Adiabatic Capture of Charged Particles in Islands of Phase Space: a New Method for Multi-Turn Extraction

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Abstract

A new method for multi-turn extraction from a circular particle accelerator is presented. It is based on adiabatic capture of particles into islands of transverse phase space generated by non-linear resonances. By appropriate use of non-linear elements, such as sextupoles and octupoles, stable islands can be created at small amplitude. By inducing an appropriate slow variation of the linear tune, particles can be captured inside these islands. This allows the beam to be split into different smaller beamlets in the transverse phase space. These beamlets can be finally transported towards higher-amplitudes to prepare them for extraction, performed by means of kicker and septum magnets. Results of numerical simulations are presented and discussed.

1 INTRODUCTION

Charged particles can be extracted from a circular machine by fast extraction (using fast dipole and septum magnets) or by slow extraction (based on the effect of a third-order resonance [1, 2]).

In some special cases, an intermediate extraction mode, called multi-turn extraction, is needed. This is the case of the transfer between the CERN Proton Synchrotron (PS) and the Super Proton Synchrotron (SPS), whose circumferences satisfy $C_{SPS} = 11C_{PS}$. Hence, to fill the SPS completely one would require ten fast-extracted pulses from the PS (the empty gap in the SPS is needed for the rise-time of the injection kicker). If the filling time has to be minimised, then the solution consists of extracting the beam over a few PS turns, instead of a single one. Such an approach is called Continuous Transfer (CT) [3]. Just before extraction, the horizontal tune Q_H is set to the value 6.25 and the closed orbit is modified so that the blade of an electrostatic septum intercepts the beam. Because of the value of the horizontal tune, four slices are shaved off the main core and extracted as a continuous ribbon over four turns. The central part is extracted last, during the fifth turn, by changing the beam trajectory so as to jump over the septum blade. In addition to the septum used to slice the beam, a kicker and a magnetic septum are used for the extraction. Although this approach allows reducing the horizontal emittance of the extracted beam, there are drawbacks with the CT: beam losses, especially at the electrostatic septum, are an intrinsic and unavoidable characteristic of this extraction process; and the extracted slices do not match the natural foliation of phase space into circles, thus generating betatronic mismatch.

2 THE NEW EXTRACTION METHOD

2.1 The principle

A novel approach was proposed, based on the use of stable nonlinear resonances [4]. In this scheme, nonlinear elements such as sextupoles and octupoles are used to generate stable islands in transverse phase space. By varying the horizontal tune, particles can be selectively trapped in the islands by adiabatic capture: some will remain in the phase space area around the origin, while others will migrate to the stable islands. As a result, the beam is split into a number of parts in transverse phase space, determined by the order of the resonance used, without any mechanical action. Finally, the separation between the islands can be increased so that enough room is available for the beam to jump over a septum blade with almost no particles lost.

2.2 The model

A simple model representing the horizontal betatron motion (the motion in the vertical plane can be safely neglected) in a circular machine under the influence of sextupole and octupole magnets was used. By assuming that the nonlinear magnets are located at the same place, and are represented in the single-kick approximation [5], the one-turn transfer map can be expressed as $\mathbf{x}_{n+1} = \mathbf{M}_n(\mathbf{x}_n)$:

$$\begin{pmatrix} x_{n+1} \\ x'_{n+1} \end{pmatrix} = R(2\,\pi\,\nu_n) \begin{pmatrix} x_n \\ x'_n + x^2_n + \kappa\, x^3_n \end{pmatrix}, \qquad (1)$$

where (x, x') are obtained from the Courant-Snyder coordinates (X, X') by means of the non-symplectic transformations [5]

$$(x, x') = \frac{K_2\, \beta_H^{3/2}}{2}\ (X, X') \qquad K_l = \frac{L}{B_0\,\rho}\,\frac{\partial^l B_y}{\partial x^l}, \quad (2)$$

K_2 (K_3) being the integrated sextupole (octupole) gradient, L the length of the nonlinear element, B_y the vertical component of the magnetic field, $B_0\,\rho$ the magnetic rigidity, and β_H the value of the horizontal beta-function at the location of the nonlinear elements. $R(2\,\pi\,\nu_n)$ is a 2×2 rotation matrix of angle ν_n, the fractional part of Q_H, and κ is expressed as

$$\kappa = \frac{2}{3}\,\frac{K_3}{\beta_H\ K_2^2}. \qquad (3)$$

The map (1) is a time-dependent system through the linear tune. The importance of introducing a time-dependence is twofold. Firstly, it allows varying the phase space topology, thus creating and moving the islands. Secondly, it allows trapping particles inside the islands, which is the necessary condition for the proposed scheme to work efficiently. A slow variation of the linear tune, adiabatic with respect to the time scale introduced by the betatron oscillations, allows particles to cross the separatrix and to be trapped inside the newly-created islands.

3 NUMERICAL RESULTS

3.1 Fourth-Order Resonance

The various phase space topologies for this novel extraction are shown in Fig. 1, evaluated by the computer code *GIOTTO* [6], where the case of a stable fourth-order resonance is shown. Each orbit consists of a set of points generated by using the map (1) where, for this special purpose, the linear tune has been kept constant, and $\kappa = -1.6$.

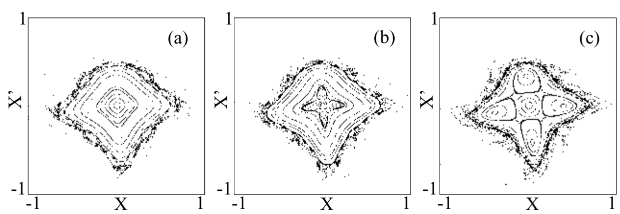

Figure 1: Phase space portrait of the map (1) for $\nu = 0.252$ (a), $\nu = 0.2492$ (b), and for $\nu = 0.2453$ (c). Each orbit consists of 2×10^3 points.

In Fig. 1 (a), the phase portrait for $\nu = 0.252$ is shown. The phase space is foliated in closed curves and no chain of islands is visible, apart from an eleventh-order one near the dynamic aperture. This configuration corresponds to the initial state where the beam is located around the origin and the dynamics is quasi-linear.

In Fig. 1 (b) the topology of the phase space is shown for $\nu = 0.2492$. A chain of four islands, corresponding to a stable fourth-order resonance, is clearly visible near the origin. At higher amplitude, the phase space is again foliated into closed curves until the last regular curve that corresponds to the border of the dynamic aperture. Under these conditions, it is possible to split the beam into five slices: one around the origin, and four inside the stable islands. The closed curve outside the chain acts as a barrier, preventing particles from moving towards higher amplitudes.

Finally, Fig. 1 (c) represents the phase space topology for $\nu = 0.2453$. The four islands are still present, but their amplitude is increased. This means that particles trapped inside islands can be transported towards the outside of phase space.

The trapping process has been simulated by using the model (1) with $\kappa = -1.6$, while the tune ν_n is varied according to the curve shown in Fig. 2. In the first part, the linear tune is decreased linearly from its initial value of 0.252 to 0.249. During this part, the capture process takes place. Then a zero-slope part follows, used to allow the beam to filament after capture, to match better the phase space topology. Then, a second linear decrease to the value 0.245 is performed which allows the islands to be moved towards higher amplitudes before extraction.

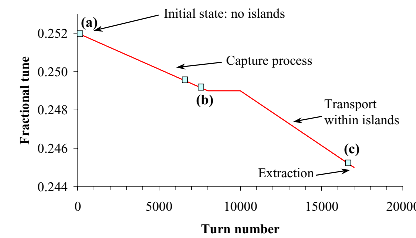

Figure 2: Linear tune ν as a function of n for the fourth-order resonance. The points on the curve labelled with (**a**), (**b**), and (**c**) correspond to the values of ν_n used in Fig 1.

A set of Gaussian-distributed initial conditions has been generated, and its evolution under the dynamics induced by the map (1) is shown in Fig. 3. The trapping process is clearly visible in the picture: it generates five beam slices, well-separated at the end of the process. No particle is lost during the trapping phase, nor when the islands are moved. The five slices have rather similar surfaces, but also their

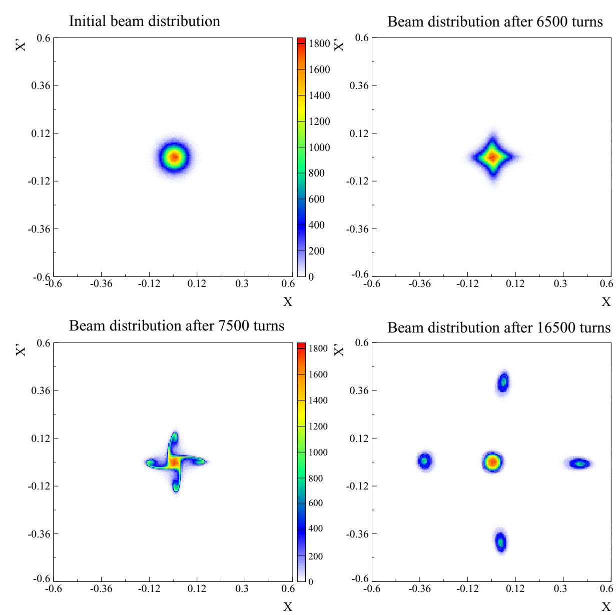

Figure 3: Evolution of the beam distribution during the trapping process with four islands. The different plots correspond to tune values represented by solid squares in Fig. 2. Each plot represents 4.9×10^5 points. The initial Gaussian distribution is centred on zero and has $\sigma = 0.045$.

shape matches the phase space topology very well, making the five parts similar as far as transverse properties are concerned. It is worthwhile pointing out that the first four extracted beam slices have exactly the same emittance, as their shape is dictated by the same phase space structure, i.e. the island along the positive x axis. In this respect, the novel approach proves to be superior to the present CT extraction mode.

The distribution functions $\rho(x)$ for the different beam slices at the end of the trapping and transport process are shown in Fig. 4. They are all Gaussian-like, as is the initial beam distribution, thus showing that the shape is almost preserved throughout the whole process.

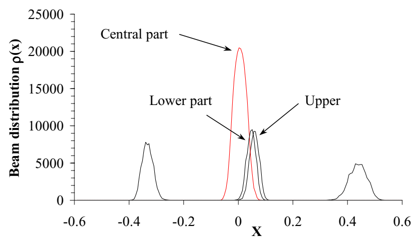

Figure 4: Beam distribution function $\rho(x)$ at the end of the capture and transport process, for all five beam slices shown in Fig 3 (lower right).

3.2 Third-Order Resonance

The proposed approach could work also for other resonances. As a proof-of-principle, the third-order resonance has been chosen. This resonance is inherently unstable [5], hence no closed curves surround the origin of phase space. This makes the adiabatic capture much more difficult, due to the risk of particle loss. In Fig. 5, the linear tune as a function of n is shown. In this case, to ensure a good

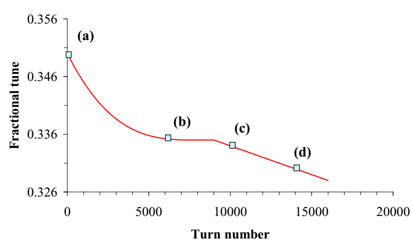

Figure 5: Linear tune ν as a function of n for the third-order resonance. The points on the curve labelled with (**a**), (**b**), (**c**), and (**d**) correspond to the values of ν_n used in Fig 6.

trapping, a polynomial dependence of order three has been selected, instead of the plain linear decrease used for the fourth-order resonance.

The various steps of the trapping process are shown in Fig 6 obtained with the system (1) and $\kappa = -5$. In spite of the different topology, three well-separated slices can be found at the end of the process. No losses occurs during the whole process and the region around the origin is almost emptied, apart from very few particles. Finally, the distribution $\rho(x)$ is shown in Fig. 7.

4 CONCLUSIONS

Numerical simulations seem to indicate that a multi-turn extraction can be envisaged by trapping particles in stable islands of transverse phase space. It is worthwhile stressing that such an approach could also used for multi-turn injection. A number of issues need further investigations, namely the quantitative relationship between the transverse emittance of the different slices and the island parameters,

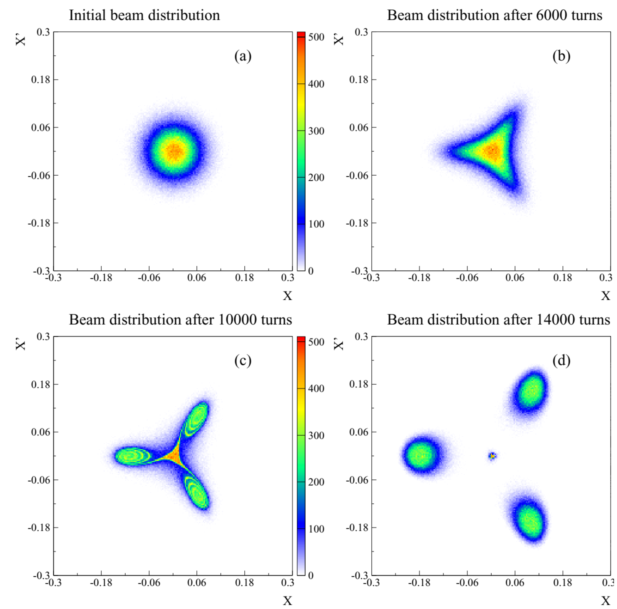

Figure 6: Evolution of the beam distribution during the trapping process with three islands. Each plot represents 4.9×10^5 points. The initial Gaussian distribution is centred on zero and has $\sigma = 0.045$.

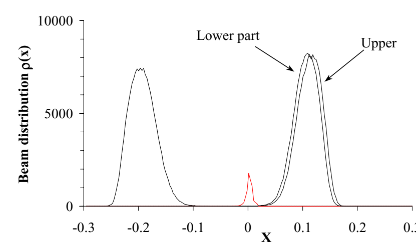

Figure 7: Beam distribution function $\rho(x)$ at the end of the capture and transport process, for all three beam slices shown in Fig. 6 (lower right).

the link between the way ν_n is varied, and the trapping efficiency. Also the influence of space charge forces should be estimated. Finally, the stability of this approach, by introducing full 4D betatron motion as well as chromaticity effects, should be checked in detail.

Experiments using the PS proton beam are already planned to verify the practical feasibility of the proposed extraction method.

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