DESTABILISING EFFECT OF LINEAR COUPLING IN THE HERA PROTON RING

E. Métral, CERN, Geneva, Switzerland
G. Hoffstaetter, F. Willeke, DESY, Hamburg, Germany

Abstract

Since the first start-up of HERA in 1992, a transverse coherent instability has appeared from time to time at the beginning of the acceleration ramp. In this process, the emittance is blown up and the beam is partially or completely lost. Although the instability was found to be of the head-tail type, and the chromaticity and linear coupling between the transverse planes was recognized as essential for the instability to occur, the driving mechanism was never clarified. An explanation of the phenomenon is presented in this paper using the coupled Landau damping theory. It is predicted that a too strong coupling can be detrimental since it may shift the coherent tune outside the incoherent spectrum and thus prevent Landau damping. Due to these features, the name "coupled head-tail instability" is suggested for this instability in the HERA proton ring.

1 INTRODUCTION

The transverse coherent amplitude growth, which is occasionally detected in the HERA proton ring at the beginning of the acceleration ramp, has been carefully studied in Ref. [1]. The characteristic behaviour of a head-tail instability *m*=0 has been clearly evidenced. At injection energy, where HERA is above transition, the transverse chromaticities are chosen slightly positive to prevent the head-tail mode *m*=0 from developing. The existence of the instability means that at least one of the transverse chromaticities becomes negative while the energy is increased. This may be due to the persistent currents in the super-conducting magnets, which increase one chromaticity and decrease the other. Furthermore, these persistent currents also introduce strong linear coupling between the transverse planes, which is identified by the tune distance on the coupling resonance $Q_x - Q_y = -1$.

In the following, all the conclusions of the observations made so far on this phenomenon are reviewed in Section 2. A simple stability criterion is then given in Section 3 for this particular HERA instability. Finally, the one- and two-dimensional theories are compared to experiments in Section 4.

2 OBSERVATIONS

After careful investigations made with controlled linear coupling and chromaticity changes on the injection flat-bottom at 40 GeV, the following conclusions can be drawn. (i) Some coupling between the transverse planes is necessary for the instability to start. (ii) It is mainly a single-bunch effect, although extra multi-bunch effects can also be present. (iii) Both transverse planes are affected, due to coupling. (iv) There is a clear chromaticity dependence, the instability appearing for small negative values of the chromaticity. (v) The most critical value of the chromaticities is $\xi_{x,y} \approx -2.5 / Q_{x,y}$, but it can also appear for positive values of one of the two chromaticities. Here, the following definition is used $\xi_{x,y} = \left(d\,Q_{x,y} / d\,p \right) \left(p_0 / Q_{x,y} \right)$, where p_0 is the design momentum and $Q_{x,y}$ are the horizontal and vertical tunes. (vi) Growth times of the order of 100 to 700 ms were measured with a single bunch and $I_b \approx 250\,\mu\text{A}$, where $I_b = N_b\, e\, f_0$ is the bunch current with N_b the number of protons in the bunch, *e* the elementary charge, and $f_0 = \Omega_0 / 2\pi$ the average revolution frequency of the particles. (vii) If the chromaticities are set to $\xi_{x,y} \approx -2.5 / Q_{x,y}$, and the particles at large amplitudes are cut by collimators partially closed, the beam is clearly unstable. (viii) If $\left| \xi_{x,y} \right|$ is set to a large value, the beam is stabilised. (ix) To suppress the instability, the chromaticities are set to $\xi_{x,y} \approx +2.5 / Q_{x,y}$ and linear coupling is compensated to have some safety margin.

3 THEORY OF COUPLED LANDAU DAMPING

The theory of coupled Landau damping reveals the possibility to stabilise a beam by sharing the transverse instability growth rates and the stabilising tune spreads, which are responsible for Landau damping [2]. It also reveals the possibility to destabilise a beam in the case of too strong a coupling. In this case, the coherent tune is shifted outside the incoherent spectrum. The Landau damping mechanism is suppressed and the beam becomes unstable. This is the phenomenon we suspect happens in the HERA proton ring.

Considering the simplest case, where the tune distributions and complex coherent tune shifts are the same in both transverse planes, the stability criterion is given by [3]

$$\Delta\omega_{\rm HWB}^{\rm spread} \geq 2\sqrt{\left(\left|U_{\rm eq}\right| + \frac{|C|\,\Omega_0}{2}\right)^2 + V_{\rm eq}^2}\,. \qquad (1)$$

Here, $\Delta\omega_{\rm HWB}^{\rm spread} = \Omega_0\,\Delta Q_{\rm HWB}^{\rm spread}$ is the transverse betatron frequency spread (Half Width at the Bottom of an elliptical spectrum), $|C| = \left|\Delta Q_{\rm normal\,modes}\right|$ is the modulus of the general complex coupling coefficient (it is the normal mode tune difference on the coupling resonance), and $U_{\rm eq}$ and $V_{\rm eq}$ are the "equivalent" dispersion relation coefficients given by

$$U_{\rm eq} = {\rm Re}\left(\Delta\omega_m^{x,y}\right), \quad V_{\rm eq} = -{\rm Im}\left(\Delta\omega_m^{x,y}\right), \qquad (2)$$

where ${\rm Re}(\ )$ and ${\rm Im}(\ )$ stand for real and imaginary parts respectively. The complex coherent frequency shifts $\Delta\omega_m^{x,y}$ are given by Sacherer's formula [4]

$$\Delta\omega_m^{x,y} = \frac{j}{|m|+1}\frac{I_b}{2\beta\tau_b\left(E/e\right)}\beta_{x,y}^{\rm av}\left(Z_{x,y}^{\rm eff}\right)_m. \qquad (3)$$

Here, $m = \ldots,-1,0,1,\ldots$ is the head-tail mode number, $j=\sqrt{-1}$ is the imaginary unit, β is the relativistic velocity factor, τ_b is the full (4σ) bunch length (in seconds), E is the total beam energy, and $\beta_{x,y}^{\rm av} = R/Q_{x,y}$ are the average transverse betatron functions, with $R = L/(2\pi)$ the average radius of the machine. The effective impedances $\left(Z_{x,y}^{\rm eff}\right)_m$ for mode m are given by

$$\left(Z_{x,y}^{\rm eff}\right)_m = \frac{\displaystyle\sum_{k=-\infty}^{k=+\infty} Z_{x,y}\left(\omega_k^{x,y}\right) h_m\left(\omega_k^{x,y} - \omega_{\xi_{x,y}}\right)}{\displaystyle\sum_{k=-\infty}^{k=+\infty} h_m\left(\omega_k^{x,y} - \omega_{\xi_{x,y}}\right)}, \qquad (4)$$

with

$$h_m\left(\omega - \omega_{\xi_{x,y}}\right) = \frac{\tau_b^2}{2\pi^4}\left(|m|+1\right)^2 \times \frac{1+(-1)^{|m|}\cos\left[\left(\omega-\omega_{\xi_{x,y}}\right)\tau_b\right]}{\left\{\left[\left(\omega-\omega_{\xi_{x,y}}\right)\tau_b/\pi\right]^2 - \left(|m|+1\right)^2\right\}^2}, \qquad (5)$$

where $Z_{x,y}$ are the transverse coupling impedances, $\omega_k^{x,y} = \left(k+Q_{x,y}\right)\Omega_0 + m\,\omega_s$ with $-\infty \leq k \leq +\infty$ and $\omega_s = 2\pi f_s$ the synchrotron angular frequency, $\omega_{\xi_{x,y}} = 2\pi f_{\xi_{x,y}} = (\xi_{x,y}/\eta)\,Q_{x,y}\,\Omega_0$ are the chromatic angular frequencies, with the slippage factor given by $\eta = \gamma_{tr}^{-2} - \gamma^{-2} = (\Delta T/T_0)/(\Delta p/p_0)$, where T is the revolution period of a particle.

If $|C|=0$, i.e. in the absence of linear coupling, the one-dimensional (Keil-Zotter's) stability criterion [5] is recovered (see Eq. (1)). If $|C|$ increases, the betatron frequency spread has to be increased according to Eq. (1). Therefore, any coupling is bad in that particular case since the stability condition of Eq. (1) is more restrictive than the one-dimensional stability criterion.

When $|C|\Omega_0/2 >> U_{\rm eq}$ and $|C|\Omega_0/2 >> V_{\rm eq}$, Eq. (1) reduces to the following simple stability criterion

$$\Delta Q_{\rm HWB}^{\rm spread} \geq \left|\Delta Q_{\rm normal\,modes}\right|. \qquad (6)$$

4 COMPARISON BETWEEN THEORY AND EXPERIMENTS

Making the numerical computations for a single-bunch beam on the injection flat-bottom, one has $I_b = 250\,\mu$A, $f_0 = 47304$ Hz, $\beta \approx 1$, $\tau_b = 2$ ns, $Q_x = 31.292$, $Q_y = 32.299$, $\xi_{x,y} \approx -2.5/Q_{x,y}$, $f_s = 30$ Hz, $E = 40$ GeV, and the momentum compaction factor is given by $\alpha_p = \gamma_{tr}^{-2} = 1.218\times10^{-3}$. The coupling impedance, which is suspected to drive the instability, is the resistive-wall impedance. In the thick-wall approximation, i.e. when the skin depth $\delta(\omega) = \sqrt{2\rho_w\varepsilon_0 c^2/\omega}$ is smaller than the wall thickness l_w, it is given by [4]

$$Z_{x,y}^{RW}(\omega) = \left[Sgn(\omega) + j\right]\frac{R}{b^3}\sqrt{\frac{2\rho_w}{\varepsilon_0|\omega|}}. \qquad (7)$$

Here, $\rho_w = 1.3\times10^{-6}\ \Omega$m is the vacuum chamber resistivity (for the warm stainless steel, which has the dominant effect), $\varepsilon_0 = 8.84\times10^{-12}$ Fm^{-1} is the permittivity of free space, c is the speed of light, $l_w = 2$mm of stainless steel in the warm sections, $Sgn(\omega) = 1$ if $\omega > 0$, -1 if $\omega < 0$, $L = 2\pi R = 6335.82$ m is the machine circumference, and $b = 4$ cm is the radius of the circular beam pipe.

An approximation is made for the weighted transverse coupling impedances $\beta_{x,y}^{\rm av}\left(Z_{x,y}^{\rm eff}\right)_m$, since the warm sections only occupy 15% of the circumference. Indeed, in the East, North, and South halls the vacuum chamber is warm in the region of 110 m to the right and to the left of the interaction points. In the West hall, it is warm 150 m to the right and to the left of the interaction point. Furthermore, the average betatron functions in the warm sections are approximately given by

$$\beta_{x,\rm warm}^{\rm av} \approx 3\,\beta_x^{\rm av}, \qquad \beta_{y,\rm warm}^{\rm av} \approx 2\,\beta_y^{\rm av}, \qquad (8)$$

where the average betatron functions are given by $\beta_{x,y}^{\rm av} = R/Q_{x,y} \approx 30$ m. Taking these two effects into account, the following results are obtained. In the absence of both linear coupling and Landau damping, the instability rise-times of the first three head-tail modes (given by $-1/{\rm Im}(\Delta\omega_m^{x,y})$) are $\tau_0 \approx 0.4$ s, $\tau_1 \approx -3$ s, and $\tau_2 \approx -24$ s for the horizontal plane, and $\tau_0 \approx 0.5$ s, $\tau_1 \approx -4$ s, and $\tau_2 \approx -34$ s for the vertical plane. Therefore, in the absence of any stabilising mechanism, a head-tail instability of mode m=0 should develop with a rise-time of about 400-500 ms, which is in perfect agreement with the observations (cf. Introduction, (iv,vi,viii) in Section 2, and Fig. 1) [1].

A typical data acquisition is shown in Fig. 1. In this case the instability develops with a rise-time of 440 ms for a few seconds, then, after quite large amplitudes are reached and a large fraction of the bunch particles has been lost, some Landau damping effect (like amplitude-dependent tune-shift) prevails, and in another few seconds the instability is damped down. Both transverse planes are affected, due to coupling.

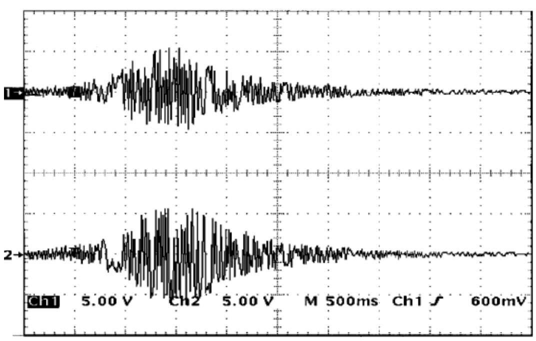

Figure 1: Growth time acquisition: the transverse position signals of the bunch center of charge from horizontal and vertical pick-ups are displayed as a function of time. Time scale: 500 ms/div.

As can be seen from Fig. 2, the most critical value of the chromaticity $\xi_{x,y} \approx -2.5 / Q_{x,y}$ found experimentally (cf. (v) in Section 2), is the most critical one theoretically if one considers the resistive-wall impedance. The lowest head-tail mode $|m| = 0$ is excited with the largest effective impedance. The resistive-wall impedance also explains the fact that it is mainly a single-bunch effect, although multi-bunch effects can be present as well (cf. (ii) in Section 2).

Applying Keil-Zotter's stability criterion, which is given by Eq. (1) with $|C| = 0$, the necessary tune transverse spreads to stabilise the head-tail mode m=0 by Landau damping are given by $\Delta Q_{x,\mathrm{HWB}}^{\mathrm{spread}} \geq 5.6 \times 10^{-5}$ and $\Delta Q_{y,\mathrm{HWB}}^{\mathrm{spread}} \geq 3.9 \times 10^{-5}$. Stability is thus ensured if such (small) tune spreads are present in the beam due to magnetic non-linearities. It is known that this is the case since, when the particles at large amplitudes are cut by collimators partially closed, the beam is clearly unstable (cf. (vii) in Section 2). In the absence of coupling and collimators, the beam should be stable, as observed (cf. (i) in Section 2). In the presence of too strong coupling, i.e. when the condition of Eq. (6) is not fulfilled, the beam becomes unstable and both planes are affected due to coupling, in agreement with the observations (cf. (iii) in Section 2). In the presence of a sufficiently small coupling, i.e. when the condition of Eq. (1) is fulfilled, the beam should be stable, even with the most critical value of the chromaticity $\xi_{x,y} \approx -2.5 / Q_{x,y}$. To have some margin during the acceleration ramp where all the parameters are not perfectly controlled, the chromaticities should be set to positive values, e.g. $\xi_{x,y} \approx +2.5 / Q_{x,y}$, since the high-order modes are more difficult to drive, and linear coupling should be reduced for Eq. (1) to be satisfied. This is also in agreement with the observations (cf. (ix) in Section 2).

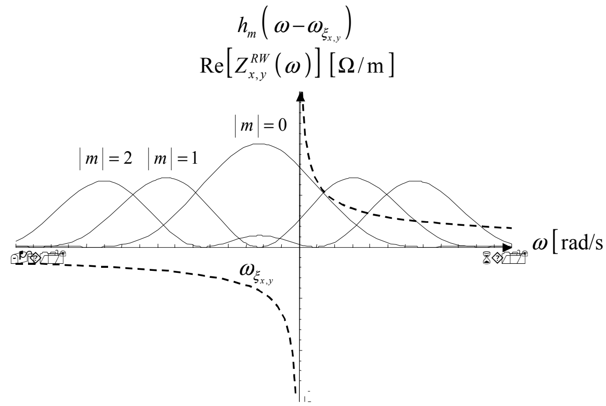

Figure 2: Transverse power spectrum for the head-tail modes $|m| = 0, 1, 2$ (solid lines) and resistive part of the resistive-wall impedance (dotted lines).

5 CONCLUSION

All the measurements performed so far on the traditionally called "Batman instability" in the HERA proton ring can be explained by the theory of coupled Landau damping of a head-tail instability due to the resistive-wall impedance. In the absence of coupling, this instability is stabilised by Landau damping in both planes whatever the chromaticity. When coupling becomes too strong, the coherent tune is shifted outside the incoherent spectrum. The Landau damping mechanism is then suppressed and the beam is unstable. To prevent the instability from developing, the normal mode tune difference on the coupling resonance has to be kept smaller than half the width at the bottom of the tune distribution.

REFERENCES

[1] F. Galluccio, F. Willeke, Proc. 5th EPAC, 1996.
[2] E. Métral, Part. Accelerators, 62(3-4), p. 259, 1999.
[3] E. Métral, CERN/PS 98-004 (CA).
[4] F. Sacherer, Proc. 9th Int. Conf. on High Energy Accelerators, Stanford 1974, p. 347.
[5] A.W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators", New York: Wiley, 1993.