

CURE FOR THE TRANSVERSE INSTABILITY DURING ELECTRON-COOLING BUNCHING WITH SKEW QUADRUPOLE MAGNETS

M. Takanaka, RIKEN, Japan

Abstract

A simulation study has predicted that during electron-cooling bunching a beam meets the transverse instability whose source is toroids of an electron-cooling device. We propose a cure for the instability with skew quadrupole magnets. We check in a multiparticle tracking simulation if the cure is effective.

1 INTRODUCTION

The Radioisotope Beam Factory (RIBF) [1] will have an electron-RI beam collider (e-RI Collider). In previous works [2], in a multiparticle tracking simulation we studied ion-beam electron-cooling (EC) bunching to prepare the required ion beam at the e-RI collider, and predicted that the beam meets the transverse instability whose source is the toroids of the EC device and that the beam can be stabilized using a transverse feedback of the bandwidth 6 GHz.

We make description about emittance behavior under the resonance due to the toroids and resonance control using skew quadrupole magnets (Q's) in Section 2. We present the simulation results to show how the resonance control is effective for curing the instability in Section 3.

The simulation program has been improved so that the longitudinal space-charge force is calculated using not only the monopole longitudinal space-charge impedance but also the dipole and the quadrupole one. Using 40,000 macroparticles, we simulate the EC bunching of $^{238}\text{U}^{92+}$ ion beams of 4 mA, or of 5.4×10^6 ions per bunch at 150 MeV/u.

2 RESONANCE

We describe the resonance due to toroid fields of an EC device and the resonance control using skew Q's.

2.1 Amplitude and Phase equation

For the case where the EC device is aligned horizontally, the main component of the toroid field that ions experience in the toroid section is the horizontal one which is integrated along the longitudinal direction (or s axis) to be approximately [2]

$$B_0 x_t (\log r_t - \log x_t) + B_0 x,$$

where the normal field is B_0 , the toroid radial center is x_t from the s axis, and the radius of the curvature of the outside inner wall is r_t . The first term expresses the effect of bending beams vertically and the second term the effect of twisting the beams.

The bending effect can be lessened using steering magnets. We try to curb the twisting effect using skew Q's. The amplitude and the phase equation are given as follows when toroids and skew Q's are treated as thin lenses;

$$\begin{aligned} \frac{d\epsilon_{x,y}}{d\theta} &= -2Q_{x,y}\epsilon_{x,y}^{\frac{1}{2}} \sum_i \beta_{x,y}^{\frac{3}{2}} g_{x,y}^i \sin \Phi_{x,y}, \\ \frac{d\Phi_{x,y}}{d\theta} &= Q_{x,y} - Q_{x,y}\epsilon_{x,y}^{-\frac{1}{2}} \sum_i \beta_{x,y}^{\frac{3}{2}} g_{x,y}^i \cos \Phi_{x,y}, \\ g_{x,y}^i &= \frac{K_{y,x}^i l^i}{Q_{x,y} \beta_{x,y}^i} \delta(\theta - \theta_{x,y}^i), \\ &= \frac{K_{y,x}^i l^i}{Q_{x,y} \beta_{x,y}^i} \frac{1}{2\pi} \left(1 + 2 \sum_n \cos n(\theta - \theta_{x,y}^i) \right) y, x, \\ x, y &= \sqrt{\epsilon_{x,y} \beta_{x,y}} \cos \Phi_{x,y}, \end{aligned}$$

for toroids

$$K_x^i l^i = \pm \frac{B_0}{p/e}, \quad K_y^i l^i = 0,$$

for skew Q's

$$K_{y,x}^i l^i = \frac{\partial B_y^i}{\partial x} l^i / \frac{p}{e},$$

where the above equation description is based on the normalized transverse phase spaces [3], the suffix i is for toroids and skew Q's, $\phi_{x,y}$ being an offset $\Phi_{x,y} = Q_{x,y}\theta + \phi_{x,y}$ is the phase on the phase space, and x, y or y, x means that it has to be x or y on the same side in respect to the commas in an equation.

When we consider only main contribution in the above differential equations to the slowly-varying terms or the $Q_x + Q_y = n$ resonance terms, the equations can be rewritten as

$$\begin{aligned} \frac{d\epsilon_{x,y}}{d\theta} &\approx -\frac{1}{2\pi} \sqrt{\epsilon_x \epsilon_y} \sum_i \sqrt{\beta_x^i \beta_y^i} K_{y,x}^i l^i \sin(\Phi + n\theta_{x,y}^i) \\ &= -\frac{1}{2\pi} \sqrt{\epsilon_x \epsilon_y} A_{x,y} \sin(\Phi + n\theta_{x,y}^{eff}), \\ \frac{d\Phi_{x,y}}{d\theta} &\approx Q_{x,y} - \frac{1}{4\pi} \sqrt{\frac{\epsilon_{y,x}}{\epsilon_{x,y}}} \sum_i \sqrt{\beta_x^i \beta_y^i} K_{y,x}^i l^i \cos(\Phi + n\theta_{x,y}^i) \\ &= Q_{x,y} - \frac{1}{4\pi} \sqrt{\frac{\epsilon_{y,x}}{\epsilon_{x,y}}} A_{x,y} \cos(\Phi + n\theta_{x,y}^{eff}), \\ A_{x,y}^2 &\equiv \left(\sum_i \sqrt{\beta_x^i \beta_y^i} K_{y,x}^i l^i \cos n\theta_{x,y}^i \right)^2 \\ &\quad + \left(\sum_i \sqrt{\beta_x^i \beta_y^i} K_{y,x}^i l^i \sin n\theta_{x,y}^i \right)^2, \end{aligned}$$

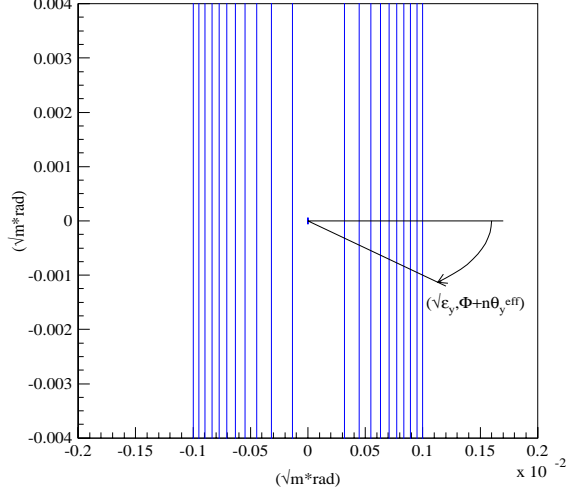


Figure 1: Resonant invariant for the emittance-increasing resonance due to the toroids in the initial condition that $\epsilon_y \leq 1 \times 10^{-6}$ m \times rad and $Q_x + Q_y = n$.

$$\sin n\theta_{x,y}^{eff} \equiv \frac{\sum_i \sqrt{\beta_x^i \beta_y^i} K_{y,x}^i l^i \sin n\theta_{x,y}^i}{A_{x,y}},$$

$$\Phi = \Phi_x + \Phi_y - n\theta.$$

2.2 Resonance due to the toroids

We consider the case where only the toroids work;

$$\frac{d\epsilon_x}{d\theta} = 0, \quad \frac{d\Phi_x}{d\theta} = Q_x,$$

$$\frac{d\epsilon_y}{d\theta} = -\frac{1}{2\pi} \sqrt{\epsilon_x \epsilon_y} A_{y\ tor}^{eff} \sin(\Phi + n\theta_{y\ tor}^{eff}),$$

$$\frac{d\Phi_y}{d\theta} = Q_y - \frac{1}{4\pi} \sqrt{\frac{\epsilon_x}{\epsilon_y}} A_{y\ tor}^{eff} \cos(\Phi + n\theta_{y\ tor}^{eff}).$$

From the above equations we derive the following resonant invariants;

$$\epsilon_x = C,$$

$$(Q_x + Q_y - n)\epsilon_y - \frac{1}{2\pi} \sqrt{\epsilon_x \epsilon_y} A_{y\ tor}^{eff} \cos(\Phi + n\theta_{y\ tor}^{eff}) = C.$$

The above equations show that the beam can be trapped into the $Q_x + Q_y = n$ resonance, as shown in Fig. 1. The type of the resonance is emittance-increasing one. The half stopband width is defined as $\sqrt{\epsilon_x / \epsilon_y} A_{y\ tor}^{eff} / 4\pi$.

2.3 Resonance control

θ_x^i is nearly equal to θ_y^i especially in the simulation model where we use the simplified lattice that consists of a constantly focusing section and a drift EC section. A_y includes terms of toroids and skew Q's, while A_x includes only terms of skew Q's. If we make A_y zero using the skew Q's, A_x becomes $A_{y\ tor}^{eff}$. We never damp down the resonance completely. We can, however, make

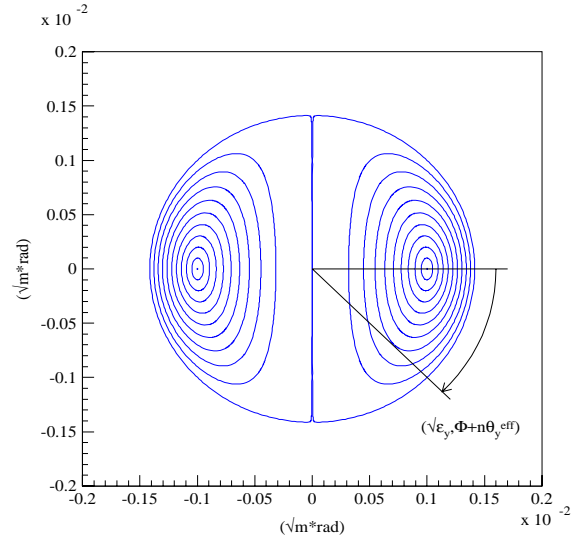


Figure 2: Resonant invariant for the emittance-beating resonance controlled using skew Q's in the initial condition that $\epsilon_x + \epsilon_y = 2 \times 10^{-6}$ m \times rad and $Q_x + Q_y = n$.

$A_x = A_y \approx A_{y\ tor}^{eff} / 2$ and $n\theta_x^{eff} - n\theta_y^{eff} = \pi$. In the case, we derive

$$\epsilon_x + \epsilon_y = \epsilon = C,$$

$$(Q_x + Q_y - n)\epsilon_y - \frac{1}{2\pi} \sqrt{(\epsilon - \epsilon_y)\epsilon_y} A_y \cos(\Phi + n\theta_y^{eff}) = C.$$

The above equations show that the beam can be trapped into the $Q_x + Q_y = n$ resonance, as shown in Fig. 2. The type of the resonance is emittance-beating one.

We expect that the instability during the bunching is cured by changing the type of the $Q_x + Q_y = n$ resonance from the emittance-increasing type to emittance-beating one using skew Q's.

3 SIMULATION RESULTS

We show two kinds of simulation results without and with the resonance control for comparison sake.

3.1 No resonance control

The evolution of the bunching is shown in Fig. 3. The bunching process is as follows. First, a coasting beam is cooled to the sixfold-rms momentum spread of 5×10^{-4} . Then, the fundamental RF voltage is increased in such a way that the momentum spread measured at the EC section is maintained at 5×10^{-4} . The RF voltage increase stops when the sixfold-rms bunch length reaches 1 m or 1/3 of the bunch spacing. When the momentum spread reaches 1.9×10^{-4} or most of the beam stays within 1m, the third-harmonic RF voltage is increased while maintaining the momentum spread.

The bunch length gets 260 (or 160) mm under the third-harmonic RF voltage 82 (or 300) kV at 32 (or 50) ms. The beam gets unstable transversely after 30 ms and is lost because of hitting the chamber wall. The source of

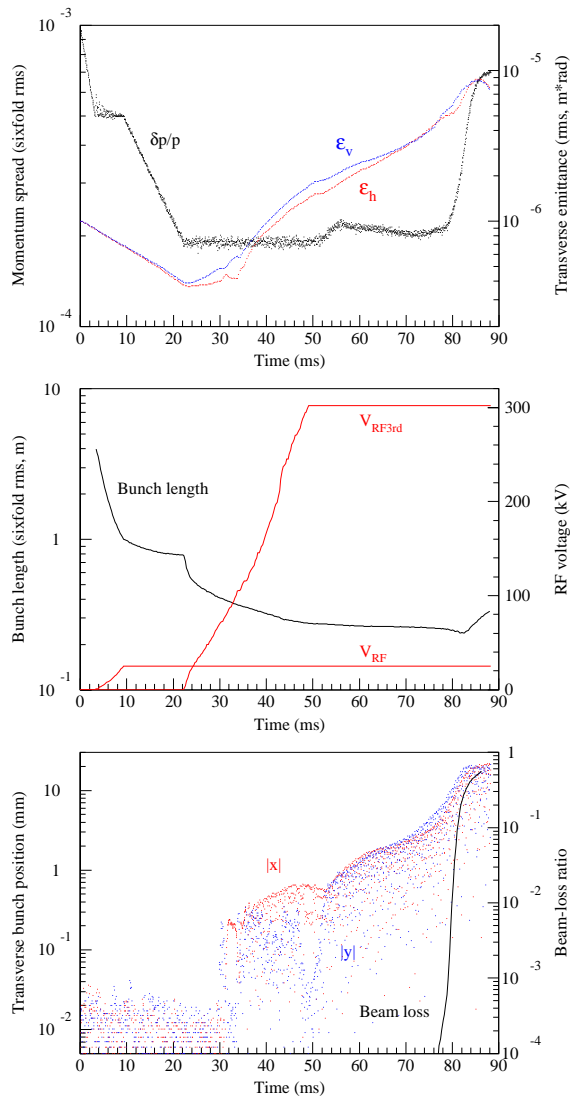


Figure 3: Evolution of the EC bunching without the resonance control.

the instability is the toroids at the EC section that drive the $Q_x + Q_y = 8$ resonance with the half stopband width $\Delta(Q_x + Q_y) = 0.005$, the bear tunes being $Q_{x0} = 4.425$ and $Q_{y0} = 3.725$,

3.2 Resonance control

We use skew Q 's to realize the emittance-beating type of the $Q_x + Q_y = n = 8$ resonance and the zero stopband width for $Q_x - Q_y = 0$. We estimate θ_x^i and θ_y^i at $Q_x + Q_y = n$ on the assumption that Q_{x0} or Q_{y0} is shifted to $n/(Q_{x0} + Q_{y0})$ of it by the space-charge effects. In the current simulation we do not take care of stopband widths of the other resonances; after the resonance control, for example, the width of the emittance-increasing $Q_x - Q_y = 1$ resonance is 0.001. The evolution of the bunching is shown in Fig. 4.

The bunch length gets 107 mm under the third harmonic RF voltage 870 kV at 74 ms. The momentum spread starts

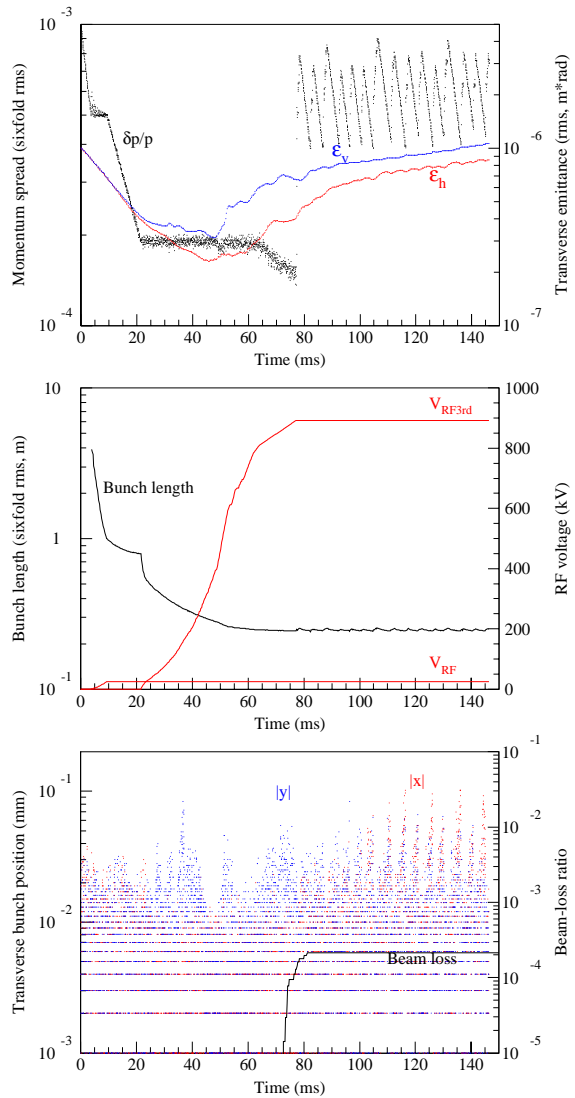


Figure 4: Evolution of the EC bunching with the resonance control.

to decrease at 61 ms, because the RF voltage-ramping rate is decreased at the time. The beam gets unstable longitudinally at 78 ms. The instability occurs around the same RF voltage when the rate is not decreased, too.

4 CONCLUSIONS

It has been proved in the multiparticle tracking simulation that the proposed cure for the transverse instability during EC bunching is effective. The much-bunched beam meets the longitudinal instability beyond the transverse instability.

5 REFERENCES

- [1] Y. Yano et al., Proc. PAC2001, 575.
- [2] M. Takanaka, Proc. PAC2001, 3897.
M. Takanaka, RIKEN Accel. Prog. Rep., Vol. 35, 312 (2002).
- [3] L.R. Evans, CERN 84-15, 319 (1984).