STABILITY CRITERIA FOR HIGH-INTENSITY SINGLE-BUNCH BEAMS IN SYNCHROTRONS

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Abstract

In circular accelerators, the interaction between the beam and its surroundings is described in terms of bunched-beam modes through Sacherer's formalism. As the bunch intensity increases, the different modes, separated by the synchrotron frequency, can no longer be treated separately. In this regime, the wake fields couple the modes together and a wave pattern travelling along the bunch is created. It leads to the longitudinal and transverse mode-coupling instabilities, aspects of which are discussed under many different names in a variety of different papers. The present work reviews and unifies these approaches. It is shown that for the transverse plane, the same intensity threshold is obtained through 5 seemingly diverse formalisms: (i) transverse mode coupling, (ii) beam break-up, (iii) fast blow-up, (iv) post head-tail, and (v) a quasi coasting-beam approach using the peak values of bunch current and momentum spread as input for the coasting-beam formula. For the longitudinal plane, a new stability criterion is derived using the mode-coupling formalism. This formula is close to the Keil-Schnell-Boussard criterion, when the space-charge impedance is much smaller than the resonator one.

1 INTRODUCTION

The Transverse Mode Coupling (TMC) instability for circular accelerators has been first described by Kohaupt [1] in terms of coupling of Sacherer's head-tail modes [2]. This extended to the transverse motion, the theory proposed by Sacherer [3] to explain the longitudinal microwave instability through coupling of the longitudinal coherent bunch modes. In linear accelerators, the Beam Break-Up (BBU) theory has been developed to explain the observed beam emittance growths and the transverse instabilities [4,5]. It has been known for some time, using a two-particle model, that the TMC instability is the manifestation in synchrotrons of the BBU mechanism observed in linacs [6,7]. The only difference comes from the synchrotron oscillation, which stabilises the beam in synchrotrons below a threshold intensity by swapping the head and the tail continuously. This effect disappears close to transition energy, or more generally when the instability rise-time is much faster than the synchrotron period. In this case, it is usually said that the concept of head-tail modes loses its meaning and that it is appropriate to use the BBU formalism to describe the interaction between the beam and its surroundings [6]. Other formalisms have also been developed to describe the instability when the rise-time is faster than the synchrotron period [8,9]. It is shown in Section 2 that using the mode-coupling formalism, for the case of a bunch interacting with a broad-band resonator impedance, and whose length is greater than the inverse of twice the resonance frequency, the same formula as in the other approachs is obtained: it is the coasting-beam formula with peak values of bunch current and momentum spread [10].

In the longitudinal plane, the microwave instability for coasting beams is well understood [10,11,12,13]. It leads to a stability diagram, which is a graphical representation of the solution of the dispersion relation depicting curves of constant growth rates, and especially a threshold contour in the complex plane of the driving impedance. When the real part of the driving impedance is much greater than the modulus of the imaginary part, a simple approximation, known as the Keil-Schnell (or circle) stability criterion, may be used to estimate the threshold curve [12]. For bunched beams, it has been proposed by Boussard [14] to use the coasting-beam formalism with local values of bunch current and momentum spread. This approximation was expected to be valid in the case of instability rise-times shorter than the synchrotron period, and wavelengths of the driving wake field much shorter than the bunch length. This empirical rule is widely used for estimations of the tolerable impedances in the design of new accelerators. A first approach to explain this instability, without coasting-beam approximations, has been suggested by Sacherer through Longitudinal Mode-Coupling (LMC) [3]. The equivalence between LMC and microwave instabilities has been pointed out by Sacherer [3] and Laclare [15] in the case of broad-band driving resonator impedances, neglecting the Potential-Well Distortion (PWD). The complete theory describing the microwave instability for bunched beams is still under development [13,16]. It is shown in Section 3 that, using the mode-coupling formalism for the case of a bunch interacting with a broad-band resonator impedance, and whose length is greater than the inverse of half the resonance frequency, a new formula is derived taking into account the PWD due to both space-charge and resonator impedances.

2 IN THE TRANSVERSE PLANE

The model used for the TMC instability is based on the mode-coupling between the two most critical head-tail modes (m and m+1) overlapping the peak of the negative resistive impedance. For zero chromaticity, the tune shifts are real. There is no Head-Tail (HT) instability, and above a threshold intensity, a TMC instability develops, with an instability rise-time faster than the synchrotron period. When the chromatic frequency is shifted positively (this is the stability criterion for the head-tail mode m=0), the simple model where the two regimes (HT and TMC) are treated separately is used. Below the threshold intensity, the standing-wave patterns (head-tail modes) are treated independently. This leads to instabilities where the head and the tail of the bunch exchange their roles (due to synchrotron oscillation) several times during the rise-time of the instability. The number of nodes on separate superimposed revolutions gives the modulus of the head-tail mode number |m|. As the intensity increases, the wake fields couple the head-tail modes together and a travelling-wave pattern is created along the bunch: this is the TMC instability. Of course, shifting the chromatic frequency positively, a head-tail instability due to e.g. the resistive-wall impedance may develop. This was the case in LEP with short bunches ($\sigma_s \approx 1$ cm), where the beam intensity was limited by a TMC instability for zero chromaticity. The head-tail mode m=1 developed as soon as the chromatic frequency was increased [17]. It was a very slow instability, with a rise-time much smaller than the synchrotron period, due to the resistive-wall impedance.

A typical TMC instability is shown in Fig. 1. It has been obtained with the CERN PS beam for the neutron Time-of-Flight facility (n-ToF) at transition (~6 GeV total energy) [18]. As can be seen from Fig. 1, the head of the bunch is stable and only the tail is unstable in the vertical plane. The particles lost at the tail of the bunch can be seen from the hollow in the bunch profile.

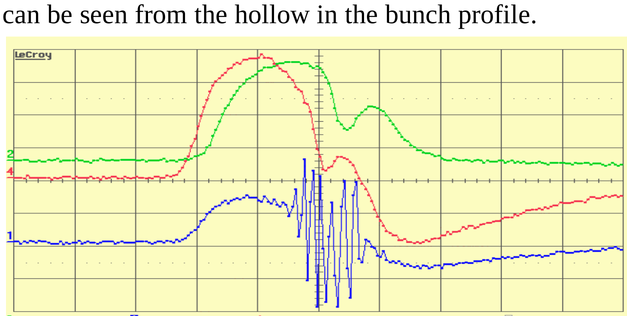

Figure 1: Single-turn signals from a wide-band pick-up. From top to bottom: Σ, Δx, and Δy. Time scale: 10 ns/div.

The threshold number of protons per bunch can be approximated by, e.g. in the vertical plane, [19,20]

$$N_b^{th} = \frac{8\pi\, Q_{y0}\, |\eta|\, \varepsilon_l}{e\, \beta^2\, c} \times \frac{f_r}{\left| Z_y^{BB} \right|} \times \left(1 + \frac{f_{\xi_y}}{f_r} \right), \qquad (1)$$

where Q_{y0} is the unperturbed tune, $\eta = \gamma_{tr}^{-2} - \gamma^{-2} = (\Delta T / T_0)/(\Delta p / p_0)$ is the slippage factor, with p the momentum and T the revolution period of a particle, e is the elementary charge, β and γ are the relativistic velocity and mass factors, c is the speed of light, $\varepsilon_l = \beta^2 E\, \tau_b (\Delta p / p_0)_{\max} \pi / 2$ is the longitudinal emittance (at 2σ, in eV.s), approximated by an elliptic area in the longitudinal phase space, with E the total beam energy, τ_b the total bunch length (in seconds), and $(\Delta p / p_0)_{\max}$ the relative momentum spread at 2σ, $f_{\xi_y} = (\xi_y / \eta)\, Q_{y0}\, f_0$ is the chromatic frequency, with $\xi_y = (\Delta Q_y / \Delta p)\, (p_0 / Q_{y0})$ the chromaticity, and f_0 the revolution frequency, and $\left| Z_y^{BB} \right|$ is the peak value of the broad-band resonator impedance, given by $Z_y^{BB}(\omega) = R_r\, (\omega_r / \omega) / \left[1 - j\, Q_r\, (\omega_r / \omega - \omega / \omega_r) \right]$, where $\omega_r = 2\pi\, f_r$ is the resonance angular frequency, $Q_r \approx 1$ the quality factor and R_r the shunt impedance (in Ω/m).

It is seen from Eq. (1) that concerning machine parameters, the intensity threshold is increased by increasing the modulus of the slippage factor, and/or the ratio between the resonance frequency and the peak value of the resonator impedance. Concerning beam parameters, the intensity threshold is increased by increasing the longitudinal emittance, and/or the chromatic frequency. The first method is used in the CERN PS to avoid the TMC instability at transition with the n-ToF bunch, and the second is used at ESRF [9]. Note that it is the longitudinal emittance which matters in the transverse plane: for a given longitudinal emittance, short and long bunches are equally stable. The PWD has no effect on the TMC instability, since the longitudinal emittance is supposed to be conserved in this mechanism.

Note that, in the case of zero chromaticity, Gareyte's conjecture for stabilisation of the beam break-up instability by the differential streaming of particles is recovered [6,19], as well as Zotter's formula [21].

3 IN THE LONGITUDINAL PLANE

The same formalism as in the transverse case is used. An additional complication comes here from the PWD, which has to be taken into account and which makes the synchrotron frequency, bunch length and momentum spread depend on the bunch intensity. The stability criterion can be approximated by Eq. (2) [22], where $I_p = 3\, e\, N_b / (2\tau_b)$ is the bunch peak current considering a parabolic line density, $(\Delta p / p_0)_{\mathrm{FWHH}}$ is the full width at half height of the relative momentum spread, $\left| Z_l^{BB}(p)/p \right|$ and $\left| Z_l^{SC}(p)/p \right|$ are the peak values of the broad-band and space-charge longitudinal impedances given respectively by

$Z_l^{BB}(p)/p = R_s(\Omega_0/\omega)/\left[1 - jQ_r\left(\omega_r/\omega - \omega/\omega_r\right)\right]$ and $Z_l^{SC}(p)/p = -jZ_0 \times \left[1 + 2\ln(b/a)\right]/(2\beta\gamma^2)$, with $p \approx \omega/\Omega_0$, R_s the shunt impedance (in Ω), $Z_0 = 377\,\Omega$ the free space impedance, a and b the average beam and effective pipe radii. Finally, $Sgn(\eta)$ denotes the sign of η: it is - below transition and + above.

$$\frac{\left|Z_l^{BB}/p\right|}{1.2} \times \left[1 - Sgn(\eta) \times \frac{3}{4}\left(\frac{\left|Z_l^{SC}/p\right|}{\left|Z_l^{BB}/p\right|} - 1\right)\right]^{1/4} \le \frac{(E/e)\beta^2|\eta|}{I_p} \times \left(\frac{\Delta p}{p_0}\right)^2_{\mathrm{FWHH}} . \quad (2)$$

Note that the Keil-Schnell-Boussard stability criterion [14] is given by Eq. (2) with the term on the left replaced by the modulus of the (total) coupling impedance at the resonance frequency. Experimentally, the most evident signature of the LMC instability is the intensity-dependent longitudinal beam emittance blow-up to remain just below threshold. A typical picture is shown in Fig. 2. It has been obtained with the CERN PS beam for LHC at extraction (25 GeV total energy) [23].

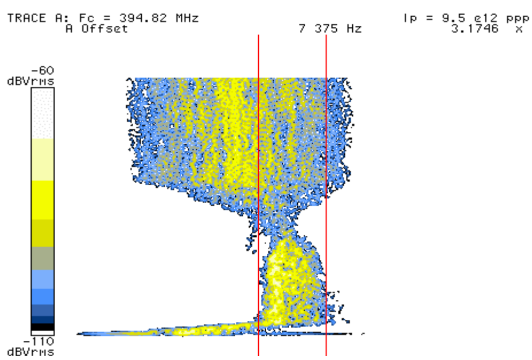

Figure 2: Longitudinal Schottky scan spectrogram during debunching. Time goes from top to bottom. Total time window is ~200 ms. In the first 100 ms the beam is still bunched by the RF voltage, which is adiabatically decreased and then switched OFF. During the debunching there is a momentum blow-up. The last "transient" is produced by the fast extraction process.

It is seen from Eq. (2) that concerning machine parameters, the threshold is increased by increasing the modulus of the slippage factor. Concerning beam parameters, the threshold is increased by increasing the energy and/or the bunch length and/or the momentum spread. Here, as opposed to the transverse case, the momentum spread is more efficient than the bunch length: for a given longitudinal emittance, short bunches are more stable than longer ones.

The stability diagram derived from Eq. (2) is shown in Fig. 3. Below transition, the space-charge impedance has a destabilising effect but, even if the space-charge impedance is much bigger than the broad-band one, the effect on the threshold is rather small due to the exponent ¼ in Eq. (2). Above transition, the space-charge impedance has a stabilising effect, as it increases the synchrotron frequency, i.e. the ratio between the momentum spread and the bunch length.

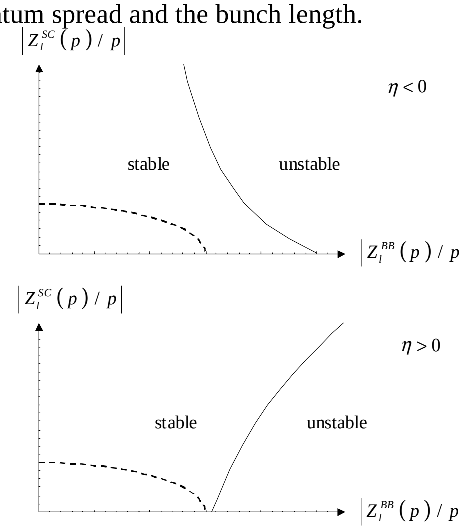

Figure 3: Stability diagram for the LMC instability below and above transition respectively. The Keil-Schnell circle is represented by the dashed curve.

4 CONCLUSION

Stability criteria for high-intensity single-bunch beams have been given, when the instability rise-times are faster than the synchrotron period. They apply for the case of a bunch interacting with a broad-band resonator impedance ($Q_r \approx 1$), and whose length is greater than the inverse of twice (half) the resonance frequency in the transverse (longitudinal) plane.

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