SIMULATIONS OF COHERENT BEAM-BEAM MODES AT RHIC

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Abstract

A recent experiment at RHIC observed the coherent dipole modes for proton bunches in collision. The collective beam-beam effect in RHIC is simulated self-consistently in 4-D (transverse) phase space. Compared to standard Monte-Carlo methods, the sampling of the distribution tails is improved. The simulations and the experimental results are compared.

1 INTRODUCTION

The beam-beam interaction can give rise to coherent transverse dipole oscillation modes. In the simplest case with only one collision there are two modes per transverse plane. The σ-mode frequency is the same as the betatron frequency without beam-beam interaction, and the π-mode frequency is shifted downwards for particles of the same charge. Alexahin pointed out that the π-mode may not be Landau damped if the beam-beam interaction is the dominant source for tune spread and the beam intensities are approximately the same [1]. The tune spread from the beam-beam interactions is the beam-beam parameter ξ while the the π mode is shifted by $Y\xi$, where $Y \approx 1.3$ is the Yokoya factor [2]. Thus the π-mode frequency can be outside the continuous frequency spectrum.

RHIC is the only existing collider with hadron beams of equal intensities and therefore the only hadron machine where π-modes can be expected. Coherent beam-beam modes may also be relevant in future hadron colliders like the LHC and VLHC. σ- and π-modes were created in a RHIC experiment [3], and were also observed in routine operation with protons. With a beam-beam parameter ξ of up to 0.0025 in proton operation and four collisions, coherent modes have not limited the collider operation so far. However, it is expected that the beam-beam parameter can be increased by a factor of 3 in the future and π-modes may become a concern.

The simulations reported here were done with the same beam parameters as in the RHIC experiment that created the π-modes. The chromatic tune spread was found to be an essential ingredient to the analysis of the data. However, a complete treatment of synchro-betatron coupling is only possible in 6D phase space, which requires a large amount of CPU time. We consider the synchrotron motion only in a simplified way in our 4D weighted macro particle tracking code `BBDeMo2C`.

2 THE EXPERIMENT

The beam-beam experiment was aimed at creating and observing coherent beam-beam modes [3]. It was carried out with protons at the injection energy of 23.4 GeV, to allow for fast refills. The machine was prepared by checking and correcting the closed orbit, the injection conditions and the chromaticities (C_x, C_y). A relatively large linear coupling remained. The longitudinal distribution of the injected beam was smoothed to avoid coherent longitudinal oscillations, which are otherwise present for long periods of time. Collisions were set up at one interaction point (IP), using orbit monitors, and collision signals. In addition, the beam-beam tune shift could be observed with a phase locked loop tune measurement system with 10^{-5} tune resolution. At all other IPs the beams were separated vertically by at least 6 rms beam sizes. The tunes in both rings were set at almost the same horizontal and vertical tunes (Q_x, Q_y). The experimental conditions are summarized in Tab. 1.

Table 1: The RHIC parameters for both beams in the experiment. The same parameters are used in the simulations.

quantity	Blue	Yellow
(Q_x, Q_y)	(0.2129,0.2412)	(0.2126,0.2392)
$\lvert Q_x - Q_y\rvert_{\min}$	0.011	0.013
(C_x, C_y)	(2.0,2.0)	(3.0,3.0)
$(\Delta p/p)_{1\sigma}$	$2.7 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
Q_s	$3.7 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$
N_b	$0.84 \cdot 10^{11}$	$0.88 \cdot 10^{11}$
$\epsilon_{x,y}$, norm. 95%	20 μm	20 μm
beam-beam ξ	**0.003**	

For the measurement of coherent modes, a single bunch of protons was filled in each ring. Transverse spectra were obtained from up to 4096 turns recorded in a beam position monitor after the blue beam experienced a small kick by the tune kicker. The beam-beam interaction can be switched on and off by separating the beams longitudinally at the IP. In Fig. 1 horizontal spectra of the Blue beam are shown with and without beam-beam interaction. The π-mode created by the beam-beam interaction is clearly visible.

3 SIMULATIONS

We simulated the experiment using the 2 degree of freedom weighted macro particle tracking code `BBDeMo2C` which we derived from `BBDeMo2D` [4] by adding the effect of chromatic tune spread.

3.1 Weighted macro particle tracking

Weighted macro particle tracking (WMPT) [4] is a method for computing the time dependent averages of

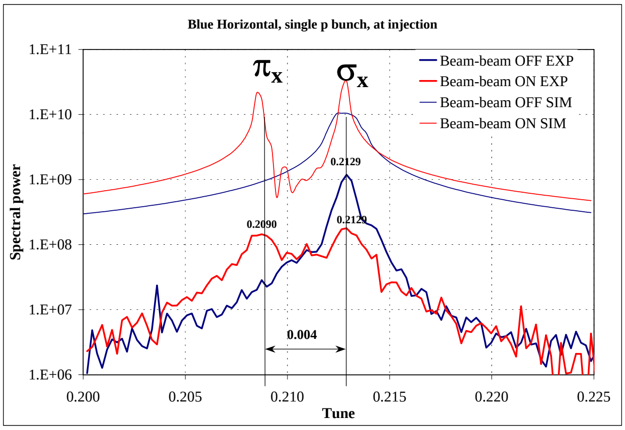

Figure 1: Transverse centroid spectra over the first 4096 turns after the kick from experimental and simulated data with beam-beam interaction on and off.

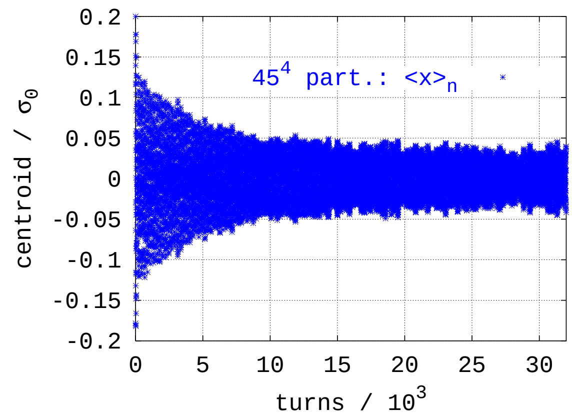

Figure 2: The simulated centroid motion of the Blue beam in time domain. See Tab.1 for parameters.

phase space functions, f, via

$$\langle f\rangle_n = \int_{\mathbb{R}^4} f(\vec{z})\Psi_n(\vec{z})\,d^4z$$
$$= \int_{\mathbb{R}^4} f(\vec{T}_{n-1}[\Psi^\star_{n-1}]\circ\ldots\circ\vec{T}_0[\Psi^\star_0](\vec{z}))\Psi_0(\vec{z})\,d^4z, \quad (1)$$

where $\vec{T}_n[\Psi^\star_n]$ is the symplectic 4D map from turn n to $n+1$ which depends explicitly on the density of the *other* beam $\Psi^\star_n$ just before the IP at turn n. Quantities with a star-superscript represent properties of the *other* beam. Given an *initial* 4D grid $\{\vec{z}_{\vec{\imath}}\}$, $\vec{\imath} := (i,j,k,l)$, $1 \le i,j,k,l \le m$, the initial density on the grid $\psi_{\vec{\imath}} := \Psi_0(\vec{z}_{\vec{\imath}})$ and a quadrature formula with weights $w_{\vec{\imath}}$, then after tracking the m^4 initial grid points to $\vec{Z}_{\vec{\imath}}(n) = \vec{T}_{n-1}[\Psi^\star_{n-1}]\circ\ldots\circ\vec{T}_0[\Psi^\star_0](\vec{z}_{\vec{\imath}})$ we obtain

$$\langle f\rangle_n \approx \sum_{\vec{\imath}} f(\vec{Z}_{\vec{\imath}}(n))\;\psi_{\vec{\imath}}\,w_{\vec{\imath}}\;. \quad (2)$$

Note that (2) samples the core and the tail regions of the distribution identically and thus can follow higher moments better than a typical Monte Carlo (*equal weight*) macro particle tracking code, which mainly samples the beam core.

3.2 The ring model

Our model of the 4D transverse one turn map includes the linear lattice with coupling, represented by the linear map $\vec{M}$, an explicitly time dependent kick map $\vec{C}_n$ for the leading order chromatic effects, and a collective kick map $\vec{K}[\Psi]$ for the strong-strong beam-beam interaction

$$\vec{z}_{n+1} = \vec{T}_n[\Psi^\star_n](\vec{z}_n)\;\;,\;\vec{T}_n[\Psi^\star] = \vec{M}\circ\vec{C}_n\circ\vec{K}[\Psi^\star] \quad (3)$$
$$\vec{z}^\star_{n+1} = \vec{T}^\star_n[\Psi_n](\vec{z}^\star_n)\;\;,\;\vec{T}^\star_n[\Psi] = \vec{M}^\star\circ\vec{C}^\star_n\circ\vec{K}^\star[\Psi]\;. \quad (4)$$

The linear part of the lattice, describing the particle motion between beam-beam interactions, is represented by $\vec{M}(\vec{z}) = \underline{R}^{1/2}\underline{S}\,\underline{R}^{1/2}\,\vec{z}$. The Courant-Snyder parameters at the IP define the uncoupled linear lattice $\underline{R} \equiv \underline{R}^{1/2}\underline{R}^{1/2}$, and the minimum attainable tune split $|Q_x - Q_y|_{\min}$ defines the matrix $\underline{S}$ that is inserted to provide linear coupling.

Synchrotron motion in conjunction with non-zero chromaticity leads to tune modulation, which is introduced in the following way. Each 4D trajectory $\vec{Z}_{\vec{\imath}}$ is assigned a random longitudinal amplitude $A_{\vec{\imath}}$ and a random longitudinal phase $\phi_{\vec{\imath}}$ at start time ($n=0$), corresponding to the longitudinal distribution of the particles in the bunch. At each turn n the relative momentum deviation $\delta := \Delta p/p$ is computed as $\delta_{\vec{\imath}}(n) = A_{\vec{\imath}}\cos(2\pi n Q_s + \phi_{\vec{\imath}})$, where Q_s is the synchrotron tune. A quadrupole-type kick $\vec{C}_n$, due to non-zero chromaticity, is applied in each plane and for each trajectory by $p_{\vec{\imath},u} \mapsto p_{\vec{\imath},u} + c_u\,\delta_{\vec{\imath}}(n)\,q_{\vec{\imath},u}$, where $u = x, y$. Here $c_u = 4\pi C_u/\beta_u$ where C_u is the chromaticity. This procedure modulates the transverse tunes (Q_x, Q_y) of each trajectory with the synchrotron tune Q_s and an amplitude $C_u A_{\vec{\imath}}$. It generates chromatic tune spreads $C_u\delta^{\rm rms}$ in both bunches, where $\eta^{\rm rms}$ is the rms momentum spread over the bunch.

The beam-beam kick due to the phase space density $\Psi^\star$ of the collision partner is represented by the kick-map $\vec{K}[\Psi^\star]$:

$$p_u \mapsto p_u + K_u[\Psi^\star](x,y)\;,\;(u = x,y) \quad (5)$$
$$K_u[\Psi^\star](x,y) \;\propto\; \int_{\mathbb{R}^4}\partial_u G(x-x',y-y')\Psi^\star(\vec{z}')\,d^4z' \quad (6)$$
$$G(x,y) \;=\; \frac{1}{2}\ln(x^2+y^2)\;. \quad (7)$$

Note that (6) is a convolution of $\vec{\nabla}G$ and $\Psi^\star_n$ which can be written as a phase space average $\langle\vec{\nabla}G(x-\cdot,y-\cdot)\rangle^\star_n$ as in Eq. (1). However, in BBDeMo2C the collective kick is not computed using (2), which would be $O(m^8)$ in computational cost, but instead by using the *Hybrid Fast Multipole Method* [4, 5], which only costs $O(m^4)$ flops.

3.3 Results of the simulations

The simulations were performed with $45^4 \approx 4\cdot10^6$ or $61^4 \approx 14\cdot10^6$ macro particles per bunch. The macro particles were initially placed on a square grid ($\vec{z}_{\vec{\imath}}$) covering $5\sigma_0$ in each phase space dimension and the initial beam density ($\psi_{\vec{\imath}}$) was assumed to be Gaussian and matched to the linear *un*-coupled lattice (round beams). The excitation through the tune kicker was simulated by giving the Blue beam an initial horizontal coherent betatron amplitude of $0.2\sigma_0$.

Using the beam parameters of the experiment, the location of the π-mode in tune space is reproduced (Fig. 1). From both experiment and simulation the Yokoya factor [2] Y is approximately 1.3. Because of the chromatic tune spread the dipole modes decohere in the simulation (Fig. 2), whereas without chromaticity the mode amplitudes would be basically stationary [4]. However, the decoherence observed in the experiment is faster (and the tune peaks are wider), which suggests the presence of additional sources of tune spread. Presumably the lattice nonlinearities, mainly from the interaction region triplets, add significantly to the tune spread.

Figure 3 shows the evolution of the horizontal emittances of the Blue beam (initially excited) and the Yellow beam (initially at rest). The total simulated emittance growth in the 16,000 turns is about 0.5%. Most of it stems from the first few hundred turns due to the initial mismatch. Then the emittances increase slowly because of the filamentation of the coherent dipole mode and finally seem to reach a (quasi)-equilibrium value. Of course, the diagnostics cannot resolve differences in emittance to these small scales and over such a short time period.

The *kurtosis* $\kappa_x := \langle (x - \langle x \rangle)^4 \rangle$ of a distribution is a measure for the content of the tails w.r.t. the core. For a Gaussian distribution in x with rms width σ the kurtosis is $3\sigma^4$. WMPT accurately models the tails and Fig. 4 shows that the tails of the distribution were not populated during the simulation over the time period simulated.

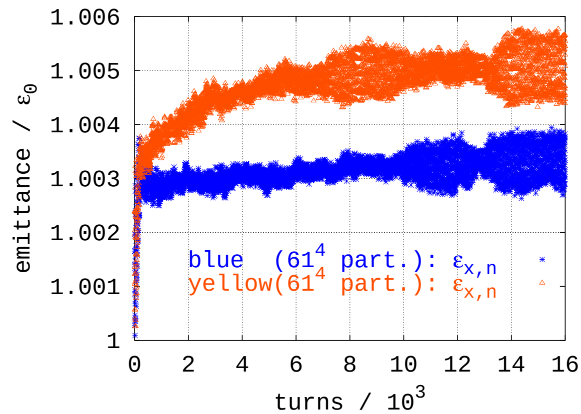

Figure 3: The simulated evolution of the emittance in time domain for the Blue and the Yellow (here:orange) beam.

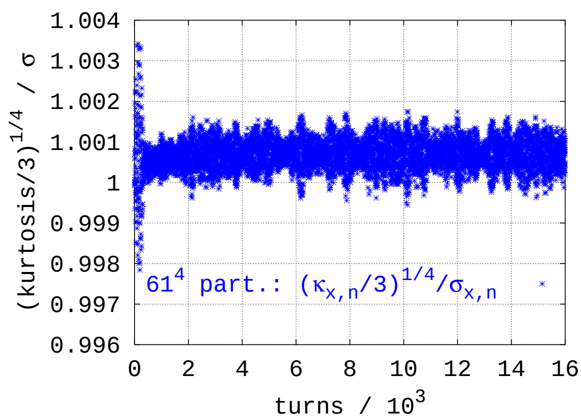

Figure 4: The simulated evolution of the kurtosis in time domain.

4 SUMMARY

Coherent beam-beam modes were observed in an experiment at RHIC. The calculation of the π-mode tune using WMPT was in good agreement. The simulations also show that the dipole modes decohere when the tune spread is increased by chromatic effects. This decoherence was also observed in the experiment, although on a faster time scale. Over the time scale of the simulation (32,000 turns) no significant emittance increase was observed. The WMPT method also allows the calculation of higher moments such as the kurtosis and this showed no halo build up.

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6 REFERENCES

[1] Y. Alexahin, “On the Landau Damping and Decoherence of Transverse Dipole Oscillations in Colliding Beams”, *Part. Accel.*,**V59**, p. 43; CERN-SL-96-064 (AP) (1996).

[2] R. E. Meller and R. H. Siemann, *IEEE Trans. Nucl. Sci.*, **NS-28**, No.3, pp.2431, (1981);
K. Yokoya, Y. Funakoshi, E. Kikutani, H. Koiso, J. Urakawa, “Tune Shift of Coherent Beam-Beam Oscillations”, p. 40, KEK Preprint 89-14 (1989).

[3] W. Fischer, L. Ahrens, M. Bai, M. Blaskiewicz, P. Cameron, R. Michnoff, F. Pilat, V. Ptitsyn, T. Sen, D. Trbojevic, and M. Vogt, “Observation of Coherent Beam-Beam Modes in RHIC”, BNL C-AD/AP note, to be published (2002).

[4] M. Vogt, T. Sen and J. A. Ellison, *Phys. Rev. ST Accel. Beams* **5**, 024401, (2002); FNAL Pub-01/096-T;
M.Vogt, J.A.Ellison, T.Sen and R.L.Warnock, *Proc.: Beam-Beam Workshop at Fermilab*, Ed.: T. Sen and M. Xiao; FERMILAB-Conf-01/390-T, pp 149, (2001)

[5] L. Greengard, *Thesis*, The MIT Press Cambridge MA London England (1988);
F. W. Jones, *Proc.: Workshop on Space Charge Physics in High Intensity Hadron Rings*, Ed.: A. U. Luccio and W. T. Weng, AIP Conf. Proc. **448**, (1998);
W. Herr and M.-P. Zorzano, *Phys. Rev. ST Accel. Beams* **4** 054402 (2001)