EMITTANCE EXCHANGE BY CROSSING A COUPLING RESONANCE

C. Carli, G. Cyvoct, M. Giovannozzi, E. Métral, G. Métral, R. Steerenberg
CERN, Geneva, Switzerland

Abstract

When the working point of a synchrotron is shifted across a second-order difference resonance in the presence of weak linear coupling, it is possible to exchange the transverse emittances. This result goes beyond the classical theory, where only emittance sharing is predicted. Systematic experimental investigations of the phenomenon have been performed at the CERN PS Complex. In the PS-Booster, the measurement of the horizontal emittance is hampered by the contribution of the momentum spread to the beam size through dispersion. Based on the new mechanism, the horizontal emittance was measured in the vertical plane, where no dispersion is present, thus improving the overall accuracy. In the PS, detailed measurements of the emittance exchange process were performed. In parallel, an analytical theory was derived. The agreement between theory and experimental results is remarkable.

1 INTRODUCTION

For several years, exchanges of the transverse emittances have been observed in the CERN Proton Synchrotron (PS) machine with different types of beams. A perfect exchange of the emittances can be seen for instance with the PS beam for LHC, as quoted in Ref. [1]. Although linear coupling between the transverse planes was suspected, the mechanism was never clarified, as the classical theory only predicts an emittance sharing.

Recently, based on theoretical considerations, the mechanism of emittance exchange has been predicted under certain circumstances (for both skew quadrupolar and longitudinal magnetic fields) [2]. This effect has been first verified in the PS-Booster (PSB), and then used for precise measurements of the horizontal emittance, which are hampered by the contribution of the momentum spread to the beam size through dispersion.

A simple theory of emittance sharing and exchange in the presence of linear betatron coupling due to skew quadrupoles has then been derived, which gives the evolution of the transverse emittances as functions of the coupling strength and the tune distance from the coupling resonance [3].

The classical and new formulae are given and discussed in Section 2. The experiments performed in the PS and PSB are then described and compared to theory in Section 3.

2 THEORY

2.1 Review of the Existing Theory

The classical formulae for the horizontal and vertical rms beam emittances in the presence of linear coupling are given by [4]

$$\varepsilon_x = \varepsilon_{x0} - \left(\varepsilon_{x0} - \varepsilon_{y0}\right) \frac{|C|^2 / 2}{\Delta^2 + |C|^2}, \qquad (1)$$

$$\varepsilon_y = \varepsilon_{y0} + \left(\varepsilon_{x0} - \varepsilon_{y0}\right) \frac{|C|^2 / 2}{\Delta^2 + |C|^2}. \qquad (2)$$

Here, $\varepsilon_{x0,y0}$ are the initial uncoupled horizontal and vertical rms emittances, given by

$$\varepsilon_{x0} = \frac{\sigma_{x0}^2}{\beta_{x0}}, \qquad \varepsilon_{y0} = \frac{\sigma_{y0}^2}{\beta_{y0}}, \qquad (3)$$

where $\sigma_{x0,y0}$ are the initial uncoupled transverse rms beam sizes, $\beta_{x0,y0}$ are the initial uncoupled transverse betatron functions, $|C|$ is the modulus of the general complex coupling coefficient (it is the tune difference on the coupling resonance), and Δ describes the tune distance from the second-order difference resonance. Near the resonance $Q_x - Q_y - l = 0$, where $Q_{x,y}$ are the transverse tunes and l is an integer, and considering only the coupling due to skew quadrupoles in the smooth approximation, Δ and $|C|$ are given by

$$\Delta = Q_y + l - Q_x, \quad |C| = \frac{R^2}{\sqrt{Q_0\left(Q_0 - l\right)}} \times \left|\underline{K}_0\right|, \quad (4)$$

where R is the average machine radius, $Q_0 \approx Q_x \approx Q_y + l$ and $\underline{K}_0 = \left(e / p_0\right)\left(\partial B_x / \partial x\right)$ is the average normalized skew gradient, with e the elementary charge, p_0 the design momentum and B_x the horizontal magnetic field.

It can be seen from Eqs. (1) and (2) that in the presence of coupling, the emittances are shared as coupling increases, the sum of the emittances being always conserved. A perfect sharing of the emittances is obtained only for "full coupling", i.e. on the resonance $\Delta = 0$ when $|C| \neq 0$.

2.2 New Formulae

New formulae have been derived in the smooth approximation, which describe the effect of linear coupling due to skew quadrupoles on the transverse emittances. As predicted by the classical formulae, the emittances are shared as coupling increases. However, as opposed to the classical formulae, the new equations also reveal the possibility to exchange the emittances as coupling decreases after the resonance crossing. These general formulae are given by [3]

$$\varepsilon_x = \varepsilon_{x0} - \left(\varepsilon_{x0} - \varepsilon_{y0}\right) \frac{\left|C\right|^2 / 2}{\Delta^2 + \left|C\right|^2 + \Delta\sqrt{\Delta^2 + \left|C\right|^2}}, \quad (5)$$

$$\varepsilon_y = \varepsilon_{y0} + \left(\varepsilon_{x0} - \varepsilon_{y0}\right) \frac{\left|C\right|^2 / 2}{\Delta^2 + \left|C\right|^2 + \Delta\sqrt{\Delta^2 + \left|C\right|^2}}. \quad (6)$$

Equations (5) and (6) are very similar to Eqs. (1) and (2): there is only the additional term $\Delta\,(\Delta^2 + \left|C\right|^2)^{1/2}$ in the denominator. This term is however very important, as it makes possible the exchange of transverse emittances after the resonance crossing.

From Eqs. (5) and (6) one sees that in the presence of very small coupling, i.e. $\Delta >> \left|C\right|$, the transverse emittances are given by

$$\varepsilon_x = \varepsilon_{x0}, \qquad \varepsilon_y = \varepsilon_{y0}. \quad (7)$$

As coupling increases, the sharing of the emittances increases and reaches its maximum value for full coupling, where the emittances are given by

$$\varepsilon_x = \varepsilon_y = \frac{\varepsilon_{x0} + \varepsilon_{y0}}{2}. \quad (8)$$

In the presence of very small coupling again, after the resonance crossing, i.e. $-\Delta >> \left|C\right|$, one has

$$\varepsilon_x = \varepsilon_{y0}, \qquad \varepsilon_y = \varepsilon_{x0}. \quad (9)$$

As can be seen from Eqs. (5) and (6), the difference between the two uncoupled emittances is transferred between the transverse planes through the following function

$$f\left(\left|C\right|, \Delta\right) = \frac{\left|C\right|^2 / 2}{\Delta^2 + \left|C\right|^2 + \Delta\sqrt{\Delta^2 + \left|C\right|^2}}, \quad (10)$$

which varies between 0 (initial uncoupled situation) and 1 (final uncoupled situation after the resonance crossing), and is equal to ½ on the coupling resonance. It is straightforward to verify that

$$\frac{\partial f}{\partial \Delta}\left(\left|C\right|, \Delta = 0\right) = -\frac{1}{2\left|C\right|}. \quad (11)$$

This implies that the smaller the coupling strength, the faster the emittance exchange with the tune distance from the coupling resonance (see Fig. 1). However, the smaller the coupling strength, the longer the time needed to cross the resonance. If $|C|$ is infinitely small, then an infinitely long time is needed to cross the resonance, for Eqs. (5) and (6) to be valid [2,3].

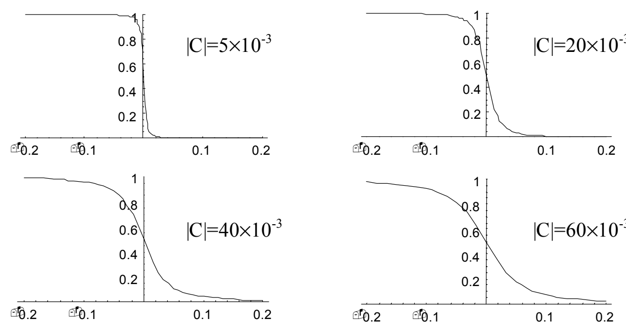

Figure 1: Plot of f as a function of the tune distance from the resonance Δ for different values of $|C|$.

3 EXPERIMENTS

3.1 In the CERN PS

The relation between the skew quadrupole current and the skew gradient of the PS at injection energy is given in Fig. 2 [5].

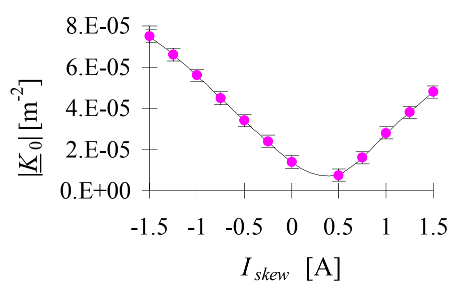

Figure 2: Modulus of the normalised skew gradient, as deduced from tune separation measurements, vs. skew quadrupole current for the PS at injection energy.

The measurements have been performed by programming the transverse tunes to slowly exchange their values within 100 ms on the injection flat-bottom. The measured normal mode tunes are shown in Fig. 3 for a particular coupling strength, where the closest-tune-approach is given by $\left|C\right|$. During the 100 ms needed to exchange the transverse tunes, the transverse emittances are measured several times with a Wire Scanner, and for different coupling strengths (see Fig. 4). As can be seen from Fig. 4, the horizontal and vertical emittances are shared until full coupling, where the emittances become equal. Then the emittances are exchanged in remarkable agreement with the theoretical predictions.

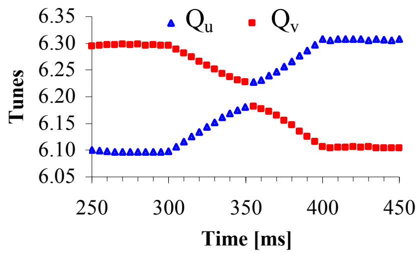

Figure 3: Measured normal mode tunes vs. time.

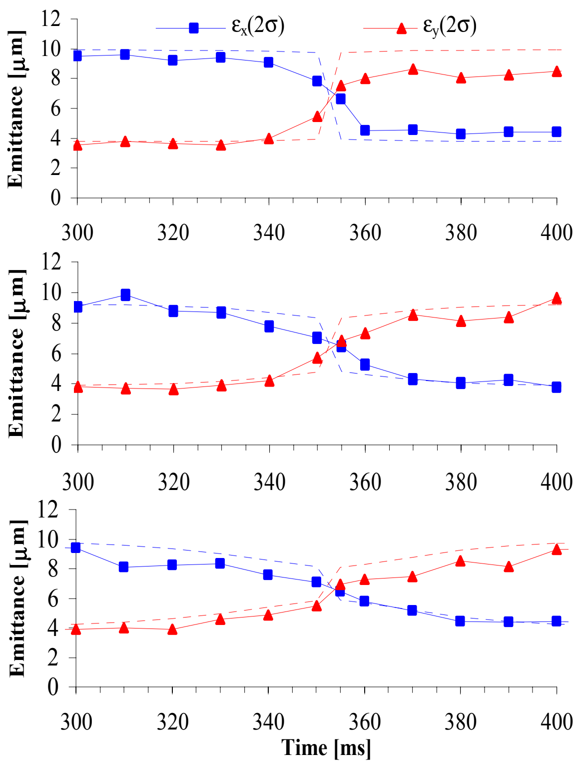

Figure 4: Transverse physical emittances vs. time for $I_{skew} \approx 0.1$ A (upper), $I_{skew} \approx -0.5$ A (centre), and $I_{skew} \approx -1.5$ A (lower). The dashed lines denote the theoretical values and the solid lines the measured ones.

3.2 In the CERN PS-Booster

For a proof of principle of the complete exchange of the emittances, the coupling resonance has been crossed several times on a long ejection flat-top, and the emittances have been measured after each crossing. Two campaigns of measurement have been carried out, the first with LHC-type beams, i.e. with small emittances and large momentum spreads, and the second with small momentum spread beams [2]. The transverse emittances have been measured using the three secondary emission monitors installed in a dedicated measurement line, and the BeamScope [6]. Here, to show the emittance exchange more explicitly, only the pictures with small momentum spread beams are presented (see Fig. 5). Furthermore, the measurements have been performed exploiting the weak coupling due to the magnet imperfections. No additional skew quadrupoles were used. As can be seen from Fig. 5, the horizontal and vertical emittances are clearly exchanged after each crossing, in agreement with the theoretical predictions.

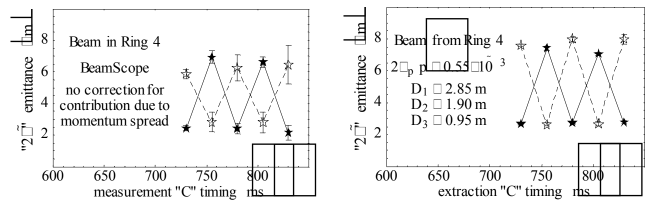

Figure 5: Emittance measurements while crossing four times the coupling resonance $Q_x - Q_y = -1$.

4 CONCLUSION

The new mechanism of emittance exchange has been predicted and described. New formulae have been derived in the smooth approximation, which describe the effect of linear coupling due to skew quadrupoles on the transverse emittances. As predicted by the classical formulae, the emittances are shared as coupling increases. However, as opposed to the classical formulae, the new equations also reveal the possibility to exchange the emittances as coupling decreases after the resonance crossing. This mechanism has been verified both in the CERN PS-Booster near the coupling resonance $Q_x - Q_y = -1$ and in the PS near the coupling resonance $Q_x - Q_y = 0$.

This effect may be of great interest in several cases, e.g. for beam transfer between two rings where the downstream machine is limited by the (vertical) aperture, as is the case between the PS and SPS for high-intensity beams. The mechanism of emittance exchange can also be used for precise measurements of the horizontal emittance, which are hampered by the contribution of the momentum spread (through dispersion) to the beam size. Furthermore, the results also mean that in theory, cooling is needed only in one plane.

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