STUDY OF ACCEPTANCE OF FFAG ACCELERATOR

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Abstract

The aim of this study is to establish the generalized procedure to design a FFAG accelerator having large transverse acceptance. Due to the large momentum and transverse acceptance, it is considered that the FFAG accelerator is quite appropriate for a phase rotator or a secondary particle accelerator [1]. Some analytical ways and tracking simulations were performed to study the problem of non-linear motion in FFAG accelerator.

1 INTRODUCTION

The guiding field of a FFAG accelerator satisfies the zero chromaticity condition with higher order multipole components. In other words, the betatron tune is independent of momentum and the chromatic aberration does not have to be considered. The feature immediately makes the FFAG advantageous as a phase rotator or a secondary particle accelerator as long as large transverse acceptance is ensured.

In a FFAG accelerator, the acceptance is mainly determined nearby resonance. Although a bare tune is carefully chosen, as the particle amplitude becomes large, the amplitude dependent tune cannot avoid one of the resonances. Then, all we have to do for obtaining the large acceptance is to investigate the behaviour of tune shift under different machine parameter regime.

2 HORIZONTAL ACCEPTANCE

2.1 The Relation between the Horizontal Acceptance and the Phase Advance

In order to survey the relation between the horizontal acceptance and the phase advance per cell, first the tracking simulations with two-dimensional field of “hard edge model” was performed. Figure 1 shows the results based on a ring with realistic radius for the periodicity 16 and 32. As shown in figure 1, the horizontal acceptance more than 10,000π mm-mrad. can be obtained with choosing the phase advance carefully.

The acceptance depends strongly upon the phase advance. Two common features are identified. One is that the acceptance becomes smaller as increasing the phase advance. The other is that the acceptance becomes small rapidly around the structure resonance corresponding to the phase advance of 2π/3, 2π/4, 2π/5.

The first feature is explained by the effect of large k value. The k value is main knob of the horizontal phase advance; the phase advance becomes larger as increasing the k value. As shown in Equation 1, the large k value enhances the higher order components of the guiding field, which causes the tune shift. Here, B_z show the vertical component of the field, B_0 is the field strength at the radius r_0 and k is the k value. In this study, positive k value is assumed.

$$B_z = B_0\left(\frac{r}{r_0}\right)^k = B_0 + B_0\frac{k}{r_0}x + B_0\frac{k(k-1)}{2!r_0}x^2 + \cdots \quad (1)$$

$(Taylor\ Expansion\ \ around\ \ r_0, r = r_0 + x)$

Second feature is obvious and no explanation is necessary. However, the direction of tune shift becomes a main concern. This will be discussed later.

In order to obtain the large horizontal acceptance, it is preferable low phase advance; however, the k value should be kept sufficiently large in order to keep the orbit excursion modest value. The orbit excursion is determined by equation 2. Here, A is the ratio of extraction momentum to injection one.

$$orbit\ excursion = r_{inj}\,(A^{\frac{1}{k+1}} - 1) \quad (2)$$

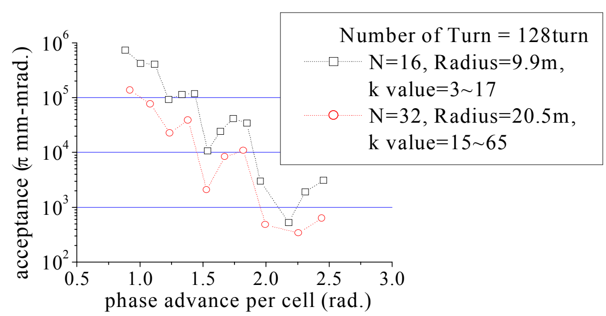


FIGURE 1. Acceptance vs. Phase Advance

2.2 Tune Shift in Horizontal Motion

When the phase advance per cell is 2π/3, the direction of tune shift is dominated by the leading term of the sextupole. According to the perturbation theory, the tune shift is provided by equations 3 and 4 [3].

$$\left\langle V_{J1} G_\phi \right\rangle = -\frac{J_1^{\,2}}{64C}\int_0^C ds\beta(s)^{3/2}S(s)\int_s^{s+C}\beta(s')S(s')ds'$$
$$\times\left\{\frac{3\cos(\psi(s')-\psi(s)-\pi\nu)}{\sin\pi\nu}+\frac{\cos 3(\psi(s')-\psi(s)-\pi\nu)}{\sin 3\pi\nu}\right\} \quad (3)$$

$$\Delta\nu = \frac{C}{2\pi}\frac{\partial\left\langle V_{J1} G_\phi\right\rangle}{\partial J_1} \quad (4)$$

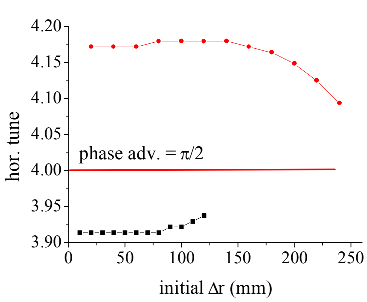
Here, V : *perturbation term of hamiltonian*
G : *generating function*
J_1 : *action variable (canonical transformed with G)*
ϕ : *angle valiable*
C : *circumference*
$\psi(s)$: *phase advance between zero and s*
$\nu(s)$: *tune*
$\langle\ \rangle$: *averaing over ϕ and s*

In equation 3, the term of $1/\sin(3\pi\nu)$ is dominant, when the phase advance is near 2π/3. The direction of the tune shift is determined whether the phase advance is above or below 2π/3. Unfortunately, the coefficient of negative sextupole component in the focusing field has larger absolute value than the positive one of in defocusing field. Therefore, the tune is always approaching the 2π/3 resonance.

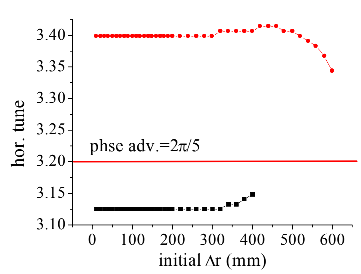
When the phase advance per cell is around 2π/4, the problem gets some complex. The direction of the first order tune shift caused by the octupole is one-way as shown in equations 5 and 6.

$$\begin{aligned} H &= J/\beta(s) - \frac{O(s)}{4} J^2 \beta^2(s) \cos^4\phi \\ &= J/\beta(s) - \frac{O(s)}{4} J^2 \beta(s)^2 \left(\frac{3}{8} + \frac{1}{2}\cos 2\phi + \frac{1}{8}\cos 4\phi\right) \\ &\equiv J/\beta(s) + V(\phi, J, s) \end{aligned} \tag{5}$$

$$\begin{aligned} \Delta\nu &= \int_0^C \frac{\partial V(\phi, J, s)}{\partial J} ds = -2J \int_0^C O(s)\beta(s)\left(\frac{3}{8} + \frac{1}{2}\cos 2\phi + \frac{1}{8}\cos 4\phi\right) ds \\ &\cong -\frac{3}{4} J \int_0^C O(s)\beta(s) ds \end{aligned} \tag{6}$$



(a) around π/2



(b) around 2π/5

FIGURE 2. Horizontal Tune Shift

However, the simulation results shown in figure 2 indicate that the direction is not always one-way. It seems that not only octupole component but also sextupole or another components should be taken into account so as to analyse the amplitude dependence of tune. The results corresponding to the case of 2π/5 is also shown in figure 2.

3 VERTICAL ACCEPTANCE

3.1 Tune Shift in Vertical Motion

In order to study the vertical motion, it is essential to introduce the three-dimensional field. The edge focusing is sometimes even stronger than the vertical focusing of the main field. The field including the fringing field can be expressed with equations from 7 to 9 [2]. Here, $E^{(n)}(\theta)$ represent the shape of the fringing field and its derivatives. Although it includes the fringing field, the zero chromaticity condition is still satisfied.

$$\begin{aligned} B_r(r,\theta,z) &= E\frac{kB_0}{r_0}\left(\frac{r}{r_0}\right)^{k-1} z \\ &\quad - \frac{1}{3!}(Ek^2 + E^{(2)})\frac{(k-2)B_0}{r_0^{\,3}}\left(\frac{r}{r_0}\right)^{k-3} z^3 + \cdots \end{aligned} \tag{7}$$

$$\begin{aligned} B_\theta(r,\theta,z) &= E^{(1)}\frac{B_0}{r_0}\left(\frac{r}{r_0}\right)^{k-1} z \\ &\quad - \frac{1}{3!}(E^{(1)}k^2 + E^{(3)})\frac{B_0}{r_0^{\,3}}\left(\frac{r}{r_0}\right)^{k-3} z^3 + \cdots \end{aligned} \tag{8}$$

$$\begin{aligned} B_z(r,\theta,z) &= EB_0\left(\frac{r}{r_0}\right)^{k} \\ &\quad - \frac{1}{2!}(Ek^2 + E^{(2)})\frac{B_0}{r_0^{\,2}}\left(\frac{r}{r_0}\right)^{k-2} z^2 \cdots \end{aligned} \tag{9}$$

It is considered that the leading term of the vertical tune shift is the normal octupole component. The effect of higher order components more than octupole is weak, unless the vertical tune is very close to the value affected these components.

Assuming that the edge focus at the focusing field is main vertical focusing force, the octupole component of azimuthal field is extracted from equation 8 as shown in equation 10.

$$B_{\theta oct} = -\frac{1}{3!}(E^{(1)}k^2 + E^{(3)})\frac{B_0}{r_0^{\,3}} z^3 \tag{10}$$

With figure 3 and equation 11, the sign of the octupole component is fixed to negative.

$$\begin{aligned} B_{\perp oct} &= -\frac{1}{3!}(E^{(1)}k^2 + E^{(3)})\frac{B_0}{r_0^{\,3}} z^3 \sin\beta \\ &\cong -\frac{E^{(1)}k^2}{3!}\frac{B_0}{r_0^{\,3}} z^3 \sin\beta = O(s) z^3 \\ &(\because O(s) < 0 \ @\ Focus) \end{aligned} \tag{11}$$

Then, the direction of the tune shift can be fixed with the equation 6 and is always upward. According to the tracking simulation with the field provided by equations from 7 to 9, that is verified as shown in figure 4. Figure 4(a) shows the direction of the vertical tune shift for various bare tunes. As a result, the vertical bare tune should not be set a little below the strong resonance line.

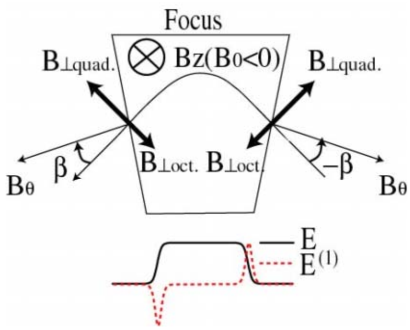
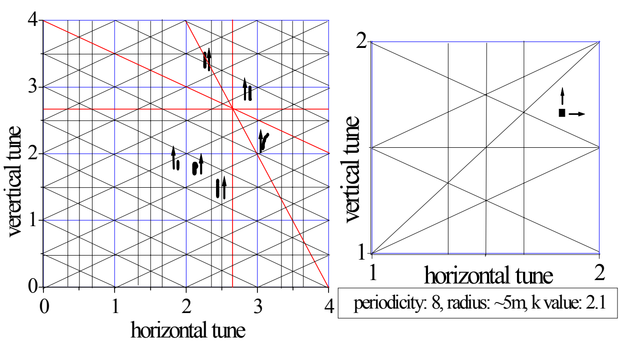


FIGURE 3. Octupole Component in Edge Focus



(a) Vertical Tune Shift (b) Tune Selection

FIGURE 4. Vertical Tune Shift and Tune Selection

3.2 Vertical Acceptance and x-y Coupling

In the case of a phase rotator or a secondary particle accelerator, the number of turn is very low, typically ten turn or so. On the other hand, the large transverse acceptance, an order of a few or ten thousand π mm-mrad., is demanded. Using the field with three dimensional field calculation code, the tracking simulation of the large emittance beam was performed.

As shown in previous sections, it is very important to choose the bare tunes carefully. Figure 4(b) shows an example of the selected tunes, taking into account up to fourth order normal resonance. There are sufficient separation above and right side.

The injected particles have the same momentum and various pair of action (Jx,Jy). One hundred particles with various initial phases were injected in each pair of action. Figure 5 shows the surviving rate of each point. The physical aperture is set to be sufficient for the beam having the emittance 20,000π and 3,000π mm-mrad. in horizontal and vertical respectively. Here, the number of turn is set 5 turns with the application to the muon phase rotator in mind. Figure 6 shows the integrated surviving rate of various emittances assuming the uniform distribution in the emittance space. When the beam emittance was 10,000π and 3,000π mm-mrad., the surviving rate more than 90 % can be achieved.

As shown in figure 5, the surviving rate can be sufficiently kept high as long as one-dimensional motion is considered. However, it drops as increasing two actions. It is considered to be the effect of the x-y coupling.

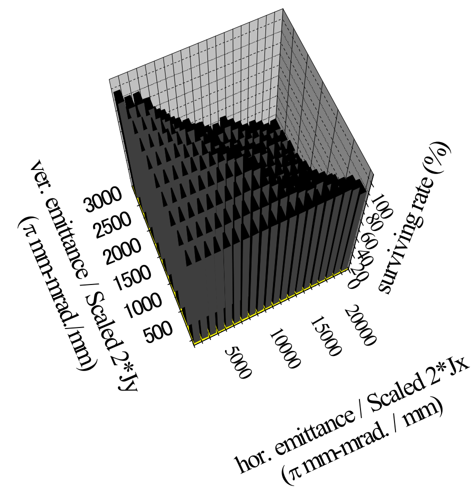


FIGURE 5. Action Dependence of Surviving Rate

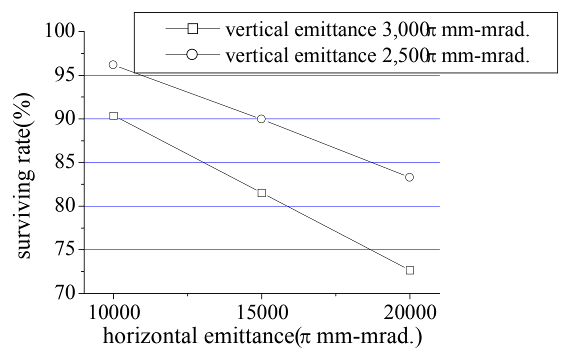


FIGURE 6. Integrated Surviving Rate

4 SUMMARY OF THE STUDY

In order to obtain the generalized procedure to design a FFAG accelerator having large transverse acceptance, the transverse motion in FFAG accelerator is studied including the non-linear terms. The direction of the tune shift is analysed in the various cases. As a result, naturally, the tunes should be chosen carefully, especially the vertical tune should not be a little below strong resonance lines. With careful tune selection, it is achieved to design a FFAG ring having the acceptance of 10.000π and 3.000π mm-mrad. in horizontal and vertical.

5 REFERENCE

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[3] R. D. Ruth, ‘SINGLE-PARTICLE DYNAMICS IN CIRCULAR ACCELERATORS’, AIP153, p152-p235