Electron-Cloud Wake Fields

G. Rumolo and F. Zimmermann, SL/AP, CERN, Geneva, Switzerland

Abstract

The electron cloud gives rise to coherent and incoherent single-bunch wake fields, both in the longitudinal and in the transverse direction, and to coherent coupled-bunch wakes. These wake fields can be computed using the simulation programs ECLOUD and HEADTAIL developed at CERN. We present the wake fields simulated for the LHC beam in the CERN SPS and at injection into the LHC in different magnetic field configurations (field-free region, dipole, and solenoid), where the magnetic field affects both the electron motion during a bunch passage and the overall electron distribution in the beam pipe.

1 INTRODUCTION

Two different aspects of the electron-cloud phenomenon are modelled and simulated by means of the programmes ECLOUD and HEADTAIL developed at CERN [1].

The first program ECLOUD simulates the build up of the electron cloud during the passage of a bunch train. It provides information on the transverse electron distribution inside the vacuum chamber, the time evolution of the total number of electrons, the energy spectrum of electrons impinging on the wall, and the corresponding heat load [2]. These outputs have already proven extremely interesting both for comparison with existing data on the electron cloud in the SPS and for extrapolation to the LHC, like the estimation of heat load or of the pressure rise in the experimental areas [1]. Sending an off-set bunch after the cloud has reached a quasi-stationary configuration and calculating the field of the perturbed cloud when the next bunch passes using the ECLOUD code, we can estimate the bunch-to-bunch wake field potentially causing a coupled bunch instability. Furthermore, the electron density value at saturation is used as an input to study single bunch phenomena, as explained below.

The HEADTAIL program models the interaction of a single bunch with an electron cloud on successive turns. The cloud is assumed to be generated by the preceding bunches, and is generally set to be initially uniform, although other initial distributions can be considered. The electrons give rise to a head-tail wake field, which can amplify any initial small deformation in the bunch shape. The resulting instability develops like a regular Transverse Mode Coupling Instability (TMCI) [3], and causes a transverse centroid motion and/or also a substantial emittance growth in the bunch. In our simulation, the interaction between beam and cloud is calculated including nonzero chromaticity, broadband impedance, space charge or beam-beam, and detuning with amplitude. Electrons can evolve in a field-free region or in different magnetic field configurations (*e.g.,* strong dipole, solenoid or combined function magnet).

HEADTAIL can be used to compute the single bunch transverse wake field, the single bunch instability threshold, and the instability growth rate above the threshold. It also contains all the necessary information to extract the longitudinal wake field and the resulting potential-well distortion. SPS and LHC parameters used in our simulations are summarized in Table 1.

Table 1: SPS and LHC parameters.

Symbol	SPS	LHC
C	6900 m	27 km
p_0	26 GeV/c	450 GeV/c
$h_{x,y}$	70/22.5 mm	22.18/22.18 mm
N_b	1.05×10^{11}	1.05×10^{11}
$\sigma_{x,y}$	3/2.3 mm	1.2/1.2 mm
σ_z	30 cm	13 cm
η	5.78×10^{-4}	3.43
Q_s	0.0022	0.006
$Q_{x,y}$	26.6	63.3
β_0	15 m	80 m
ρ_e	10^{12} m^{-3}	10^{12} m^{-3}

2 TRANSVERSE AND LONGITUDINAL WAKE FIELDS IN THE CERN SPS

Single-bunch wake fields can be calculated by using the HEADTAIL code. In the simulation, we displace one longitudinal bunch slice (for instance, vertically by an amount $\Delta y \propto \sigma_y$), and then we evaluate the electron cloud response in terms of electric field on axis ($x = y = 0$). Normalizing this field by the amount of displacement and the number of particles contained in the displaced slice, we obtain the dipole wake function on axis (in $\Omega\mathrm{s}^{-1}\mathrm{m}^{-1}$, after multiplication by the factor $\mathrm{m}\gamma\mathrm{c}^2/\mathrm{e}^2$). As the field on axis is not directly related to the force exerted by the cloud on the slices that follow the displaced one, we can also evaluate an averaged dipole wake function from the net force caused by a displaced slice on later portions of the beam. In this case, instead of looking only at the field on axis, we calculate the overall force exerted by the distorted cloud on all the particles contained in one slice, and then divide by the total charge in that slice to obtain an effective electric field. Shapes in the two cases appear quite different, as shown in Figs. 1 and 2. These plots correspond to an almost round beam in an SPS field-free region and are calculated for a longitudinally uniform bunch distribution. Wake functions on axis reach much larger values and exhibit a spiky structure that is smoothed out to a more regular profile when the transverse integration over the bunch slice is carried out. It is worth noting two remarkable features that make the electron cloud wake field differ from a conventional dipole wake field: first, the two above definitions of wake do not yield the same result, and second, the shape of the wake depends on the longitudinal location of the displaced slice. In a dipole region electrons accumulate in the form of one or two vertical stripes. The horizontal wake tends to vanish, and the vertical one becomes also weaker. We show in Figs. 3 and 4 the averaged dipole wake functions of a Gaussian bunch distribution in a dipole field region and for different initial electron distributions (one stripe $4\sigma_x$ wide and then two $2\sigma_x$ wide stripes located at increasing distances from the center). The frequency of the wake decreases as the separation between the two electron stripes is increased. Finally, the effect of a solenoid field of 10 mT is shown in Fig. 5. Coupling introduced by the solenoid is evident: when the bunch head is horizontally displaced the wake appears also in the vertical plane and vice versa.

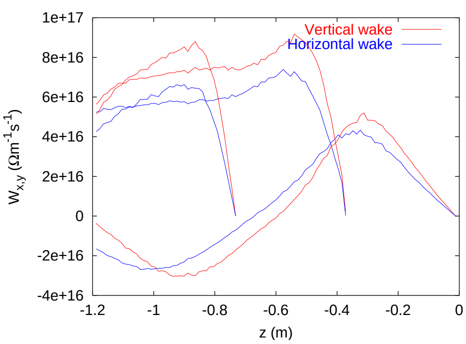

Figure 1: Horizontal and vertical averaged dipole wake functions for a uniform SPS bunch with line density $\lambda = N_b/(4\sigma_z)$, evaluated displacing three different bunch slices at $t = 0, 6/5\sigma_z/\mathrm{c}, 12/5\sigma_z/\mathrm{c}$ (field-free region).

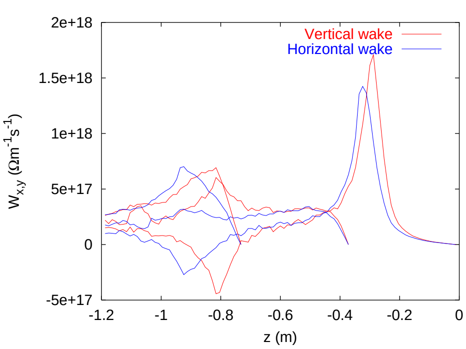

Figure 2: Horizontal and vertical wake functions on axis for a uniform SPS bunch with line density $\lambda = N_b/(4\sigma_z)$, evaluated displacing three different bunch slices at $t = 0, 6/5\sigma_z/\mathrm{c}, 12/5\sigma_z/\mathrm{c}$ (field-free region).

Even though the kick approximation allows us to use a two-dimensional model to study transverse effects, the electron cloud is in reality distributed more or less uniformly around the ring, and thus generates a longitudinal wake field which may give rise to potential well distortion and eventually micro-wave instability. The longitudinal field arises primarily from the accumulation of electrons near the center of the bunch during its passage. The 3-dimensional electron distribution can be reconstructed by

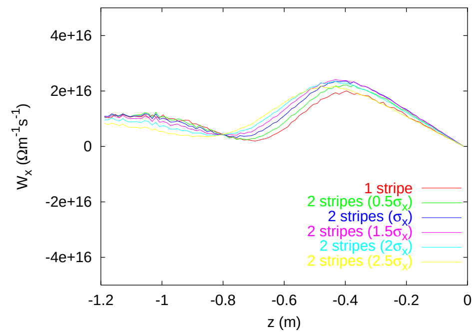

Figure 3: Horizontal averaged dipole wake functions for a Gaussian SPS bunch in a dipole (different initial electron distributions).

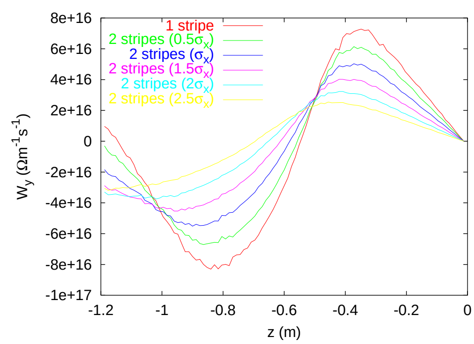

Figure 4: Vertical averaged dipole wake functions for a Gaussian SPS bunch in a dipole (different initial electron distributions).

identifying the time during the bunch passage with the longitudinal position along the bunch. This distribution can then be post-processed in order to compute the longitudinal electric field on a 3-dimensional grid. We assume that the initial electron distribution is unperturbed and uniform, and assign a homogeneous charge distribution to the region of the grid which lies in front of the bunch. The electric field is calculated on the 3-D grid points using a cloud-in-cell algorithm, and is then multiplied by the factor $\Delta z/C$ to account for the fact that the real electrons are distributed all around the circumference C. Figure 6 displays the longitudinal electric field $E_z(z)$ due to the electron cloud simulated for a Gaussian bunch in the SPS. The bunch profile is also indicated. The field is negligibly small (in agreement with the estimation from a full 3-D plasma physics code [1]), less than 10 V/m. To estimate the possible bunch distortion due to this field, we assume a Gaussian energy distribution and compute the longitudinal bunch profile expected for the electron-cloud potential well using

$$\rho(z) = \rho_0 \exp\left[-\frac{1}{2}\left(\frac{\omega_s z}{\eta \mathrm{c}\sigma_\delta}\right)^2 - \frac{r_0}{\eta\sigma_\delta^2\gamma C}\int_0^z \mathrm{d}z' W(z')\right], \tag{1}$$

where the longitudinal wake $W(z)$, wake function from a Gaussian bunch, is related to the longitudinal electric field estimated from the HEADTAIL code by

$$W(z) \approx \frac{E_z(z)}{\mathrm{e}}\left(\frac{4\pi}{Z_0\mathrm{c}}\right) C\,.$$

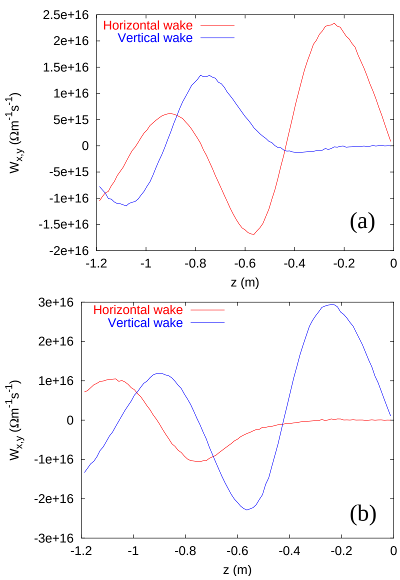

Figure 5: Averaged dipole wake functions for a Gaussian SPS bunch in a 10 mT solenoid, evaluated for horizontal offset alone (a), and vertical offset (b).

We like to call Eq. (1) the *quasi-Haissinski solution*. Unlike the real Haissinski equation [4] for an ordinary wake field, Eq. (1) is not self-consistent, since the field $E_z(z)$ on the RHS varies with the beam distribution in an unknown way. Normally, the wake field can be expressed as a convolution between the distribution function and a Green function wake W_0: the equation can then be solved numerically for $\rho(z)$. For the electron cloud case, such Green function W_0 is not known, and it may not even exist owing to violations of linearity and time invariance.

Nevertheless, we can use Eq. (1) to compute the bunch

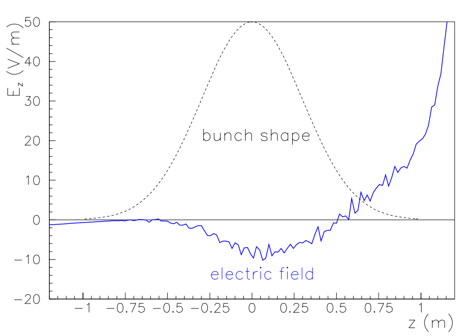

Figure 6: Longitudinal electric field due to the electron cloud for a Gaussian bunch in the SPS. Bunch head is on the left.

profile which would be formed under the influence of the additional electric field $E_z(z)$ (neglecting its dependence on the perturbed bunch profile itself), and compare this with the initial distribution. A discrepancy would indicate a significant potential-well distortion, requiring further iterations to determine the self-consistent bunch profile. However, Fig. 7 shows that the initial and predicted distributions are very similar, and hence we do not expect a large effect of the electron cloud on the longitudinal bunch

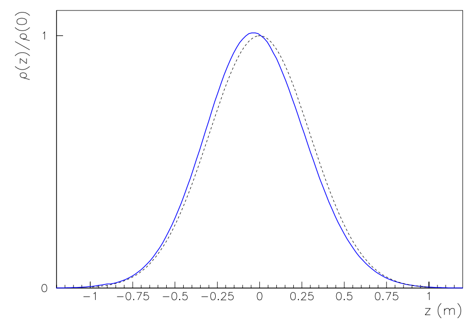

Figure 7: Equilibrium bunch density computed from the wake for a Gaussian bunch in the SPS.

shape in the SPS. We note that the modified distribution is shifted slightly forward, which compensates for the additional energy loss due to the cloud.

3 WAKE FIELDS IN THE LHC AT INJECTION ENERGY

The bunch-to-bunch wake field W for LHC at injection (for parameters, see Table 1) has been evaluated with the ECLOUD code, and results are displayed in Table 2.

Table 2: Bunch-to-bunch wake fields in LHC at injection.

δ_{max}	Region	W (MΩ·m^{-1}·s^{-1})	$1/\tau$ (s^{-1})
1.4	Field-free	$\leq 1.05 \times 10^7$	≤ 0.113
1.4	Dipole	$\leq 6.71 \times 10^7$	≤ 0.71
1.5	Field-free	$\leq 3.1 \times 10^7$	≤ 0.331
1.5	Dipole	$\leq 8.03 \times 10^7$	≤ 0.86

4 CONCLUSIONS AND OUTLOOK

Single bunch transverse and longitudinal wake functions have been calculated for the SPS by means of the HEAD-TAIL code. Transverse wakes can be of great interest for predicting the instability threshold from the TMCI theory, although this theory can only be applied in first approximation, and needs to be adapted to this particular case. The electron cloud wake field is not conventional. Work is being carried out to take into account its features in the theory [1]. The longitudinal wake field can only slightly affect the bunch shape and is not likely to be responsible for any microwave instability.
The long range wake field due to the electron cloud evaluated with the ECLOUD code for LHC at injection shows a weakly destabilizing effect.

5 REFERENCES

[1] V. Baglin, B. Jenninger, M. Jimenez, T. Katsouleas, E. Perevedentsev, A. Rossi, G. Rumolo, F. Zimmermann, Proc. of ECLOUD'02, Yellow Report CERN-2002-001

[2] G. Rumolo and F. Zimmermann, "Practical User Guide for ECLOUD" CERN-SL-Note-2002-016 (AP)

[3] K. Ohmi,, F. Zimmermann and E. Perevedentsev, Phys. Rev. E **65** (2002) 016502

[4] J. Haissiniski, Nuovo Cimento **18B**, 72 (1973)