AN IMPROVED IMPEDANCE MODEL OF METALLIC COATINGS*

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Abstract

A metal coating is required on the inner surface of the ceramic injection kicker vacuum chambers of the Spallation Neutron Source (SNS) for two reasons. First, a coating shields the ceramic surface from the beam to reduce the secondary emission (TiN is the candidate for the coating because of the low secondary emission coefficient). Secondly, a coating is required to suppress penetration of the beam fields into the kicker at dangerous beam frequencies (about 1 MHz). The choice for the metal thickness is determined by the resulting impedance and eddy-current limitations. Here we present an improved model for the coating impedance, which shows significant deviation from the handbook expressions at low frequencies.

1 INFINETELY LONG PIPE

In earlier papers (see [1] and references therein) the coating impedance was estimated in the simple model of an infinitely long pipe. Figure 1 shows the layers of this pipe. A thin metallic layer with conductivity σ and thickness d_m is on the inner surface of a ceramic layer, ceramic layer (yellow in the figure) has permittivity equal to ε and thickness d_c. The next layer is either air or air and a superconducting surface. These two cases give the same result.

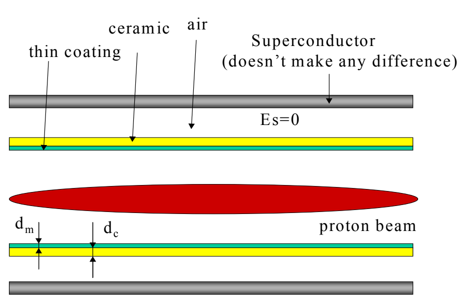

Figure 1 Infinitely long pipe used before for the coating impedance calculations

Here we present longitudinal impedance from formula (8) in [1], p. 203:

$$\frac{Z_{\parallel}}{L} = Z_{met}\frac{A+\tanh(\kappa d_m)}{1+A\tanh(\kappa d_m)}, \qquad (1)$$

where $Z_{met} = \frac{1-i\,\mathrm{sgn}(\omega)}{bc}\sqrt{\frac{|\omega|}{2\pi\sigma}}$,

$A = \frac{(1-i\,\mathrm{sgn}(\omega))d_c}{d_s}(1-\frac{1}{\varepsilon})$, L is the piece length.

2 FINITE LENGTH CASE - CAVITY, SHIELDED FROM THE BEAM BY A COATING LAYER

In this section we show that at least for low frequencies (wavelength much larger than the dimensions of the kicker) formula (1) should be significantly modified. Figure 2 shows the configuration and the notations. All the parts of the assembly are considered to be azimuthally symmetric for simplicity (though in practice the kicker yoke is not) in order to get a first approximation for the impedance.

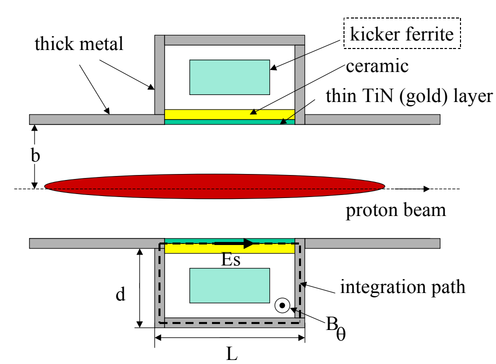

Figure 2. Simplified injection kicker model with coating.

Let's start with the layer-by-layer solutions for the fields. The vacuum chamber field and the coating layer has the same form of the solutions:

$$\begin{aligned} &I.B_\theta = a_1 r + \frac{b_1}{r}, && \text{vacuum chamber} \\ &II.B_\theta = a_2 e^{\kappa\Delta r} + b_2 e^{-\kappa\Delta r}, && \text{metal} \\ &I.E_s = -2a_1/ik, && \text{vacuum chamber} \\ &II.E_s = \frac{c\kappa}{4\pi\sigma}(a_2 e^{\kappa\Delta r} - b_2 e^{-\kappa\Delta r}), && \text{metal} \end{aligned} \quad . \qquad (2)$$

Inside the cavity at the coating-ceramic boundary electric to magnetic field ratio can be estimated from low frequency approximation through the Faraday's law:

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$$\int_{gap} E_s ds \approx E_s L = -\frac{1}{c}\dot{\Phi} = \frac{i\mu\omega B_\theta L d}{c}, \qquad (3)$$

where μ is the average permeability of the cavity.

Using the relation $\frac{Z_\parallel}{L} = -\frac{Z_0}{2\pi b}\frac{E_s}{B_\theta}$ for the impedance determination, one can get for the impedance:

$$Z_\parallel = LZ_{met}\frac{\frac{Z_{cavity}}{Z_{met}} + \tanh(\kappa d_m)}{1 + \frac{Z_{cavity}}{Z_{met}}\tanh(\kappa d_m)}, \qquad (4)$$

where $Z_{met} = \frac{1 - i\,\mathrm{sgn}(\omega)}{2\pi b d_s \sigma}$, $Z_{cavity} \approx -i\omega Z_0 \frac{\mu d}{2\pi c b}$ are impedances per unit length of the metal and the cavity.

If we have two layers, a similar analysis gives the following impedance:

$$Z_\parallel = LZ_{met1}\frac{\frac{Z_{in}}{Z_{met1}} + \tanh(\frac{(1-i\,\mathrm{sgn}(\omega))d_{m1}}{d_{s1}})}{1 + \frac{Z_{in}}{Z_{met1}}\tanh(\frac{(1-i\,\mathrm{sgn}(\omega))d_{m1}}{d_{s1}})},$$
$$Z_{in} = Z_{met2}\frac{\frac{Z_{cavity}}{Z_{met2}} + \tanh(\frac{(1-i\,\mathrm{sgn}(\omega))d_{m2}}{d_{s2}})}{1 + \frac{Z_{cavity}}{Z_{met2}}\tanh(\frac{(1-i\,\mathrm{sgn}(\omega))d_{m2}}{d_{s2}})} \qquad , (5)$$

where subscripts 1,2 stand for the first (nearest to the beam) and the second coating layer, respectively. When both skin depths are large compared to the layer thickness and the cavity impedance is much larger than the resistance of the layers, formula (5) gives the simple result:

$$Z_\parallel = L\frac{Z_1 Z_2}{Z_1 + Z_2}, \qquad (6)$$

where $Z_{1,2} = \frac{1}{2\pi b d_{m1,2}\sigma}$ are just DC resistances of the layers per unit length. It means that the layers impedance is equivalent to that of a parallel circuit of the two resistances.

Figure 3 shows the results of these calculations compared with the Piwinski result. Shown is the longitudinal impedance of a TiN layer (resistivity 45 μΩ cm, thickness 18 μm) on the inner surface of the SNS ceramic chambers (relative permittivity 10, inner radius 8 cm, thickness 1.25 cm and total length of 5m). Three cases are shown. The first shows the Piwinski (infinite geometry) result. The second shows the realistic case displayed in figure 2 for a 0.1m cavity height and relative permeability of 1000. The third curve shows the case in which the external "cavity" is formed by a perfect conductor on the outer surface of the ceramic chamber. We see that the Piwinski result and the case of the perfect external conductor show good agreement over the frequency range considered. The more realistic case of the kicker geometry gives an impedance which equals the DC resistance of the coating over the entire range of frequencies considered. This can be easily understood by realizing that the resulting impedance is the parallel combination of the coating resistance and the external circuit, which is quite large for all but the very smallest frequencies. Therefore, the image current flows through the metallic coating and the resulting impedance is simply the DC resistance of the coating.

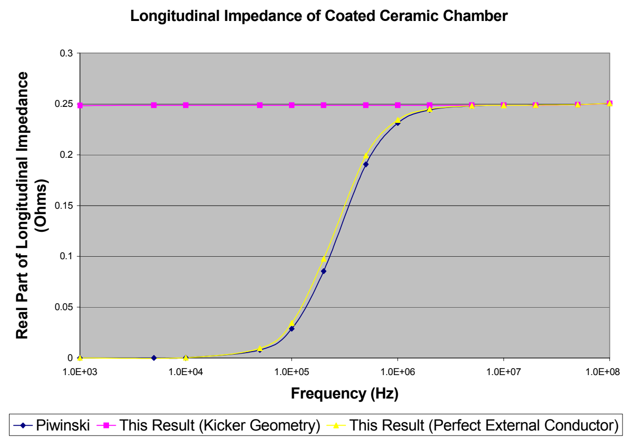

Figure 3: Real part of the longitudinal impedance of a ceramic chamber with thin metallic coating. Three cases are shown: i) the Piwinski result for an infinitely long chamber, ii) this result with a kicker model as shown in figure 2, and iii) this result with a perfect conductor on the outside of the ceramic chamber

3 TRANSVERSE IMPEDANCE

The transverse resistive impedance can be found using the quasi-static approximation, where responses to the beam electric and magnetic dipoles are calculated separately [3,4]. For the magnetic dipole, only longitudinal component of the vector potential is non-zero; when its dependence on the longitudinal coordinate is neglected, it automatically satisfies the Coulomb gauge. The contribution of the electric dipole is trivial, since we assume that the transverse electric field is shielded at the coating radius. The magnetic dipole produces the vector potential $A_z \propto \cos\theta \exp(-i\omega t)$:

$$I.\, A_z = c_1 r + \frac{2I_0 x_0}{cr}, \quad \text{vacuum}$$
$$II.\, A_z = c_2 e^{\kappa(r-a)} + c_3 e^{-\kappa(r-a)}, \quad \text{metal} \qquad (8)$$

where x_0 is the displacement of a point current I_0 from the axis. Using again the quasi-stationary relation between E_s and B_θ :

$$\int_{gap} E_s ds \approx E_s L = -\frac{1}{c}\dot{\Phi} = \frac{i\mu\omega B_\theta L d}{c}$$ leads to the following condition for the vector potential at the coating outer boundary:

$$A = -\mu d \frac{\partial A_z}{\partial r}. \quad (9)$$

Two other conditions are given by the continuity of the potential and its radial derivative at the inner boundary. Along with condition (13) they produce the following set of equations for coefficients c_i:

$$\begin{aligned} &I. \quad \frac{2I_0 x_0}{cb} + c_1 b = c_2 + c_3, \\ &II \quad -\frac{2I_0 x_0}{cb^2} + c_1 = \kappa(c_2 - c_3), \\ &III. \quad c_2 e^{\kappa d_m} + c_3 e^{-\kappa d_m} = -\kappa d\mu (c_2 e^{\kappa d_m} - c_3 e^{-\kappa d_m}). \end{aligned} \quad (10)$$

Including contribution of the electric dipole, the transverse impedance is calculated as:

$$\begin{aligned} Z_\perp &= -i\left(\frac{c_1}{I_0 x_0} + \frac{2}{cb^2}\right) = \\ &-i\frac{Z_0}{\pi\kappa b^3}\frac{\mu\kappa d + \tanh(\kappa d_m)}{1 + \mu\kappa d\tanh(\kappa d_m) + \mu d / b} \end{aligned}, \quad (11)$$

where we neglected term $\tanh(\kappa d_m)/\kappa b$ in the denominator. The result (11) transforms to one with radially unbounded magnetic [4] after a substitution $d \to b$.

The formulae (7) and (11) agree at high frequency but deviate at low frequency. Namely, in the denominator of formula (11) a term $\mu d/b$ starts to dominate the hyperbolic tangent term when the skin depth approaches $\sqrt{ad_m}$. It happens for the region about 100 kHz for a 10 microns TiN coating. Another form of the transverse impedance (11) is:

$$Z_\perp = \frac{2cLZ_{met}}{\omega b^2}\frac{\frac{Z_{cavity}}{Z_{met}} + \tanh(\kappa d_m)}{1 + \frac{Z_{cavity}}{Z_{met}}\tanh(\kappa d_m) + \mu d / b}, \quad (12)$$

where Z_{met} and Z_{cavity} are given after formula (4).

Figure 4 shows these results for the transverse impedance compared with the Piwinski result. The same three cases shown for the longitudinal impedance are shown here. Our result for a perfect external conductor on the surface of the ceramic chamber agrees with the Piwinski result for infinite geometry. The result for the realistic kicker geometry shown in figure 2 is higher than the Piwinski result by a factor of 2-3 in the frequency range of importance for transverse resistive wall instabilities (0.2-0.7 MHz). At lower frequencies, our results show transverse impedance, which is more than an order of magnitude larger than that predicted by the infinite geometry result.

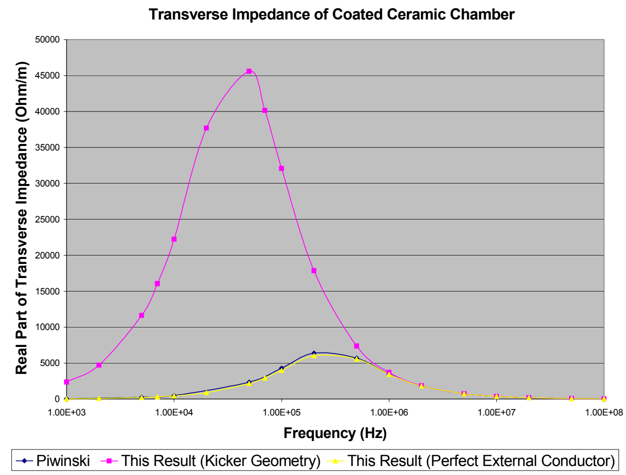

Figure 4: Real part of the transverse impedance of a ceramic chamber with thin metallic coating. Three cases are shown: i) the Piwinski result for an infinitely long chamber, ii) this result with a kicker model as shown in figure 2, and iii) this result with a perfect conductor on the outside of the ceramic chamber.

4 CONCLUSION

New formulae for the coating impedance are obtained. They show that in order to estimate the impedance, a knowledge of the surrounding impedance is needed. For most of the interesting frequencies (around 1 MHz) the image current squeezes through the coating, shielding the outer chambers from the beam fields. But for very low frequencies (below 100 kHz in our case when the skin depth reaches $\sqrt{ad_m}$ value), the transverse impedance rapidly goes to zero, which eases the problem of closed orbit deviations due to collective fields.

ACKNOWLEDGEMENTS

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