MODELLING NONLINEAR OPTICS IN THE CERN SPS

A. Faus-Golfe, IFIC, Valencia, Spain;
G. Arduini, P. Collier, F. Zimmermann, CERN, Geneva, Switzerland

Abstract

Nonlinear fields arising from eddy currents in the vacuum chamber and remanent fields in the magnets of the CERN SPS vary with time and with the acceleration cycle. We describe a procedure of constructing a nonlinear optics model for the SPS, by considering sextupolar, octupolar, and decapolar field errors in the dipole and quadrupole magnets, respectively, whose strengths are adjusted so as to best reproduce the measured nonlinear chromaticities up to third order in the momentum deviation. Applying this procedure to SPS chromaticity measurements taken at 26 GeV/c, we have obtained a refined optics model. The tune shifts with the transverse amplitude predicted by this model are consistent with a direct measurement.

1 INTRODUCTION

The nonlinear chromaticity and amplitude-dependent betatron tune shifts in the CERN SPS were measured during a series of machine studies in the years 2000 and 2001 [1, 2]. This study was motivated by two observations: (1) The measured detuning with amplitude significantly varied from year-to-year and also after changes to the SPS supercycle. These often required re-optimization of sextupoles and Landau-damping octupoles. (2) Measurements of the nonlinear chromaticity revealed a large cubic component.

The changes were attributed to time-dependent variations of the remanent fields in the SPS dipoles and possibly quadrupoles. The SPS accommodates two types of dipoles, called MBB and MBA with different chamber heights and aspect ratio (the full width of MBA chambers is 152 mm, and the full height 34.5 mm; for the MBB chambers the corresponding numbers are 129 mm, and 48.5 mm).

We developed a procedure where we fit normal sextupole and decapole errors b_3 and b_5 to the two types of dipole magnets and octupole field components b_4 to the focusing and defocusing quadrupoles so as to reproduce the measured nonlinear chromaticities through third order. The nonlinear SPS model so obtained can be used, *e.g.*, to predict the detuning with amplitude, whose direct measurement probes the validity of the model.

2 MEASUREMENTS

Optics data were taken with a 48-bunch LHC-type beam on October 18–20, 2000, and July 17–19, 2001. The number of protons per bunch varied from $1.1 - 1.8 \times 10^{10}$. The synchrotron tune was about 250 Hz (2 MV rf voltage at 200 MHz) and the Landau damping octupoles were switched off, unless noted otherwise.

2.1 Chromaticity

Typical measurements of horizontal and vertical chromaticity are shown in Figs. 1 and 2. The beam momentum was varied by changing the rf frequency. Its change $\Delta\delta$ is related to the average shift in the horizontal closed orbit ΔR [m] via $\Delta\delta \approx 0.4\Delta R$. Note that the δ ($\equiv \Delta p/p$) values in the experimental curves were inferred from the shift in the horizontal BPM readings. The discrete sampling of the orbit at the BPMs gives rise to a correction factor 0.8 with respect to the average orbit change, which is included on the horizontal axis.

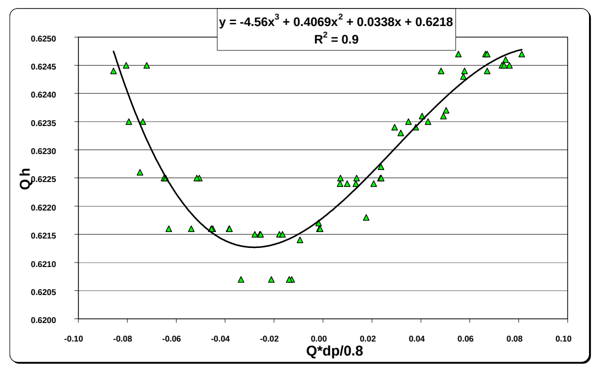

Figure 1: Measured horizontal tune versus product of momentum offset and tune, $Q\delta$, on 19th July 2001 (series 2).

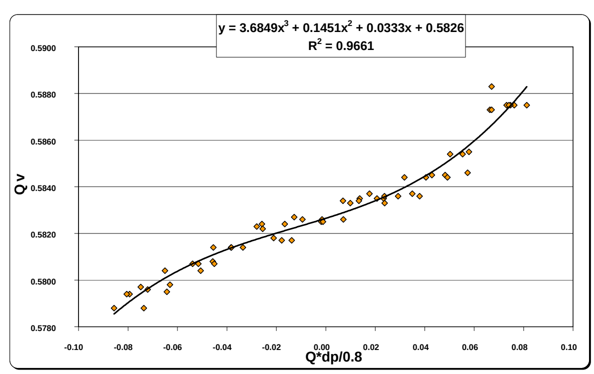

Figure 2: Measured vertical tune versus product of momentum offset and tune, $Q\delta$, on 19th July 2001 (series 2).

The measurement reveals a significant third-order chromaticity. From a fit of the measured Q_x and Q_y vs. $\delta p/p$, we obtain the linear, quadratic and cubic components of the two chromaticities, as

$$Q_{x,y} = Q_{0x,y} + Q'_{x,y}\delta + \frac{Q''_{x,y}}{2}\delta^2 + \frac{Q'''_{x,y}}{6}\delta^3 .$$

Table 1 lists the values obtained for the year 2001.

MDs	Q_{0x} Q_{0y}	Q'_x Q'_y	1/2 Q''_x 1/2 Q''_y	1/6 Q'''_x 1/6 Q'''_y	R^2_x R^2_y
17/07	0.623	4.5	500	-156000	0.98
	0.582	1.75	-313	156000	0.98
18/07	0.622	1.085	480	-167000	0.87
(Series 1)	0.583	1.937	-17	48000	0.99
18/07	0.622	0.992	420	-146000	0.89
(Series 2)	0.583	2.027	-100	-49000	0.97
18/07	0.622	1.358	423	-245000	0.88
(Series 3)	0.581	3.987	343	-86000	0.98
19/07	0.622	1.424	584	-218000	0.91
(Series 1)	0.580	0.934	-38	185000	0.93
19/07	0.622	1.125	451	-168000	0.90
(Series 2)	0.583	1.106	160	135000	0.97

Table 1: Linear and nonlinear chromaticity components obtained from a polynomial fit of the measured betatron tune as a function of rf frequency.

2.2 *Detuning*

The horizontal detuning with amplitude was measured on 25th July for three different octupole settings, namely for (-2,0), (0,0), (2,0), where the pairs of strengths quoted are in units of [m^{-4}] and refer to the horizontal and vertical octupoles, respectively. The measurement of Q_x vs. A_x is depicted in Fig. 3 By determining the parabolic dependence $\frac{1}{2}(\partial Q_x^2/\partial A_x^2)$ of each data set using a fit, the optimum horizontal octupole setting, for zero detuning, was inferred to be roughly -0.875 m^{-4}. An analogous measurement was performed in the vertical plane [2].

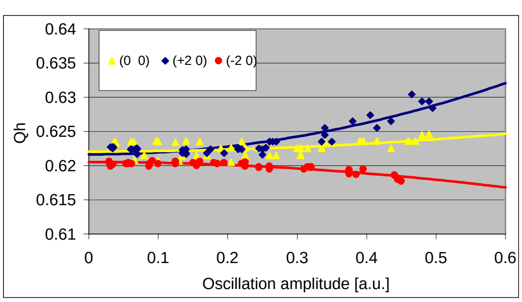

Figure 3: Measured horizontal tune versus amplitude in [a.u.] on 25th July 2001, for various settings of the horizontal octupoles in [m^{-4}]. The vertical octupoles were held constant at 0.0 m^{-4}. Note that 0.1 [a.u.] on the horizontal scale corresponds to 3.652 mm in the horizontal pick-up ($\beta_x = 100$ m).

Note that the geometric beam emittance is about $\epsilon_{x,y} \approx$ 140 nm. The rms beam sizes at the pick up ($\beta_{x,y} \approx 100$ m, $D_x \approx 3$ m, $\sigma_p/p \approx 1.1 \times 10^{-3}$) are $\sigma_x \approx 5.2$ mm and $\sigma_y \approx 3.8$ mm. Thus, the maximum amplitude in Fig. 3 corresponds to about 3.5σ_x. Note that for amplitudes below about 2σ a refined analysis of the detuning would be required, *e.g.*, using the formalism developed in Ref. [3], according to which the measured detuning ΔQ is enhanced approximately by the factor $(1 + 1/(2Z^2))^2$, where Z denotes the kick amplitude in units of σ. At 2σ the enhancement is 28%. At 1σ it amounts to a factor of 2.6. For 3.5σ the correction can be neglected.

3 OPTICS MODEL

The nonlinear model is constructed as follows. For each data set, we first set the tunes to the measured values $Q_{0x,y}$ by adjusting the main quadrupole families. The changes are a fraction of per mill.

In order to obtain the measured chromaticities we insert a b_3 component at the center of each of the two kinds of dipole and we match its value to obtain the best possible agreement with the measured linear chromaticity ($Q'_{x,y}$ from Table 1). The resulting values for the b_3 components are summarised in Table 2, where the parameter b_{3a} refers to MBA and b_{3b} to MBB. The values for b_{3b} are fairly reproducible with a variation of about $\pm$ 20%, but its average is about three times the value measured in the year 2000, which may be related to the different cycle. The value of b_{3a} shows a scatter by up to a factor of 2. However on average it appears to be closer to the value found in 2000. Also, we note that b_{3a} and b_{3b} are of opposite sign for all the measurements.

The variation of the sextupole component in the dipoles with beam energy contains a field-proportional term, a constant remanent field, and an eddy current term ($\propto \dot{B}$). The last one is zero in our case. Using the historical magnet data in [4, 5] at 26 GeV/c we would expect $b_{3a} \approx 1.05 \times 10^{-3}$m^{-2} and $b_{3b} \approx 1.8 \times 10^{-4}$m^{-2}. The value of b_{3a} agrees well with our fit. The expected value of b_{3b} is almost zero, while the fit yields a large negative value. This might be the consequence of the different cycles considered for our measurements and for the historical data. The variation in b_{3b} observed between 2000 and 2001 seems to indicate that this component, b_{3b}, is more sensitive to remanent fields.

Next we add a b_4 component at the center of each kind of quadrupole and match its value to reproduce the measured second order chromaticity ($Q''_{x,y}$ from Table 1). The resulting values are summarised in table 2. The multipole component b_{4f} refers to the horizontally focusing quadrupoles ($QF1.F$, $QF1A.F$, $QF2.F$ and $QF2A.F$) and b_{4d} to the horizontally defocusing quadrupoles ($QD.F$ and $QDA.F$) We observe that b_{4f} is reproducible and even comparable to the value found in 2000. On the other hand, there is a large scatter in the value of b_{4d}, which may indicate that it is not well constrained by the fit.

Adding the same b_5 component for the two kinds of dipole [1], we match its value to get the best possible agreement with the measured third order chromaticity ($Q'''_{x,y}$ from Table 1). Also these results are summarised in Table 2. The maximum spread in b_5 is less than 50%. The average is about twice the value fitted in 2000, which again could be a feature of the different SPS cycle.

element	**dipoles**	**quadrupoles**	**dipoles**
MDs	b_{3a} b_{3b}	b_{4f} b_{4d}	b_5
units	$10^{-3}[\mathrm{m}^{-2}]$	$10^{-1}[\mathrm{m}^{-3}]$	$[\mathrm{m}^{-4}]$
18/10/2000	1.366 -0.826	0.808 -2.551	-5.833
17/07/2001	1.024 -2.774	0.974 2.470	-17.956
18/07/2001 (Series 1)	0.985 -3.188	1.056 -0.491	-16.779
18/07/2001 (Series 2)	1.662 -3.567	0.884 0.703	-14.399
18/07/2001 (Series 3)	1.665 -3.360	1.080 -4.357	-25.417
19/07/2001 (Series 1)	0.815 -2.452	1.305 -0.509	-24.936
19/07/2001 (Series 2)	0.759 -2.466	1.056 -2.422	-19.304

Table 2: Matched multipole components for the nonlinear chromaticity measurements in 2000 [1] and 2001.

To simulate the detuning with amplitude for the different Landau damping octupole settings we followed the same procedure as described above in order to match the tunes and the linear chromaticity (by adjusting b_{3a} and b_{3b}). Recognizing the spread in the fitted higher-order nonlinear field components, for computation in MAD we then assumed the values b_{4f}, b_{4d} and b_5 for a typical case in Table 2. Taking the nonlinear chromaticity components from 19^{th} July 2001 (series 1), we calculated with MAD the horizontal tune versus the horizontal amplitude for the same octupole settings as used in the measurements. The results are displayed in Fig. 4. Figures 3 and 4 show good agreement between the measured and fitted detuning. A similar agreement was obtained in the vertical plane [2].

4 CONCLUSIONS

We fitted nonlinear field errors up to decapolar components in the SPS dipole and quadrupole magnets so as to reproduce the nonlinear chromaticity measured at 26 GeV. We find that at this energy the sextupolar fields b_3 inside the two types of SPS dipoles need to have opposite sign to explain the measurements, which is roughly consistent with expectation. The values of these b_3 components in the main dipoles are in general quite reproducible with the exception of those measured on 18/07/2001 (b_{3a} namely). Nevertheless, the quality of the fit to the data is not particularly good, especially for the horizontal tune dependence, as indicated by the low value of R_x^2 in Table 1. To match the measured second order chromaticities an octupolar component was introduced in each kind of quadrupole magnet. While the fitted b_4 component for the focusing quadrupoles is fairly reproducible from measurement to measurement, this is much less the case for the horizontally defocusing quadrupoles, where the fitted b_4 values show a large variation and even a change in sign. As a last step, we determined the decapole component in the two dipole families which would yield the measured third order chromaticities. The b_5 value is similar for all the measurements, varying by about $\pm 25\%$. Using the fitted values of the b_3, b_4 and b_5 field components in dipoles and quadrupoles, we then predicted the amplitude-dependent detuning. The result is in good agreement with the observations, for various settings of the Landau-damping octupoles. In particular, the setting of the Landau-damping octupoles resulting in zero detuning inferred from the model agrees with that determined in a direct measurement. Therefore, it may be possible to optimize the octupole settings by fitting a measurement of the nonlinear chromaticity. The latter could be obtained quickly and for the entire cycle at once [6].

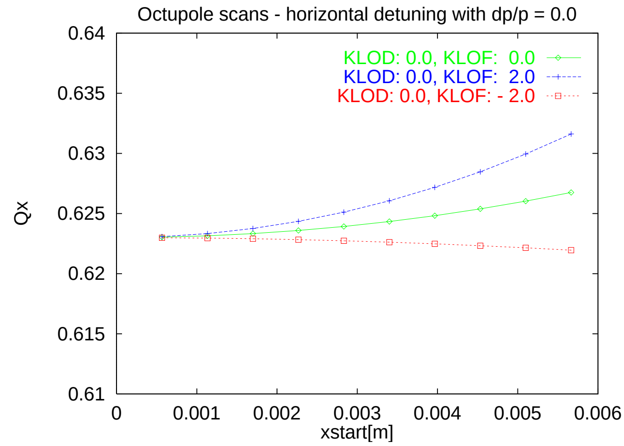

Figure 4: Predicted horizontal tune versus amplitude for different octupole settings, after fitting to linear chromaticity measured on 25^{th} July and to nonlinear chromaticity from 19^{th} July 2001 (series 1). Note that 1 mm at the starting point corresponds to 0.971 mm in the pick-up.

5 REFERENCES

[1] G. Arduini, F. Zimmermann, A. Faus-Golfe, SL Note 2001-030 MD (2001).

[2] G. Arduini, P. Collier, F. Zimmermann, A. Faus-Golfe, SL Note 2001-049 MD (2001).

[3] R.E. Meller, et al., SSC-N-360 (1987).

[4] V. Hatton, A. Riche, A. Swift, Lab II/MA-Int.75-2 (1975).

[5] M. Cornacchia, Lab. II-DI-PA/Int. 75-8 (1975).

[6] J. Wenninger, private communication (2001).