GLOBAL COMPENSATION OF LONG-RANGE BEAM-BEAM INTERACTIONS WITH MULTIPOLE CORRECTORS

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Abstract

To control the nonlinear effects of long-range beam-beam interactions due to a large number of non-localized parasitic collisions, we propose a global compensation scheme for long-range beam-beam interactions by using magnetic multipole correctors based on a minimization of nonlinearities in one-turn maps of a storage ring collider. Simulation study on a test model that is similar to LHC showed that this global compensation of the long-range beam-beam interactions is effective in increasing the dynamic aperture and improving the linearity of the phase-space region occupied by beams.

1 INTRODUCTION

In large storage ring colliders such as LHC and Tevatron, the long-range beam-beam interaction could be a major factor that reduces the beam lifetime and limits the luminosity. For LHC, wire compensation scheme has been proposed to compensate the long-range beam-beam perturbations due to parasitic collisions inside interaction regions [1]. Simulation studies showed that the wire compensation is very effective to such localized long-range beam-beam perturbations [2, 3]. In the case of multi-bunches operation in Tevatron, however, serious long-range beam-beam effects are due to many parasitic beam collisions that are distributed around the ring. For such non-localized long-range beam-beam perturbations, it is difficult to apply the wire compensation scheme. The electron-beam compensation of beam-beam tune spread, on the other hand, has been developed for the elimination of the bunch-to-bunch tune variation due to the long-range beam-beam interactions in Tevatron RUN IIB [4]. The nonlinear beam-beam perturbations due to the parasitic collisions could, however, still pose a serious problem that causes a beam-size growth and limits the luminosity even after the elimination of the bunch-to-bunch tune variation. Note that the use of the electron-beam compensation to reduce the beam-beam tune spread within a bunch may not improve and could even damage the beam stability [5]. In order to control adverse effects of long-range beam-beam interactions in such cases involving a large number of non-localized parasitic collisions, we propose a new compensation scheme: the global compensation of long-range beam-beam interactions by using multipole correctors based on a minimization of nonlinearities in one-turn maps of a storage ring collider.

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2 GLOBAL COMPENSATION SCHEME

Without beam-beam interactions, the nonlinear beam dynamics in a storage ring can be described by a one-turn map that contains all global information of nonlinearities in the system. By minimizing nonlinear terms of a one-turn map order-by-order with a few groups of multipole correctors, one can reduce the nonlinearity of the system globally [6, 7]. To include long-range beam-beam interactions into the map for the global compensation, one should recognize that a large beam separation is typical at parasitic beam collisions. In both LHC and Tevatron, for example, the beam separation at parasitic collisions is in a scale of 10σ, where σ is the nominal beam size. In the phase-space region occupied by beams, therefore, the long-range beam-beam interactions can be approximated with the strong-weak formula that can further be expanded into a Taylor series around the beam separation and be included into the one-turn map for the global compensation of the nonlinearities of the system.

Considering round Gaussian beams, the momentum kicks in transverse phase space due to the long-range beam-beam interaction can be written as

$$\Delta\vec{p} = G_0 \frac{\vec{r}+\vec{r}_0}{|\vec{r}+\vec{r}_0|^2}\left(1 - e^{-\frac{|\vec{r}+\vec{r}_0|^2}{2\sigma^2}}\right) \tag{1}$$

where $\vec{r}$ is transverse coordinate and $\vec{r}_0$ the horizontal and vertical beam separation. The kick strength G_0 is related to the beam-beam parameter ξ by $G_0 = 8\pi\sigma^{*2}\xi/\beta^*$ where σ^* and β^* are the nominal beam size and beta function at interaction point. In order to easily computer the Taylor expansion of $\Delta\vec{p}$ with our code, we rewrite $\Delta\vec{p}$ as

$$\Delta\vec{p} = G_0 \vec{F}_1 F_2 F_3 \tag{2}$$

where the expansions of $\vec{F}_1$, F_2, and F_3 around $\vec{r}_0$ are

$$\vec{F}_1 = \vec{r}+\vec{r}_0 \tag{3}$$

$$F_2 = \frac{1}{|\vec{r}+\vec{r}_0|^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{r_0^{2(n+1)}}\left(|\vec{r}+\vec{r}_0|^2 - r_0^2\right)^n \tag{4}$$

$$\begin{aligned} F_3 &= 1 - e^{-\frac{|\vec{r}+\vec{r}_0|^2}{2\sigma^2}} \\ &= 1 - e^{-\frac{r_0^2}{2\sigma^2}} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n\sigma^{2n} n!}\left(|\vec{r}+\vec{r}_0|^2 - r_0^2\right)^n \end{aligned} \tag{5}$$

With the method of differential algebra [8] or Lie algebra [9], the expansion of $\Delta\vec{p}$ can be easily calculated through $\vec{F}_1$, F_2, and F_3 to any desired order. Neglecting the transverse and longitudinal coupling, a 4-dimensional one-turn map for the transverse motion including the long-range beam-beam interactions can then be calculated in the form of Dragt-Finn Factorization [9]

$$M_0 = R_0\, e^{:H_3:} e^{:H_4:} \cdots e^{:H_n:} \cdots. \tag{6}$$

where R_0 is a linear transformation and H_n a homogeneous polynomial of degree n that is associated with the (n-1)th-order nonlinearity in the map,

$$H_n = \sum_{i+j+k+l=n} u^{(n)}_{ijkl}\, x^i\, p_x^j\, y^k\, p_y^l. \tag{7}$$

The global compensation of nonlinearities in the system is based on an assumption that with a few groups of multipole correctors, $\{H_n \mid n \geq 3\}$ can be minimized order-by-order and, consequently, the dynamics of the system can be substantially improved. In order to minimize $\{H_n \mid n \geq 3\}$ with a few parameters of the global correctors, we postulate that the nth-order nonlinearity in a one-turn map can be characterized by the magnitude of its nth-order coefficients which is defined by

$$\lambda_n = \left\{ \sum_{i+j+k+l=n+1} \left[u^{(n+1)}_{ijkl} \right]^2 \right\}^{1/2} \qquad \text{for } n \geq 2. \tag{8}$$

Note that in the case of $n = 2$, sextupoles for the chromaticity correction need to be excluded in the calculation of H_3. For convenience, we define the nth-order global correction when all λ_i with $i = 2, ..., n$ are minimized order-by-order using the multipole correctors up to the nth order.

3 TEST MODEL

The test lattice used in this study is the LHC collision lattice version 5.0. The fractional parts of horizontal and vertical tunes are $(\nu_x,\ \nu_y) = (0.31,\ 0.32)$. Head-on and long-range beam-beam interactions at two high luminosity interaction points (IP1 and IP5, $\beta^* = 0.5$ m) were included. The crossing angle of two counter-rotating beams was taken to be 300 μrad with vertical crossing at IP1 and horizontal at IP5. Both head-on and long-range beam-beam interactions were calculated with the approximation of round Gaussian beams (strong-weak model). For the long-range beam-beam interactions, there are 15 parasitic collisions on the each side of a IP.

Due to the beam separation and large beta-functions inside the inner triplets of IP1 and IP5, nonlinearities of the collision lattice are dominated by the field errors of superconducting high gradient quadrupoles (MQX) of the inner triplets. In this study, multipole field errors of MQXs up to the 10th-order were included. The random multipole components of MQXs were chosen with Gaussian distributions centered at zero and truncated at $\pm 3\sigma_{b_{n+1}}$ or $\pm 3\sigma_{a_{n+1}}$ where $\sigma_{b_{n+1}}$ and $\sigma_{a_{n+1}}$ are the rms value of the nth-order normal and skew multipole coefficient, respectively. Reference harmonics of version 2.0 for the Fermilab MQXs and 3.0 for the KEK MQXs were used [10]. In this study, we used the mixed configuration of the inner triplets in which the Fermilab quads are at Q2A and Q2B and the KEK quads at Q1 and Q3. No local correctors were used for the field errors in the triplets

To test the global compensation of the nonlinearities in the ring, we included four corrector packages symmetrically located in arcs. Each corrector package contains thin-lens kicks with normal and skew components of a desired multipole corrector.

4 TESTING RESULTS

To study the effect of the global compensation of the long-range beam-beam interactions, dynamic aperture (DA) of the system without and with the compensation is calculated with 10^5-turn 4-dimensional element-by-element tracking. The tracking has been done without momentum deviations.

In the case of the nonlinear lattice with the head-on and long-range beam-beam interactions, we used 50 different samples of random multiple components generated with different seed numbers in a random number generator routine to improve the statistical significance of the simulations. Fig. 1 plots the DA of the 50 random samples without or with the global compensation of the nonlinearities in the system when $\xi = 0.0064$. Because in our simulation we considered beam-beam interactions only in two interaction regions instead of four in LHC, we intentionally used ξ that is twice as large as the LHC design value in order to be more close to the LHC situation. Without including any beam-beam interactions, the smallest and average DA of these 50 samples is 7.7σ and 9.8σ, respectively. With both the head-on and long-range beam-beam interactions but without the global compensation, the smallest and average DA of the 50 samples is found to be 5.9σ and 6.6σ, respectively. After the 6th-order global compensation in this case, the smallest and the average DA increase to 8.3σ and 9.0σ, respectively, which is a more than 40% gain in the DA. Moreover, the standard deviation of the DA among the 50 samples becomes much smaller after the global compensation. This indicates that the system after compensation is much more linear in the phase-space region inside the DA.

In this case of the nonlinear lattice with beam-beam interactions, the global compensation reduces overall nonlinearities due to both the long-range beam-beam interactions and nonlinear field errors in the lattice. In order to isolate the effect of the global compensation on the long-range beam-beam interactions, we studied the case that contains only the beam-beam interactions and otherwise a linear lattice. In this case, the nonlinearities of the system are from head-on collisions at two IPs and 15 parasitic collisions on the each side of an IP. Fig. 2 plots the DA as a function of ξ before and after the global compensation of the long-range beam-beam interactions. It again shows that the global compensation improves the DA significantly. With-

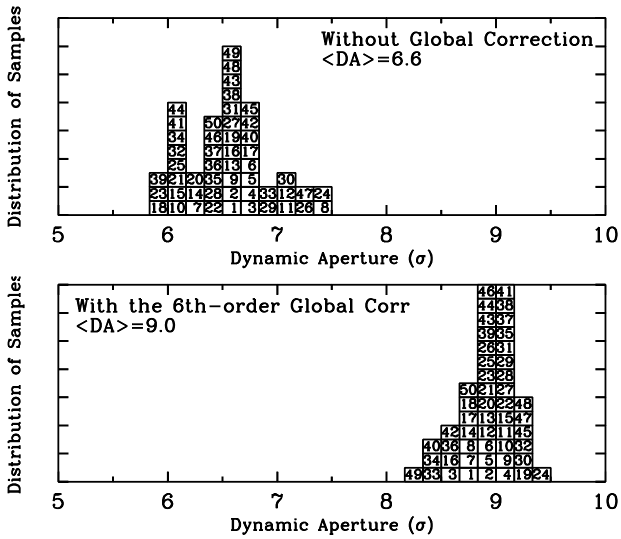

Figure 1: Dynamic aperture of fifty samples of the LHC collision lattice without or with the global compensation of the long-range beam-beam interactions. The number in each block identifies each sample.

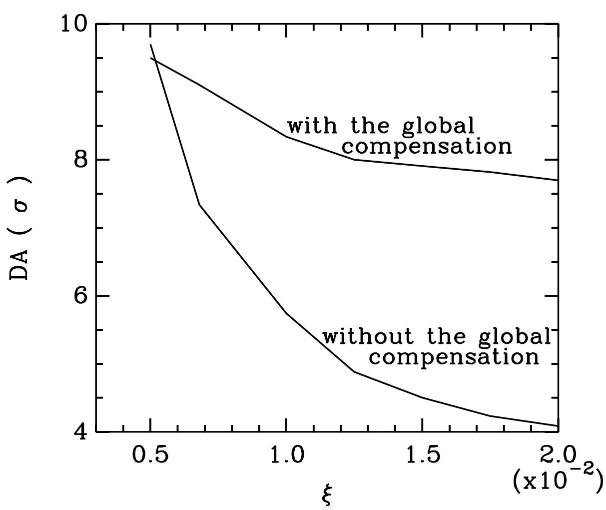

Figure 2: Dynamic aperture vs. beam-beam parameter without or with the global compensation of the long-range beam-beam interactions in the linear lattice.

out the compensation the DA decreases with ξ quickly. After the compensation, such reduction of the DA becomes much slower as the more nonlinear the system, the larger the increase of the DA after the compensation.

It should be noted that in all the cases we studied with either linear or nonlinear lattice the best DA achieved with the global compensation is always around 9σ that is about the beam separation at parasitic collision points. This is understandable since beyond this region the expansions for the long-range beam-beam interaction in Eqs. (4) and (5) are invalid and the multipole correctors for the global compensation could make the system even more nonlinear there. In the case that the original system is already quite linear, the original DA could be larger than or close to the beam separation, such as the cases when $\xi < 0.005$ in Fig. 2, and the global compensation of the long-range beam-beam interactions will not be effective in improving the DA. As a matter of fact, in such cases we don't expect to do any correction for the DA. In the phase-space region occupied by beams, on the other hand, the expansion of the long-range beam-beam interactions in Eqs. (4) and (5) is always good and, therefore, the compensation is effective in improving the linearity of the phase-space region near the closed orbit. An improvement of the linearity in this region could result in a longer beam life time and a slower luminosity decay.

5 SUMMARY

The global compensation of long-range beam-beam interactions with multipole correctors based on the minimization of nonlinearities in a one-turn map is an effective means to suppress long-range beam-beam effects. With a few groups of multipoles correctors, nonlinear terms in a one-turn map including the long-range beam-beam interactions can be minimized order-by-order and, consequently, the nonlinearity of the system in the phase-space region of interest is significantly reduced. The unique features of this global compensation scheme includes: (a) long-range beam-beam effects due to a large number of non-localized parasitic collisions can be effectively controlled; and (b) the overall nonlinearities in the system including both the long-range beam-beam interactions and magnetic field errors in the lattice can treated systematically with same groups of multipole correctors.

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