TRANSITION EFFECTS IN ROUND PIPE WITH FINITE CONDUCTING WALLS

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Abstract

The study of the transit resistive wake field excitation by relativistic electron beam in round pipe is given. Two models have been considered: the transition of the round pipe from perfectly to finite conducting walls and the semi-infinite resistive pipe. In the first model, the excited fields have been calculated by using the modal expansion in frequency domain. The second model has been studied based on the image charge representation technique.

1. INTRODUCTION

The radiation excited by the relativistic charged particle in the infinite round pipe with finite conductivity wall material is well understood and presented in number of Refs. [1-6]. These results are successfully used for evaluation of the resistive wakefield effects in accelerators. However, for the very small bunch length required for the SASE-FEL [7], the transition effects can play an essential role for the correct simulation of the resistive wakefield effects. As an example, for the TESLA-FEL [4] transfer line, a simple geometrical consideration shows that for the bunch length of 25 μm the transition length for steady state resistive wakes can reach few hundred meters. This distance corresponds to time interval when the scattered fields from the skin depth of the surface reach the driving bunch.

The exact knowledge of the longitudinal resistive wakes in transition regime results on the better understanding of the particle-environment interaction, the exact calculation of the induced energy spread as well as can lead to better performance of the surrounding vacuum chamber during the beam acceleration or transfer.

2 TRANSITION FROM INFINITE TO FINITE CONDUCTIVITY IN ROUND PIPE

The ultrarelativistic point charge passing through infinite round pipe along the z-axes of symmetry is studied. The pipe walls are the perfectly conducting material at $z < 0$, and has the finite conductivity at $z > 0$. The pipe radius b is constant and the wall thickness is infinity. The boundary between the two metallic walls is in $z = 0$ plane. The charge passes from the pipe with infinite conducting walls to the finite one in $z \to 0^+$ direction. It is well known that in perfectly conducting pipe, the field lines of the ultrarelativistic moving charge are perpendicular to pipe surface and charge does not radiate. For the finite conducting walls the self-fields of the charge are delay, the charge is radiate and in steady state regime the excited electromagnetic fields are

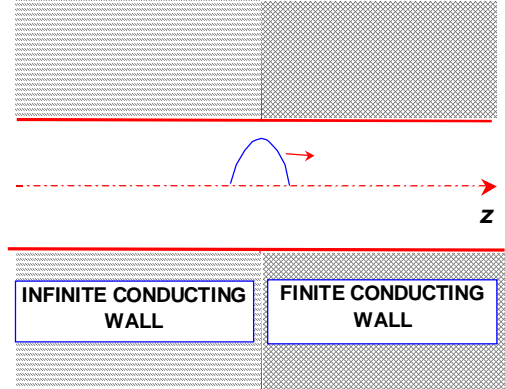_____
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Figure 1. The geometry of the problem

propagates in the pipe with the phase velocity small below velocity of light. These two limiting case, the perfect conducting pipe and steady state regime in finite conducting pipe, are coupled by the transition part where the re-arrangement of charge fields take place that accompany by the transition radiation.

The solution for the transition radiation is presented in terms of the modal expansion.

The charge field components in the pipe $(0 < r < b)$ with perfectly conducting walls $(z < 0)$ are given by

$$E_r^{q_1}(r,z) = H_\theta^{q_1}(r,z)/\varepsilon_0 c = \frac{q}{2\pi r \varepsilon_0 c} e^{-jkz}. \quad (1)$$

with the wave number $k = \omega/c$.

The charge field in the pipe $(0 \le r \le b)$ with the wall finite conductivity $(z > 0)$ is given by [1]

$$E_r^{q_2}(r,z) = \left(\frac{jkF_z r}{2} + \frac{q}{2\pi r \varepsilon_0 c} \right) e^{-jk_0 z},$$

$$E_z^{q_2}(r,z) = F_z e^{-jk_0 z} \quad (2)$$

with $H_\theta^{q_2}(r,z) = E_r^{q_2}(r,z)\varepsilon_0 c$, and

$$F_z = \frac{Z_0 q}{2\pi b}\left(\frac{v}{k} + j\frac{kb}{2} \right)^{-1}, \quad v^2 = -j\omega\sigma_c\mu_0 \quad (3)$$

As follow from the expressions (1) and (2), the charge field gets the breakage at the entering into the transition between two parts of pipe. The fields that radiated in the transition part should match these fields. The transition radiation fields in the both part of pipe are presented in modal form. The free (without charge) field components in the round pipe (for $0 \le r \le b$) can be written as:

$$E_r = \sum_{n=1}^{\infty} A_n^{\pm} e^{j(p_n^{\pm} z + \omega t)} \frac{jp_n^{\pm}}{u_n^{\pm}} J_0'\left(u_n^{\pm} r\right),$$

$$E_z = \sum_{n=1}^{\infty} A_n^{\pm} e^{j(p_n^{\pm} z + \omega t)} J_0\left(u_n^{\pm} r\right), \qquad (4)$$

$$H_\theta = -\sum_{n=1}^{\infty} A_n^{\pm} e^{j(p_n^{\pm} z + \omega t)} \frac{jk_\pm^2}{\mu\omega u_n^{\pm}} J_0'\left(u_n^{\pm} r\right)$$

with $u_n^{\pm} = j_{0,n}^{\pm}/b$, $k_\pm^2 = k^2 - j\mu\omega\sigma_c$, $p_n^{\pm} = (k_\pm^2 - (j_{0,n}^{\pm}/b)^2)^{1/2}$. The values $j_{0,n}^{-} = j_{0,n}$ are the roots of the zero order Bessel function and correspond to perfect conducting pipe, while $j_{0,n}^{+}$ to finite conducting pipe and are given by the equation:

$$\frac{k^2 b}{\mu_0 j_{0,n}^{+}} \frac{J_0'\left(j_{0,n}^{+}\right)}{J_0\left(j_{0,n}^{+}\right)} = j \frac{k_+^2}{\mu v_n} \qquad (5)$$

The index (+) indicates the waves propagating in forward direction, while (−) in opposite one. The coefficients $A_n^{\pm}$ are given by the matching of field components E_r, E_z and H_θ at $z = 0$ cross section inside the pipe (for $0 < r < b$)). Using the orthogonality of the Bessel functions $J_0(u_n^- r) = J_0(j_{0,n} r)$ one obtain two linearly independent equations:

$$A_k^{-} \frac{p_k^{-}}{k_0} \frac{J_1(j_{0,k})}{2 j_{0,k}} + \sum_{n=1}^{\infty} A_n^{+} \frac{p_n^{+}}{k_0} \frac{J_0\left(j_{0,n}^{+}\right)}{j_{0,k}^2 - \left(j_{0,n}^{+}\right)^2} = \frac{F_z}{j_{0,k}^2},$$

$$A_k^{-} \frac{J_1(j_{0,k})}{2 j_{0,k}} - \sum_{n=1}^{\infty} A_n^{+} \frac{J_0\left(j_{0,n}^{+}\right)}{j_{0,k}^2 - \left(j_{0,n}^{+}\right)^2} = \frac{F_z}{j_{0,k}^2}. \qquad (6)$$

The linear combination of equations (6) results on the infinite system of linear equations for the coefficients A_n^{+}:

$$\sum_{n=1}^{\infty} A_n^{+} \frac{J_0\left(j_{0,n}^{+}\right)}{\left(p_n^{+} - p_k^{-}\right)} = \frac{F_z}{k_0 + p_k^{-}}, \quad k = 1,2,3\ldots\infty. \qquad (7)$$

This system is typical for the closed electrodynamic problems and is solved exactly by the methods of direct reversion [8]:

$$A_k^{+} = -\frac{F_z}{J_0\left(j_{0,k}^{+}\right)} \prod_{n=1}^{\infty} S_n,$$

$$S_n = \begin{cases} \dfrac{p_n^{-} - p_n^{+}}{k_0 + p_n^{+}} & \text{for } n = k \\[2ex] \dfrac{p_n^{-} - p_k^{+}}{p_n^{-} - p_k^{-}} \dfrac{p_n^{+} + k_0}{p_n^{-} + k_0} & \text{for } n \neq k \end{cases}. \qquad (8)$$

The uniform convergence of the infinity product in (8) follows from the asymptotic form of solution given by eq. (5), which in the case of $|\kappa|^{3/2}/j_{0,k}^2 << 1$ ($\kappa = ks_0$ - dimensionless wave number, $s_0 = (2cb^2\varepsilon_0/\sigma_c)^{1/3}$ - characteristic size of pipe) can be written as

$$j_{0,k}^{+} = j_{0,k} + (sign(\kappa) - j)|\kappa|^{3/2}/2 j_{0,k} \qquad (9)$$

In the low frequency range $\left(\kappa^{3/2} << j_{0,1}^2\right)$ the solution (8) may be presented as

$$A_k^{\pm} = \frac{2F_z}{j_{0,k} J_1(j_{0,k})} \frac{k_0 \mp p_k^{\mp}}{p_k^{+} + p_k^{-}} \qquad (10)$$

This asymptotic solution is applicable to the bunches with comparatively large rms length. As an example, for Gaussian bunch rms length $\sigma_z = 1$cm and pipe radius $b = 10$cm, the maximal value of κ is equal to *0.05*.
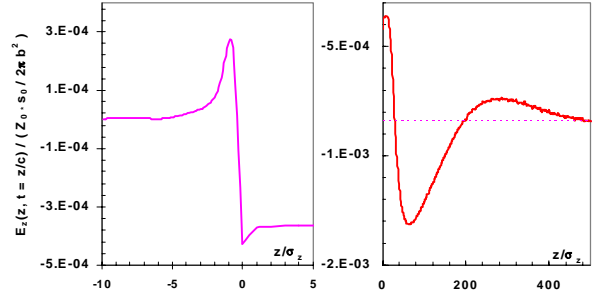

Figure 2. Longitudinal electrical component of the bunch field in the pipe with the finite conducting walls during (left) and after (right) passing the transition at z=0. Dotted curve presents the same value for the infinity pipe with finite conducting walls.

Fig.2 shows the longitudinal electric field at the bunch center during and after the bunch passing the transition at location z=0. The bunch rms length is 1cm, the tube radius 10cm. As it follows, the substantial radiation behind the bunch arises when the considerable number of bunch particle was passed through the transition aperture.
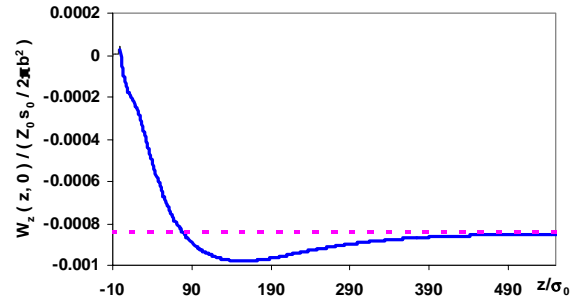

Figure 3. Longitudinal wake potential at the bunch center before $z < 0$ and after $z > 0$ passing the transition. Dotted curve presents the same value for the infinity pipe with finite conducting walls.

The longitudinal wake potential at the bunch center is presented in Fig.3. The wake potential is given

$$W_z(z,s) = \frac{1}{|z - z_1|} \int_{z_1}^{z} E_z\left(z, t = \frac{z + s}{c}\right) dz \qquad (11)$$

3 TRANSITION RADIATION WITH FINITE CONDUCTIVITY

Charged particle transition radiation through infinite perfectly conducting plane into the semi-infinite round pipe with finite conducting walls is considered (Fig.4). The point charge moves along the z-axes of symmetry.
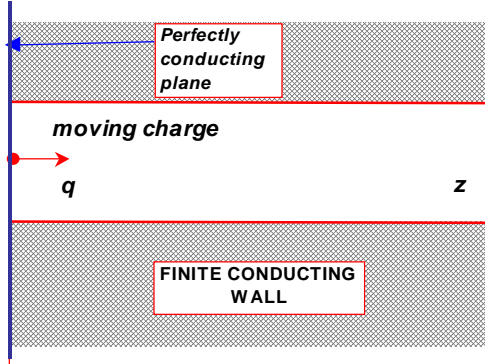
Figure 4. The geometry of the problem

The electromagnetic field in the waveguide after entering the particle consists of the sum of charge field in infinite waveguide with finite conducting walls $\vec{E}^{q_2}$ given by form. (2) and field excited at the crossing point $\vec{E}^r$. From the boundary condition at z=0 plane $E_r^r(r, z=0) = -E_r^{q_2}$. Other components are given by Maxwell equations $E_z^r = F_z e^{jk_0 z}$, $H_\theta^r = -c\varepsilon_0 E_r^r$. Thus the longitudinal component of the electrical field on the axe of the waveguide may be written as:

$$E_z(z,t) = f(-z + ct) + f(z + ct) \qquad (12)$$

where [5]

$$f(s) = -\frac{Z_0 cq}{3\pi b^2}\left\{4e^{-\frac{s}{s_0}}\cos(\sqrt{3}\,s/s_0) + \xi(i\sqrt{2\,s/s_0}) - \right.$$
$$\left. -\xi(e^{i\pi/6}\sqrt{2\,s/s_0}) - \xi(-e^{-i\pi/6}\sqrt{2\,s/s_0})\right) \qquad (13)$$

and $\xi(z)$ is a complex error function.

The field experienced by the driving charge ($t = z/c$) is given by

$$E_z(z, t = z/c) = f(0) + f(2z/s_0) \qquad (14)$$

Note, that at the transition point the retarding electric field is twice as large as the retarding field of the point charge in finite conducting pipe for steady state regime.
The longitudinal field component experienced by charge (retarding potential) as it moves along the z-direction is given in fig.4. The second term in (14) decreases during the particle motion and becomes equal to zero at $z \to \infty$ (Fig.4). In limiting case for large z the solution well agreed with the solution for the infinitely resistive pipe.
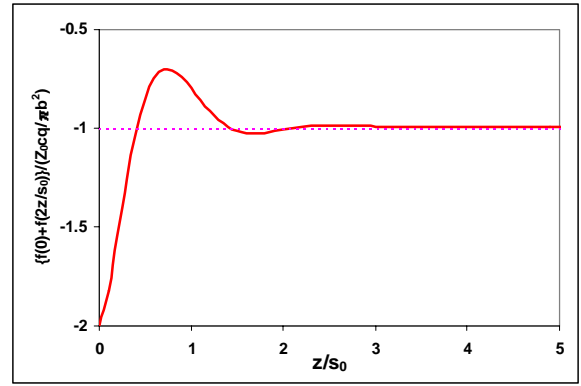

Figure 5. The point charge transition radiation longitudinal field component in semi-infinite waveguide with the finite conducting walls. Dotted curve presents the same value for infinity pipe with finite conducting walls.

4 CONCLUSION

The transition radiation of the particles in a finite conducting pipe has been studied. The two models have been considered: the transition between perfectly conducting and resistive pipes, the semi-infinite resistive tube. In both case, the exact solutions for the induced fields have been derived. The comparison of the asymptotic behaviour of the induced fields with well-known solutions in resistive pipe gives a good agreement.

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