implies, according to reference[4], a reduction of  $\sim 14\%$  in the fifth harmonic peak. The tolerance for the phase error accepted was 1 degree, corresponding to a degradation of less than 1% of the harmonic peaks. It was achieved after approximately 100 thousand iterations (Fig.1).

## CONCLUSIONS

Tolerances for the three optimization parameters considered were easily obtained for the prototype of the LNLS EPU with the magnet blocks from Table 1. The sorting code took only a few minutes of computer time to accomplish this. The number of operations needed to achieve

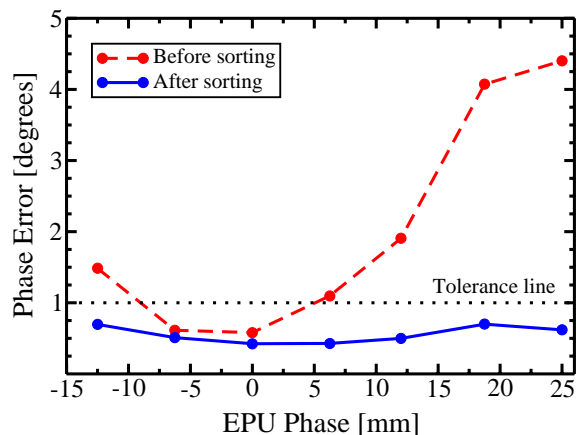


Figure 1: Phase errors for various polarization states before and after sorting.

specified tolerances is proportional to two factors: the first is the number of operations involved in the computation of the optimization parameters for each trial configuration of magnets. This factor is linearly proportional to the number of magnet blocks. The second factor comes from the size of the optimization parameters for typical random configurations. This factor is stochastic in nature and it scales as  $\sqrt{N}$  with the number of blocks  $N$ , just like the r.m.s. deviation of a 1D random walk. The number of operations for the sorting algorithm is therefore proportional to  $N^{3/2}$  and the code is expected to take a few hours to optimize the ordering of the blocks for the full 56-period undulator.

Despite the fact that interaction between magnet blocks change the optimization parameters when the EPU is set at different phases, a final configuration of the blocks were attained in which the field tolerances are satisfied in any phase and everywhere in the relevant spatial region around the axis of the device.

Depending on how small the magnetic gap of the undulator is, magnetization inhomogeneities of the blocks may have a non-negligible effect on optimization parameters. With the Helmholtz coils characterization of the blocks, their inhomogeneities are averaged in the magnetization vectors and hence they are not taken into account in the sorting algorithm. Currently work is in progress in order to overcome this problem.

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