FIXED FIELD ALTERNATING GRADIENT LATTICE DESIGN WITHOUT OPPOSITE BEND*

D. Trbojevic, M. Blaskiewicz, and E. D. Courant, BNL, Upton, N.Y. 11973, USA
A. Garren, LBL, Berkeley, CA, USA

Abstract

This report presents an attempt of the lattice design with a fixed field alternating gradient (FFAG) magnets without the usual opposite bends. It should allow particle acceleration through a small aperture. An example was made for the muon beam acceleration in an energy range 10-20 GeV with distributed RF cavities. The dispersion function for the central energy of 15 GeV has maximum value of the order of 7 cm. The lattice is composed of a combined function elements and sextupoles. We present the magnet configuration, orbit, chromaticities, tunes, and betatron function dependence on momentum (energies) during acceleration. For the lattice design we used SYNCH an MAD programs. For these large momentum offsets δp/p=+-33% we found discrepancies between analytical and codes' results. This will be corrected in the new versions of codes (MAD-X). Because of uncertainties of the programs MAD and SYNCH some details of the presented results might not be correct.

1 INTRODUCTION

A lattice design of the FFAG, a relatively old concept developed in 1954 by K. R. Symon [1], has started to get the attention again. Advantages of the FFAG design are the fixed magnetic field, large range of particle energy, simple RF; power supplies are simple, and there is no transition energy. But drawbacks are an increase in size due to reverse bending magnets, and a large aperture size. The larger size increases the cost of the ring. Recently some modified FFAG lattice designs have been described at KEK where the amount of opposite bending was significantly reduced [2]. This FFAG design does not have opposite bends, the required aperture size is significantly reduced, and a difference in the path length for different energies should be small enough to allow easy RF acceleration. The energy range of a lattice example is ΔE=+-50%. The presented results were obtained by using the lattice design codes MAD8 [3] and SYNCH [4]. This extremely large acceptance of the momentum range immediately raises questions of the codes accuracy. We are presently studying results by using a simplified example and comparing analytical with the codes' results. There are clear indications the codes will need corrections. These first results are going to be checked with other additional lattice codes. The basic concept and results of an example will be presented in the next few sections. It is important to note that if this concept would reduce dramatically the cost of the acceleration for the muon neutrino factory or for the muon collider.

2 SELECTION OF A LATTICE

A goal is set to provide a lattice with very small dispersion (the maximum size of D_{max}~6-7 cm) together with small horizontal betatron functions ($\beta_{x\ max}$~4 m) to allow the large momentum acceptance. Limitations on the dispersion were set as an arbitrary aperture size of +-35 mm. This limitation is presented by the inequality between the aperture and the dispersion function:

$$\Delta x = D\frac{\delta p}{p}\ ;\ D \le \frac{\pm 35mm}{\pm 0.5} \le 70\,mm \qquad (1)$$

The dispersion function D oscillates within a well defined limits defined by the H function:

$$H = \xi^2 + \zeta^2,\ \xi = D'\sqrt{\beta} + \frac{\alpha \cdot D}{\beta},\ \zeta = \frac{D}{\sqrt{\beta}}. \qquad (2)$$

A choice for the lattice is *"the minimum emittance lattice"* we presented earlier [5], because it provides the minimum of the H function. The betatron functions within two basic cells are presented in figure1.

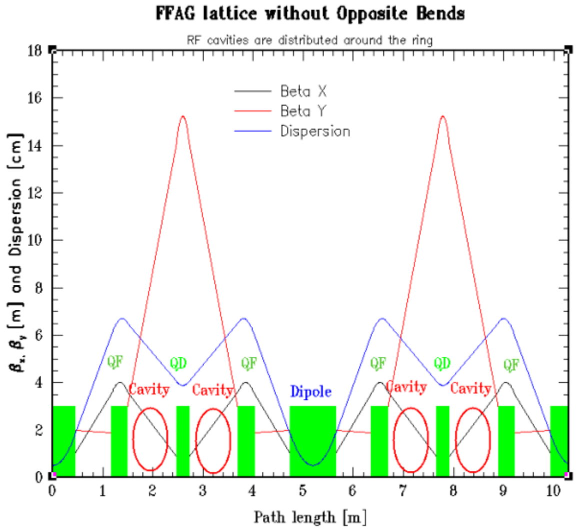

Figure 1. Betatron functions within two basic cells. Dispersion function is in cm.

The defocusing quadrupole *QD* and a dipole are combined function elements. The sextupoles are placed at the centers of the quadrupoles. They correct the chromaticity to zero at the central energy of 15 GeV. A drift space between the focusing quadrupole QF a combine function quadrupole QD (it has a dipole with a field of B=1.6 T).

3 LATTICE PROPERTIES

During acceleration dependence of the tunes on momentum offsets is presented in figure 2. The horizontal and vertical betatron tunes change during acceleration, as presented in fig. 2., but because particles pass only twenty turns the resonance conditions would not occur and the beam will not be lost.

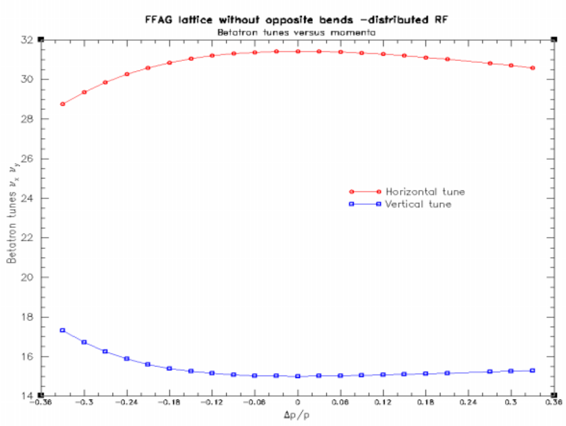

Figure 2. Horizontal and vertical betatron tunes dependence on momentum.

The chromaticity and momentum compaction dependence on momentum are presented in figure 3 and figure 4, respectively. The maximum of the horizontal and vertical betatron functions through acceleration is presented in figure 5.

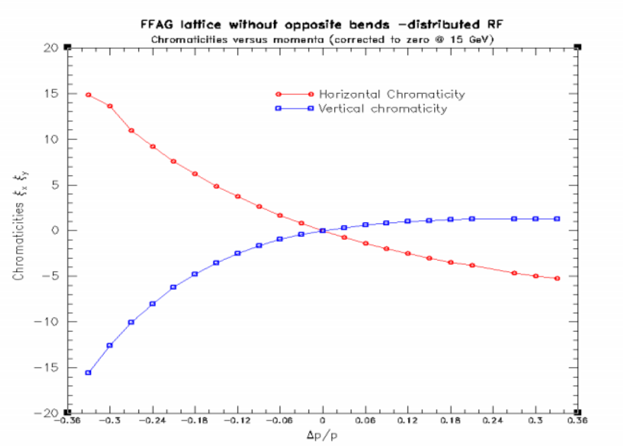

Figure 3. Chromaticity dependence on momentum.

The momentum compaction changes a sign as particles are accelerated. Bellow the negative momentum offset of dp/p<–0.09 the momentum compaction is smaller than zero (the lattice has imaginary γ_t). The RF slipping factor η is also crosses a value of zero during acceleration. The dispersion function dependence on momentum shows that the maximum does not exceed values above 7.6 cm through the whole acceleration. Maximum of the dispersion function dependence on momentum is presented in figure 6. The orbit offsets in the horizontal plane during acceleration are presented in fig. 6. The path length during acceleration is presented in figure 7.

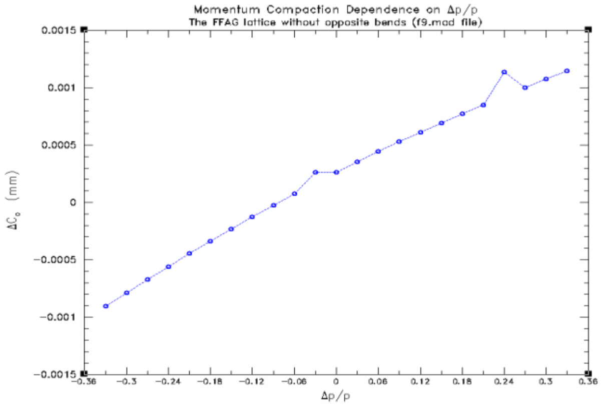

Figure 4. The momentum compaction dependence on momentum.

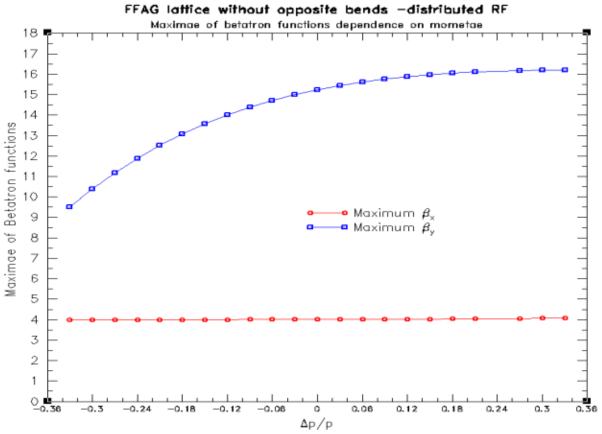

Figure 5. Maximum of the horizontal and vertical betatron functions during acceleration.

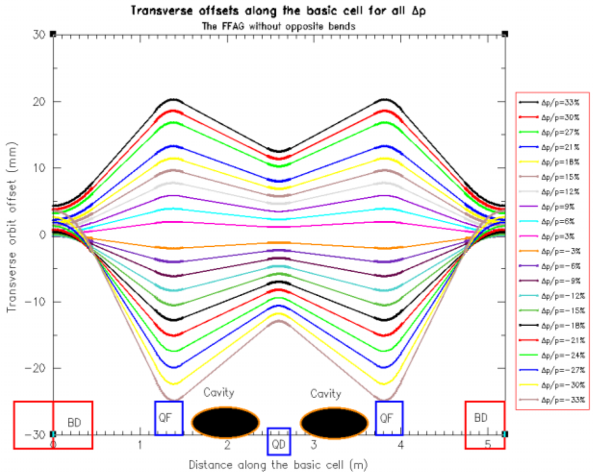

Figure 6. Orbit offsets in the horizontal plane during acceleration.

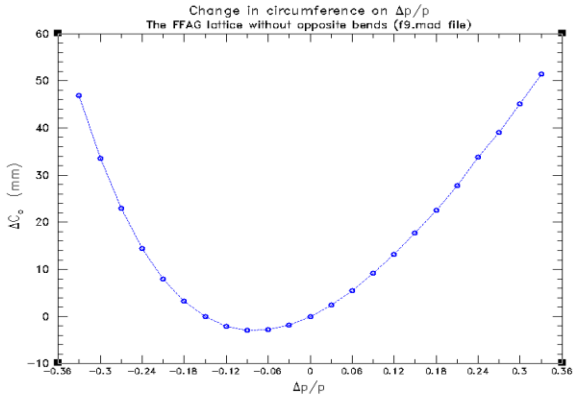

Figure 7. Difference in path length dependence on momentum.

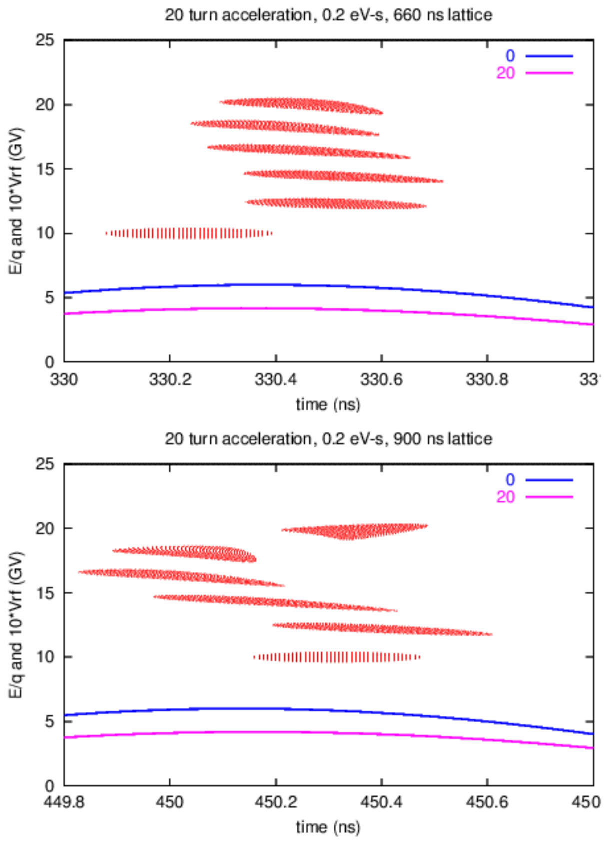

Figure 8. Twenty turn acceleration for two examples of the FFAG lattices.

The frequency of the RF cavities is 200 MHz. The 1-D update equations are:

$$\tau_{n+1}=\tau_n+T(E_n) \tag{3}$$

$$\left(\frac{R}{Q}\right)I(t)=\frac{1}{\omega_{rf}}\frac{dV(t)}{dt}+\omega_{rf}\int_0^t dt_1 V(t_t) \tag{4}$$

$$E_{n+1}=E_n+qV(\tau_{n+1}), \tag{5}$$

where *I(t)* smoothed by 0.5 ps and *V(t)* updated with Δt=0.15 ps. For 6.25 10^{12} muons the total charge is 1 μC. Assuming a factor of two voltage drop, the initial stored energy in the RF cavities is *U* = 10 GV x 1 μC x 4/3 = 13 kJ. Taking a total voltage of 500 MV and the *rf* frequency of ω_{rf}= 2π x 200MHz, one obtains *(R/Q)*=7.6 kΩ. The simulations used this impedance and V= 600 MV so the voltage dropped to 400 MV at the end of the cycle. Taking an average acceleration of 10 MV/m per cavity the requisite *R/Q* per cavity is 126 Ω. The stored energy per cavity is 300 J. For *E*=10 MV/m the volume is 0.7 m^3. The average gradient was assumed to be 10 GeV/m, and the cavity length is ~75 cm, while a drift length for the whole cavity is ~ 1 meter long. The circumferences of the ring in two examples are 198 m and 270 m.

4 SUMMARY

An attempt for a new FFAG lattice design without opposite bends is presented. Muon acceleration from 10 GeV to 20 GeV is being considered in the presented example. The orbit offsets where smaller than +-30 mm. Sixty RF cavities are distributed around the ring. Promising results with very small change of circumference for a very large energy-momentum range were obtained by the lattice design codes MAD and SYNCH. Due to extremely large momentum offsets, results of the codes are being carefully analyzed. Analytical results in a simpler example with very large momentum range have shown disagreement with the results of the two lattice design codes. With corrections of codes' errors we hope the basic concept would still provide similar results.

5 REFERENCES

[1] K. R. Symon, "The FFAG Synchrotron – MARK I", MURA-KRS-6, November 12, 1954, pp. 1-19.

[2] R. Ueno et al, "Multi Orbit Synchrotron with FFAG Focusing For Acceleration of High Intensities Hadron Beams", PAC 99, New York, 1999, 2271-2273.

[3] F. C. Iselin and H. Grote, MAD, "Methodical Accelerator Design", CERN/SL, 90-13 (12991).

[4] A. Garren, A. S. Kenney, E. D. Courant, A. D. Russell, and M. J. Syphers, SYNCH-A Program for Design and Analysis of Synchrotrons and Beam Lines, User's Guide 1993.

[5] Trbojevic, D., Courant, E., Low emittance lattices for electron storage rings revisited, 4th European Particle Accelerator Conference, London, England, June 27-July 1, 1994, pp. 1000-1002.